2.9: Basis and Dimension

Defn: Let S be a subspace of R^k. A set $\mathcal{T}$ is a **basis** for S if

i.) $\mathcal{T}$ is linearly independent and

ii.) $\mathcal{T}$ spans S.

Examples

a.) $\{\begin{bmatrix}1\\2\\0\end{bmatrix}, \begin{bmatrix}3\\4\\0\end{bmatrix}\}$ is a basis for $span\{\begin{bmatrix}1\\2\\0\end{bmatrix}, \begin{bmatrix}3\\4\\0\end{bmatrix}\}$

b.) $\{\begin{bmatrix}1\\2\\0\end{bmatrix}, \begin{bmatrix}3\\4\\0\end{bmatrix}, \begin{bmatrix}4\\6\\0\end{bmatrix}\}$ is NOT a basis for $span\{\begin{bmatrix}1\\2\\0\end{bmatrix}, \begin{bmatrix}3\\4\\0\end{bmatrix}\}$

c.) $\{\begin{bmatrix}1\\2\\0\end{bmatrix}\}$ is NOT a basis for $span\{\begin{bmatrix}1\\2\\0\end{bmatrix}, \begin{bmatrix}3\\4\\0\end{bmatrix}\}$

Defn: A vector space is called **finite-dimensional** if it has a basis consisting of a finite number of vectors. Otherwise, V is **infinite dimensional**.

Thm: All basis for a finite-dimensional vector space have the same number of elements.

Defn: $dim(V)$ = the **dimension** of a finite-dimensional vector space V = the number of vectors in any basis for S. If $dim(V) = n$, then V is said to be n-dimensional.

rank A = Rank of a matrix A = dimension of Col A
= number of pivot columns of A.

nullity of A = dimension of Nul A = number of free variables.

Basis theorem: Let H be a p-dimensional subspace of R^n.

i.)If $H = \text{span}\{w_1, ..., w_p\}$, then $\{w_1, ..., w_p\}$ is a basis for H.

ii.) If $v_1, ..., v_p$ are linearly independent vectors in H,
then $\{v_1, ..., v_p\}$ is a basis for H.

Rank(A) + nullity(A) = Number of columns of A.

Ex. 1) Suppose A is a $5X7$ matrix.

If Rank(A) = 4, then nullity(A) =

$A\mathbf{x} = \mathbf{0}$ has ______________________ solutions.

$A\mathbf{x} = \mathbf{b}$ has ______________________ solutions.

If Rank(A) = 5, then nullity(A) =

$A\mathbf{x} = \mathbf{0}$ has ______________________ solutions.

$A\mathbf{x} = \mathbf{b}$ has ______________________ solutions.

If Rank(A) = 5, the column space of A =