

Math 2418 Linear Algebra Quiz #6
Oct. 17-18, 2001

[10] 1.) Show that the line $\mathbf{x} = t \begin{bmatrix} 4 \\ 8 \end{bmatrix}$ is a subspace of R^2 .

A.) Show that if $\mathbf{v}, \mathbf{w} \in S$, then $\mathbf{v} + \mathbf{w} \in S$

$\mathbf{v} \in S$ implies $\mathbf{v} = t_1 \begin{bmatrix} 4 \\ 8 \end{bmatrix}$ for some real number t_1

$\mathbf{w} \in S$ implies $\mathbf{w} = t_2 \begin{bmatrix} 4 \\ 8 \end{bmatrix}$ for some real number t_2

$$\mathbf{v} + \mathbf{w} = t_1 \begin{bmatrix} 4 \\ 8 \end{bmatrix} + t_2 \begin{bmatrix} 4 \\ 8 \end{bmatrix} = (t_1 + t_2) \begin{bmatrix} 4 \\ 8 \end{bmatrix}$$

Thus $\mathbf{v} + \mathbf{w} \in S$

B.) Show that if $\mathbf{v} \in S$, then $c\mathbf{v} \in S$

$\mathbf{v} \in S$ implies $\mathbf{v} = t_1 \begin{bmatrix} 4 \\ 8 \end{bmatrix}$ for some real number t_1

$$c\mathbf{v} = c(t_1 \begin{bmatrix} 4 \\ 8 \end{bmatrix}) = (ct_1) \begin{bmatrix} 4 \\ 8 \end{bmatrix}$$

Thus $c\mathbf{v} \in S$

Thus S is a subspace of R^2

[10] 2.) Write $\begin{bmatrix} 5 \\ -4 \\ -5 \end{bmatrix}$ as a linear combination of $\begin{bmatrix} 4 \\ 1 \\ 2 \end{bmatrix}$, $\begin{bmatrix} 1 \\ 2 \\ 3 \end{bmatrix}$, and $\begin{bmatrix} 6 \\ 5 \\ 8 \end{bmatrix}$

$$\begin{bmatrix} 4 & 1 & 6 & 5 \\ 1 & 2 & 5 & -4 \\ 2 & 3 & 8 & -5 \end{bmatrix} \rightarrow \begin{bmatrix} 1 & 2 & 5 & -4 \\ 4 & 1 & 6 & 5 \\ 2 & 3 & 8 & -5 \end{bmatrix} \rightarrow \begin{bmatrix} 1 & 2 & 5 & -4 \\ 0 & -7 & -14 & 21 \\ 0 & -1 & -2 & 3 \end{bmatrix} \rightarrow \begin{bmatrix} 1 & 2 & 5 & -4 \\ 0 & -1 & -2 & 3 \\ 0 & -7 & -14 & 21 \end{bmatrix} \rightarrow$$

$$\begin{bmatrix} 1 & 2 & 5 & -4 \\ 0 & -1 & -2 & 3 \\ 0 & 0 & 0 & 0 \end{bmatrix} \rightarrow \begin{bmatrix} 1 & 0 & 1 & 2 \\ 0 & 1 & 2 & -3 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

$$\text{Thus } \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 2 - s \\ -3 - 2s \\ s \end{bmatrix}$$

Let $s = 0$

$$\text{Answer 2.) } \begin{bmatrix} 5 \\ -4 \\ -5 \end{bmatrix} = 2 \begin{bmatrix} 4 \\ 1 \\ 2 \end{bmatrix} - 3 \begin{bmatrix} 1 \\ 2 \\ 3 \end{bmatrix} + 0 \begin{bmatrix} 6 \\ 5 \\ 8 \end{bmatrix}$$