

Math 2418 Linear Algebra Quiz #7
Oct. 24-25, 2001

Circle T for True and F for false.

[2] 1.) $Span\left\{\begin{bmatrix} 4 \\ 1 \end{bmatrix}, \begin{bmatrix} 2 \\ 3 \end{bmatrix}, \begin{bmatrix} 7 \\ 5 \end{bmatrix}\right\} = R^2.$ T

The vectors are not multiples of each other, thus they span something at least 2-dimensional. Since they live in R^2 , they span something at most 2-dimensional. Since the only 2-dimensional plane in R^2 is R^2 , $Span\left\{\begin{bmatrix} 4 \\ 1 \end{bmatrix}, \begin{bmatrix} 2 \\ 3 \end{bmatrix}, \begin{bmatrix} 7 \\ 5 \end{bmatrix}\right\} = R^2.$

[2] 2.) $\left\{\begin{bmatrix} 4 \\ 1 \end{bmatrix}, \begin{bmatrix} 2 \\ 3 \end{bmatrix}, \begin{bmatrix} 7 \\ 5 \end{bmatrix}\right\}$ is linearly independent. F

3 vectors in a 2-dimensional space cannot be linearly independent.

[2] 3.) $\left\{\begin{bmatrix} 4 \\ 1 \end{bmatrix}, \begin{bmatrix} 2 \\ 3 \end{bmatrix}, \begin{bmatrix} 7 \\ 5 \end{bmatrix}\right\}$ is a basis for $R^2.$ F

Since they are not linearly independent, they are not a basis.

[2] 4.) $Span\left\{\begin{bmatrix} -2 \\ -3 \end{bmatrix}, \begin{bmatrix} 4 \\ 6 \end{bmatrix}, \begin{bmatrix} 0 \\ 0 \end{bmatrix}\right\} = R^2.$ F

The last two vectors are multiples of the first. Thus they span a (1-dimensional) line going through the origin and $(-2, -3)$ [and $(4, 6)$].

[2] 5.) $\left\{ \begin{bmatrix} -2 \\ -3 \end{bmatrix}, \begin{bmatrix} 4 \\ 6 \end{bmatrix}, \begin{bmatrix} 0 \\ 0 \end{bmatrix} \right\}$ is linearly independent. F

3 vectors in a 2-dimensional space cannot be linearly independent.

Since at least one of the vectors is a linear combination of the other vectors (since its a multiple of one of the vectors), they are not linearly independent.

[2] 6.) $\left\{ \begin{bmatrix} -2 \\ -3 \end{bmatrix}, \begin{bmatrix} 4 \\ 6 \end{bmatrix}, \begin{bmatrix} 0 \\ 0 \end{bmatrix} \right\}$ is a basis for R^2 . F

Since they are not linearly independent, they are not a basis.

Since they don't span all of R^2 , they are not a basis for R^2 .

[2] 7.) $\text{Span}\{7 + 4t, 2 - 3t\} = \mathbf{P}_1 =$ the set of all polynomials of degree at most 1. T

The vectors are not multiples of each other, thus they span something at least 2-dimensional. Since they live in \mathbf{P}_1 , they span something at most 2-dimensional. Since the only 2-dimensional subspace in \mathbf{P}_1 is \mathbf{P}_1 , $\text{Span}\{7 + 4t, 2 - 3t\} = \mathbf{P}_1$

[2] 8.) $\text{Span}\{3 + t, 5 - 2t\} = \text{Span}\{1 - t, 4 + 2t\}$. T

Since they both span a 2-dimensional subspace of \mathbf{P}_1 and the only 2-dimensional subspace in \mathbf{P}_1 is \mathbf{P}_1 , $\text{Span}\{3 + t, 5 - 2t\} = \mathbf{P}_1 = \text{Span}\{1 - t, 4 + 2t\}$.

[2] 9.) $\{3 + 2t^2, 4 - t, 5 - 2t + t^2\}$ is a basis for $\text{Span}\{7 - t + 2t^2, 9 - 3t + t^2\}$. F

3 vectors cannot be a basis for a 2-dimensional subspace. Either, they are not linearly independent or they span a larger 3-dimensional space.

[2] 10.) $Span\{7 - t + 2t^2, 9 - 3t + t^2\} = Span\{2 - 2t - t^2, 16 - 4t + 3t^2\}$. T

By looking carefully at these spaces, you can determine that they span the same space since the vectors in each set is a linear combination of the vectors in the other set.

[2] 11.) $Span\{7 - t + 2t^2, 9 - 3t + t^2\} = Span\{2 - 2t - t^2, 16 - 4t + 2t^2\}$. F

By looking carefully at these spaces, you can determine that they do NOT span the same space since at least one of the vectors in each set is NOT a linear combination of the vectors in the other set.