Math 2418 Linear Algebra Quiz #3
Sept. 12-13, 2001

1.) True or False

A.) If A is a square matrix, $Ax = b$ has a unique solution. F

B.) If A is an invertible square matrix, $Ax = b$ has a unique solution. T

C.) If a square matrix A is not invertible, then $Ax = b$ cannot have a unique solution.

T

D.) If A is not invertible, then $Ax = b$ cannot have a unique solution. F

2.) Suppose $\begin{bmatrix} 2 & 3 & 4 \\ 6 & 10 & 17 \\ 10 & 15 & 24 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ 3 & 1 & 0 \\ 5 & 0 & 1 \end{bmatrix} \begin{bmatrix} 2 & 3 & 4 \\ 0 & 1 & 5 \\ 0 & 0 & 4 \end{bmatrix}$.

Use LU factorization to solve:

$$\begin{aligned} 2x_1 + 3x_2 + 4x_3 &= 2 \\ 6x_1 + 10x_2 + 17x_3 &= 16 \\ 10x_1 + 15x_2 + 24x_3 &= 14 \end{aligned}$$

Goal: Solve $Ax = L(Ux) = b$ for x. $L(Ux) = b$. Let $Ux = y$. Then $Ly = b$.

Step 1: Solve $Ly = b$ for y.

$$\begin{bmatrix} 1 & 0 & 0 & 2 \\ 3 & 1 & 0 & 16 \\ 5 & 0 & 1 & 14 \end{bmatrix} \xrightarrow[R_2 - 3R_1,\ R_3 - 5R_1]{} \begin{bmatrix} 1 & 0 & 0 & 2 \\ 0 & 1 & 0 & 10 \\ 0 & 0 & 1 & 4 \end{bmatrix}.$$ Thus $y_1 = 2,\ y_2 = 10,\ y_3 = 4$.

Step 2: Solve $Ux = y$ for x.

$$\begin{bmatrix} 2 & 3 & 4 & 2 \\ 0 & 1 & 5 & 10 \\ 0 & 0 & 4 & 4 \end{bmatrix} \xrightarrow[\frac{R_3}{4}]{} \begin{bmatrix} 2 & 3 & 4 & 2 \\ 0 & 1 & 5 & 10 \\ 0 & 0 & 1 & 1 \end{bmatrix} \xrightarrow[R_1 - 4R_3,\ R_2 - 5R_3]{} \begin{bmatrix} 2 & 3 & 0 & -2 \\ 0 & 1 & 0 & 5 \\ 0 & 0 & 1 & 1 \end{bmatrix}$$

$$\xrightarrow[R_1 - 3R_2]{} \begin{bmatrix} 2 & 0 & 0 & -17 \\ 0 & 1 & 0 & 5 \\ 0 & 0 & 1 & 1 \end{bmatrix} \xrightarrow[\frac{R_1}{2}]{} \begin{bmatrix} 1 & 0 & 0 & -\frac{17}{2} \\ 0 & 1 & 0 & 5 \\ 0 & 0 & 1 & 1 \end{bmatrix}$$

Answer: 2.) $x_1 = -\frac{17}{2},\ x_2 = 5,\ x_3 = 1$

Check:
$$\begin{aligned} 2(-\tfrac{17}{2}) + 3(5) + 4(1) &= 2 \\ 6(-\tfrac{17}{2}) + 10(5) + 17(1) &= 16 \\ 10(-\tfrac{17}{2}) + 15(5) + 24(1) &= 14 \end{aligned}$$