Math 2418 Linear Algebra Quiz #8 Nov 7-8, 2001

[8] 1.) Show that $< a_0 + a_1 t,\ b_0 + b_1 t > = \ b_0 - a_1 b_1$ is NOT an inner product on P_1.

$< 1,\ 2 + 3t >= 2 - (0)(3) = 2$
$< 2 + 3t, 1 >= 1 - (3)(0) = 1.$
Thus, $< 1,\ 2 + 3t > \neq < 2 + 3t, 1 >$.
Since this operation is not commutative, it is not an inner product.

Alternate answer:
$< 2 + 3t, 1 >= 1 - (3)(0) = 1.$
$< 2, 1 > + < 3t, 1 >= [1 - (0)(0)] + [1 - (3)(0)] = 2.$
Thus, $< 2 + 3t, 1 > \neq < 2, 1 > + < 3t, 1 >$, and thus this operation is not an inner product.

Alternate answer:
$5 < 1,\ 2 + 3t >= 5(2 - (0)(3)) = 10$
$< 5(1),\ 2 + 3t >= 2 - (0)(3) = 2$
Thus, $5 < 1,\ 2 + 3t > \neq < 5(1),\ 2 + 3t >$, and thus this operation is not an inner product.

Alternate answer:
$< 1 + 2t, t >= 0 - (2)(1) = -2$. An inner product is never negative, and thus this operation is not an inner product.

Alternate answer:
$< 1 + t, 1 + t >= 1 - (1)(1) = 1$. But $1 + t \neq 0$. Thus this operation is not an inner product.

2.) Let P_2 have the inner product $< a_0 + a_1 t + a_2 t^2,\ b_0 + b_1 t + b_2 t^2 > = \ a_0 b_0 + a_1 b_1 + a_2 b_2$

[4] 2a.) $||3 + 10t^2|| \ = \ \underline{\sqrt{109}}$

$< 3 + 10t^2,\ 3 + 10t^2 >= (3)(3) + (0)(0) + (10)(10) = 109.$

$||3 + 10t^2|| = \sqrt{< 3 + 10t^2, 3 + 10t^2 >} = \sqrt{109}$

[4] 2b.) $< 4 - 8t + t^2,\ 3 + t - 4t^2 > = \ \underline{0}$

$< 4 - 8t + t^2,\ 3 + t - 4t^2 >= (4)(3) + (-8)(1) + (1)(-4) = 0$

[2] 2c.) Is $4 - 8t + t^2$ orthogonal to $3 + t - 4t^2$? $\underline{YES}$