Math 2418 Linear Algebra Quiz #4 (OPEN BOOK, OPEN NOTES)
Sept. 20-21, 2001

[10] 1.) Prove by giving a specific counter-example that $det(A+B) \neq detA + detB$.

$$det(\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} + \begin{bmatrix} -1 & 0 \\ 0 & -1 \end{bmatrix}) = det(\begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}) = 0$$

$$det(\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}) + det(\begin{bmatrix} -1 & 0 \\ 0 & -1 \end{bmatrix}) = 1 + 1 = 2.$$

Thus, $det(\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} + \begin{bmatrix} -1 & 0 \\ 0 & -1 \end{bmatrix}) \neq \text{det}(\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}) + det(\begin{bmatrix} -1 & 0 \\ 0 & -1 \end{bmatrix})$

Note, this is only one possible answer. There are many other correct answers.

[10] 2.) Let $A = \begin{bmatrix} 3 & -2 & 1 \\ 5 & 6 & 2 \\ 1 & 0 & -3 \end{bmatrix}$. Suppose $Adj\,A = \begin{bmatrix} x & -6 & -18 \\ y & -10 & -1 \\ -6 & -2 & 28 \end{bmatrix}$.

Find x, y, and $detA$, and use this information to find A^{-1}.

$$A^T = \begin{bmatrix} 3 & 5 & 1 \\ -2 & 6 & 0 \\ 1 & 2 & -3 \end{bmatrix}.$$

$$x = (-1)^{1+1} det(\begin{bmatrix} 6 & 0 \\ 2 & -3 \end{bmatrix}) = -18 \text{ and } y = (-1)^{2+1} det(\begin{bmatrix} 5 & 1 \\ 2 & -3 \end{bmatrix}) = -(-15-2) = 17$$

$$A = \begin{bmatrix} 3 & -2 & 1 \\ 5 & 6 & 2 \\ 1 & 0 & -3 \end{bmatrix} \xrightarrow{R_2 + 3R_1} \begin{bmatrix} 3 & -2 & 1 \\ 14 & 0 & 5 \\ 1 & 0 & -3 \end{bmatrix}$$

Thus $detA = -(-2) det(\begin{bmatrix} 14 & 5 \\ 1 & -3 \end{bmatrix}) = 2[(14)(-3) - (1)(5)] = 2[-42-5] = 2[-47] = -94$

$$A^{-1} = \frac{1}{detA}(adj A)$$

Answer 2.) $x = -18$, $y = 17$, $detA = -94$, $A^{-1} = \begin{bmatrix} \frac{18}{94} & \frac{6}{94} & \frac{18}{94} \\ -\frac{17}{94} & \frac{10}{94} & \frac{1}{94} \\ \frac{6}{94} & \frac{2}{94} & -\frac{28}{94} \end{bmatrix}$.