Math 2418 Linear Algebra Final
December 8, 2002 SHOW ALL WORK

[8] 1.) Solve the following system of equations (if the answer is no solution, then state "no solution"):

1a.) If $\begin{bmatrix} 0 & 1 & 2 & 0 & 4 & 3 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix} \mathbf{x} = \begin{bmatrix} 0 \\ 1 \\ 2 \end{bmatrix}$, then **no solution**.

1b.) If $\begin{bmatrix} 0 & 1 & 2 & 0 & 4 & 3 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix} \mathbf{x} = \begin{bmatrix} 2 \\ 1 \\ 0 \end{bmatrix}$, then $\mathbf{x} = \begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \\ x_5 \\ x_6 \\ x_7 \end{bmatrix} = \begin{bmatrix} v \\ 2-2u-4t-3s \\ u \\ 1 \\ t \\ s \\ r \end{bmatrix}$

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[5] 2a.) The eigenvalues of $A = \begin{bmatrix} 0 & 2 & 0 \\ 1 & 4 & 3 \\ 0 & 1 & 0 \end{bmatrix}$ are $\underline{0, 5, -1}$.

$$\begin{vmatrix} \lambda & -2 & 0 \\ -1 & \lambda-4 & -3 \\ 0 & -1 & \lambda \end{vmatrix} = \lambda \begin{vmatrix} \lambda-4 & -3 \\ -1 & \lambda \end{vmatrix} + 2 \begin{vmatrix} -1 & -3 \\ 0 & \lambda \end{vmatrix} = \lambda[(\lambda-4)(\lambda)-3] + 2[-\lambda-0]$$

$$= \lambda[\lambda^2 - 4\lambda - 3] - 2\lambda = \lambda[\lambda^2 - 4\lambda - 3 - 2] = \lambda[\lambda^2 - 4\lambda - 5] = \lambda(\lambda-5)(\lambda+1) = 0.$$

Hence $\lambda = 0, 5, -1$

[3] 2b.) Is A diagonalizable? *yes*

since all eigenvalues have algebraic multiplicity one.

[3] 2c.) Is A invertible? *no*

$\lambda = 0$ is an eigenvalue.

[2, extra credit from,9.5] 2d.) If $||\mathbf{x}|| = 1$, then $\mathbf{x}^T A\mathbf{x} \geq\ -1$

3.) Suppose A is a 8×4 matrix with rank 3

[1] 3a.) Dimension of the Nullspace of A is $\underline{1}$.

[1] 3b.) The domain of $T(\mathbf{x}) = A\mathbf{x}$ is $\underline{R^4}$.

[1] 3c.) The codomain of $T(\mathbf{x}) = A\mathbf{x}$ is $\underline{R^8}$.

[1] 3d.) Is $T(\mathbf{x}) = A\mathbf{x}$ one-to one? $\underline{No}$.

[1] 3e.) Is $T(\mathbf{x}) = A\mathbf{x}$ onto? $\underline{No}$.

For the following two problems, circle the best answer:

[1] 3f.) $A\mathbf{x} = \mathbf{0}$ has

v.) infinite number of solutions

[1] 3g.) $A\mathbf{x} = \mathbf{b}$ where b is an arbitrary vector has
vi.) no solution or an infinite number of solutions

4.) Suppose C is a 3×3 matrix with rank 3.

[1] 4a.) Dimension of the Nullspace of C is $\underline{0}$.

[1] 4b.) The domain of $T(\mathbf{x}) = C\mathbf{x}$ is $\underline{R^3}$.

[1] 4c.) The codomain of $T(\mathbf{x}) = C\mathbf{x}$ is $\underline{R^3}$.

[1] 4d.) Is $T(\mathbf{x}) = C\mathbf{x}$ one-to one? $\underline{Yes}$.

[1] 4e.) Is $T(\mathbf{x}) = C\mathbf{x}$ onto? $\underline{Yes}$.

For the following two problems, circle the best answer:

[1] 4f.) $C\mathbf{x} = \mathbf{0}$ has

iii.) exactly one solution

[1] 4g.) $C\mathbf{x} = \mathbf{b}$ where b is an arbitrary vector has

iii.) exactly one solution

[7] 5a.) Prove that $T(x,y) = (2y, 0, x+3y)$ is a linear transformation by using theorem 4.3.2.

$T(x,y) + T(a,b) = (2y, 0, x+3y) + (2a, 0, a+3b) = (2y+2a, 0, x+3y+a+3b)$.

$T((x,y)+(a,b)) = T((x+a, y+b)) = (2(y+b), 0, (x+a)+3(y+b)) = (2y+2b, 0, x+a+3y+3b) = (2y+2a, 0, x+3y+a+3b)$.

Hence $T(x,y) + T(a,b) = T((x,y)+(a,b))$

$rT(x,y) = r(2y, 0, x+3y) = (2ry, 0, rx+3ry)$

$T(r(x,y)) = T(rx, ry) = (2ry, 0, rx+3ry)$

Hence $rT(x,y) = T(r(x,y))$

Hence T is linear.

[4] 5b.) Show that $T(x,y) = (2y, 0, x+3y)$ is a linear transformation by finding the matrix A such that $T(\mathbf{x}) = A\mathbf{x}$

$$\begin{bmatrix} 0 & 2 \\ 0 & 0 \\ 1 & 3 \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 2y \\ 0 \\ x+3y \end{bmatrix}$$

$$\text{A} = \underline{\begin{bmatrix} 0 & 2 \\ 0 & 0 \\ 1 & 3 \end{bmatrix}}$$

[5] 5c.) Show that $T(x,y) = (2y, 1, x+3y)$ is NOT a linear transformation by showing that theorem 4.3.2 does not hold for a specific example (use only real numbers).

$T(0,0) = (0,1,0) \neq (0,0,0)$ hence T is not linear.

OR

$T(1,1) + T(1,1) = (2,1,4) + (2,1,4) = (4,2,8) \neq (4,1,8) = T(2,2)$ hence T is not linear.

OR etc.

[10] 6.) Suppose the characteristic equation of $A = \begin{bmatrix} 1 & -1 & 1 & -1 \\ -1 & 1 & -1 & 1 \\ 1 & -1 & 1 & -1 \\ -1 & 1 & -1 & 1 \end{bmatrix}$ is $\lambda^3(\lambda - 4) = 0$.

Find a matrix P that orthogonally diagonalizes A, and determine P^{-1} and $P^{-1}AP$.

$\lambda^3(\lambda - 4) = 0$ implies $\lambda = 0$ is an eigenvalue with multiplicity 3 and $\lambda = 4$ is an eigenvalue with multiplicity 1. Hence $D = P^{-1}AP = \begin{bmatrix} 4 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}$

$\lambda = 4:$

$$\begin{bmatrix} 3 & 1 & -1 & 1 \\ 1 & 3 & 1 & -1 \\ -1 & 1 & 3 & 1 \\ 1 & -1 & 1 & 3 \end{bmatrix} \to \begin{bmatrix} 1 & 3 & 1 & -1 \\ 3 & 1 & -1 & 1 \\ -1 & 1 & 3 & 1 \\ 1 & -1 & 1 & 3 \end{bmatrix} \to \begin{bmatrix} 1 & 3 & 1 & -1 \\ 0 & -8 & -4 & 4 \\ 0 & 4 & 4 & 0 \\ 0 & -4 & 0 & 4 \end{bmatrix} \to \begin{bmatrix} 1 & 3 & 1 & -1 \\ 0 & 1 & 0 & -1 \\ 0 & -2 & -1 & 1 \\ 0 & 1 & 1 & 0 \end{bmatrix} \to$$

$$\begin{bmatrix} 1 & 3 & 1 & -1 \\ 0 & 1 & 0 & -1 \\ 0 & 0 & -1 & -1 \\ 0 & 0 & 1 & 1 \end{bmatrix} \to \begin{bmatrix} 1 & 3 & 1 & -1 \\ 0 & 1 & 0 & -1 \\ 0 & 0 & 1 & 1 \\ 0 & 0 & 0 & 0 \end{bmatrix} \to \begin{bmatrix} 1 & 3 & 0 & -2 \\ 0 & 1 & 0 & -1 \\ 0 & 0 & 1 & 1 \\ 0 & 0 & 0 & 0 \end{bmatrix} \to \begin{bmatrix} 1 & 0 & 0 & 1 \\ 0 & 1 & 0 & -1 \\ 0 & 0 & 1 & 1 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

$$\begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \end{bmatrix} = \begin{bmatrix} -t \\ t \\ -t \\ t \end{bmatrix} = t \begin{bmatrix} -1 \\ 1 \\ -1 \\ 1 \end{bmatrix}$$

Hence a basis for the eigenspace corresponding to $\lambda = 4$ is $\left\{ \begin{bmatrix} -1 \\ 1 \\ -1 \\ 1 \end{bmatrix} \right\}$

Normalize: $\begin{bmatrix} -1 \\ 1 \\ -1 \\ 1 \end{bmatrix} \cdot \begin{bmatrix} -1 \\ 1 \\ -1 \\ 1 \end{bmatrix} = 4, \ \sqrt{4} = 2.$

Thus an orthonormal basis for the eigenspace corresponding to $\lambda = 4$ is $\left\{ \begin{bmatrix} -\frac{1}{2} \\ \frac{1}{2} \\ -\frac{1}{2} \\ \frac{1}{2} \end{bmatrix} \right\}$

$\lambda = 0$:

$$\begin{bmatrix} 1 & -1 & 1 & -1 \\ -1 & 1 & -1 & 1 \\ 1 & -1 & 1 & -1 \\ -1 & 1 & -1 & 1 \end{bmatrix} \to \begin{bmatrix} 1 & -1 & 1 & -1 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

$$\begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \end{bmatrix} = \begin{bmatrix} r - s + t \\ r \\ s \\ t \end{bmatrix} = r \begin{bmatrix} 1 \\ 1 \\ 0 \\ 0 \end{bmatrix} + s \begin{bmatrix} -1 \\ 0 \\ 1 \\ 0 \end{bmatrix} + t \begin{bmatrix} 1 \\ 0 \\ 0 \\ 1 \end{bmatrix}$$

Hence a basis for the eigenspace corresponding to $\lambda = 0$ is $\left\{ \begin{bmatrix}1\\1\\0\\0\end{bmatrix}, \begin{bmatrix}-1\\0\\1\\0\end{bmatrix}, \begin{bmatrix}1\\0\\0\\1\end{bmatrix} \right\}$

We must now use G.S. and normalize to find an orthonormal basis.

$$\begin{bmatrix}1\\1\\0\\0\end{bmatrix} \cdot \begin{bmatrix}1\\1\\0\\0\end{bmatrix} = 2, \quad \begin{bmatrix}1\\1\\0\\0\end{bmatrix} \cdot \begin{bmatrix}-1\\0\\1\\0\end{bmatrix} = -1. \text{ Hence } proj_{\begin{bmatrix}1\\1\\0\\0\end{bmatrix}} \begin{bmatrix}-1\\0\\1\\0\end{bmatrix} = -\tfrac{1}{2}\begin{bmatrix}1\\1\\0\\0\end{bmatrix}.$$

Thus $\begin{bmatrix}-1\\0\\1\\0\end{bmatrix} - -\frac{1}{2}\begin{bmatrix}1\\1\\0\\0\end{bmatrix} = \begin{bmatrix}-\frac{1}{2}\\\frac{1}{2}\\1\\0\end{bmatrix}$ is perpendicular to $\begin{bmatrix}1\\1\\0\\0\end{bmatrix}$. Thus $\begin{bmatrix}-1\\1\\2\\0\end{bmatrix}$ is also $\perp$ to $\begin{bmatrix}1\\1\\0\\0\end{bmatrix}$.

$$\begin{bmatrix}1\\1\\0\\0\end{bmatrix} \cdot \begin{bmatrix}1\\0\\0\\1\end{bmatrix} = 1, \quad \begin{bmatrix}-1\\1\\2\\0\end{bmatrix} \cdot \begin{bmatrix}-1\\1\\2\\0\end{bmatrix} = 6, \quad \begin{bmatrix}-1\\1\\2\\0\end{bmatrix} \cdot \begin{bmatrix}1\\0\\0\\1\end{bmatrix} = -1$$

Hence $proj_{span\left\{\begin{bmatrix}1\\1\\0\\0\end{bmatrix}, \begin{bmatrix}-1\\1\\2\\0\end{bmatrix}\right\}} \begin{bmatrix}1\\0\\0\\1\end{bmatrix} = \frac{1}{2}\begin{bmatrix}1\\1\\0\\0\end{bmatrix} + -\frac{1}{6}\begin{bmatrix}-1\\1\\2\\0\end{bmatrix} = \begin{bmatrix}\frac{2}{3}\\\frac{1}{3}\\-\frac{1}{3}\\0\end{bmatrix}$

Thus $\begin{bmatrix}1\\0\\0\\1\end{bmatrix} - \begin{bmatrix}\frac{2}{3}\\\frac{1}{3}\\-\frac{1}{3}\\0\end{bmatrix} = \begin{bmatrix}\frac{1}{3}\\-\frac{1}{3}\\\frac{1}{3}\\1\end{bmatrix}$ is $\perp$ to $span\left\{\begin{bmatrix}1\\1\\0\\0\end{bmatrix}, \begin{bmatrix}-1\\1\\2\\0\end{bmatrix}\right\}$. Thus $\begin{bmatrix}1\\-1\\1\\3\end{bmatrix}$ is also $\perp$ to $span\left\{\begin{bmatrix}1\\1\\0\\0\end{bmatrix}, \begin{bmatrix}-1\\1\\2\\0\end{bmatrix}\right\}$.

Normalize: $\begin{bmatrix}1\\-1\\1\\3\end{bmatrix} \cdot \begin{bmatrix}1\\-1\\1\\3\end{bmatrix} = 12$

$$\begin{bmatrix}1\\1\\0\\0\end{bmatrix} \rightarrow \begin{bmatrix}\frac{1}{\sqrt{2}}\\\frac{1}{\sqrt{2}}\\0\\0\end{bmatrix}, \quad \begin{bmatrix}-1\\1\\2\\0\end{bmatrix} \rightarrow \begin{bmatrix}-\frac{1}{\sqrt{6}}\\\frac{1}{\sqrt{6}}\\\frac{2}{\sqrt{6}}\\0\end{bmatrix}, \quad \begin{bmatrix}1\\-1\\1\\3\end{bmatrix} \rightarrow \begin{bmatrix}\frac{1}{\sqrt{12}}\\-\frac{1}{\sqrt{12}}\\\frac{1}{\sqrt{12}}\\\frac{3}{\sqrt{12}}\end{bmatrix}$$

$$\underline{P = \begin{bmatrix}-\frac{1}{2} & \frac{1}{\sqrt{2}} & -\frac{1}{\sqrt{6}} & \frac{1}{\sqrt{12}}\\ \frac{1}{2} & \frac{1}{\sqrt{2}} & \frac{1}{\sqrt{6}} & -\frac{1}{\sqrt{12}}\\ -\frac{1}{2} & 0 & \frac{2}{\sqrt{6}} & \frac{1}{\sqrt{12}}\\ \frac{1}{2} & 0 & 0 & \frac{3}{\sqrt{12}}\end{bmatrix}} \quad \underline{P^{-1} = P^T = \begin{bmatrix}-\frac{1}{2} & \frac{1}{2} & -\frac{1}{2} & \frac{1}{2}\\ \frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} & 0 & 0\\ -\frac{1}{\sqrt{6}} & \frac{1}{\sqrt{6}} & \frac{2}{\sqrt{6}} & 0\\ \frac{1}{\sqrt{12}} & -\frac{1}{\sqrt{12}} & \frac{1}{\sqrt{12}} & \frac{3}{\sqrt{12}}\end{bmatrix}} \quad P^{-1}AP = \underline{\begin{bmatrix}4&0&0&0\\0&0&0&0\\0&0&0&0\\0&0&0&0\end{bmatrix}}.$$

[10] 7.) The least squares straight line fit to the three points

(0, 2), (1, 0), (2, 1) is $\underline{y = \frac{3}{2} - \frac{1}{2}x}$.

$y = b + mx$

$2 = b + 0m$

$0 = b + 1m$

$1 = b + 2m$

$$\begin{bmatrix} 1 & 0 \\ 1 & 1 \\ 1 & 2 \end{bmatrix} \begin{bmatrix} b \\ m \end{bmatrix} = \begin{bmatrix} 2 \\ 0 \\ 1 \end{bmatrix}$$

To find the least squares solution:

$$\begin{bmatrix} 1 & 1 & 1 \\ 0 & 1 & 2 \end{bmatrix} \begin{bmatrix} 1 & 0 \\ 1 & 1 \\ 1 & 2 \end{bmatrix} \begin{bmatrix} b \\ m \end{bmatrix} = \begin{bmatrix} 1 & 1 & 1 \\ 0 & 1 & 2 \end{bmatrix} \begin{bmatrix} 2 \\ 0 \\ 1 \end{bmatrix}$$

$$\begin{bmatrix} 3 & 3 \\ 3 & 5 \end{bmatrix} \begin{bmatrix} b \\ m \end{bmatrix} = \begin{bmatrix} 3 \\ 2 \end{bmatrix}$$

$$\begin{bmatrix} 3 & 3 & 3 \\ 3 & 5 & 2 \end{bmatrix} \to \begin{bmatrix} 1 & 1 & 1 \\ 3 & 5 & 2 \end{bmatrix} \to \begin{bmatrix} 1 & 1 & 1 \\ 0 & 2 & -1 \end{bmatrix} \to \begin{bmatrix} 1 & 1 & 1 \\ 0 & 1 & -\frac{1}{2} \end{bmatrix} \to \begin{bmatrix} 1 & 0 & \frac{3}{2} \\ 0 & 1 & -\frac{1}{2} \end{bmatrix}$$

Thus, $b = \frac{3}{2}$ and $m = -\frac{1}{2}$.

[5] 8a.) Write $-2+t^2$ as a linear combination of $4t+2t^2$, $1+3t+t^2$, and $-1+t+t^2$.

$$\begin{bmatrix} 0 & 1 & -1 & -2 \\ 4 & 3 & 1 & 0 \\ 2 & 1 & 1 & 1 \end{bmatrix} \rightarrow \begin{bmatrix} 2 & 1 & 1 & 1 \\ 4 & 3 & 1 & 0 \\ 0 & 1 & -1 & -2 \end{bmatrix} \rightarrow \begin{bmatrix} 2 & 1 & 1 & 1 \\ 0 & 1 & -1 & -2 \\ 0 & 1 & -1 & -2 \end{bmatrix} \rightarrow \begin{bmatrix} 2 & 1 & 1 & 1 \\ 0 & 1 & -1 & -2 \\ 0 & 0 & 0 & 0 \end{bmatrix} \rightarrow \begin{bmatrix} 2 & 0 & 2 & 3 \\ 0 & 1 & -1 & -2 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$
$$\begin{bmatrix} 1 & 0 & 1 & \frac{3}{2} \\ 0 & 1 & -1 & -2 \\ 0 & 0 & 0 & 0 \end{bmatrix} \rightarrow$$

Hence $\frac{3}{2}(4t+2t^2) - 2(1+3t+t^2) = 6t + 3t^2 - 2 - 6t - 2t^2 = -2 + t^2$.

Answer 8a: $\underline{\frac{3}{2}(4t+2t^2) - 2(1+3t+t^2) + 0(-1+t+t^2)}$

[3] 8b.) Is $\{4t+2t^2, 1+3t+t^2, -1+t+t^2\}$ a basis for P_2?$\underline{No}$.

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[4] 9.) Find an LU factorization of $A = \begin{bmatrix} 3 & 6 \\ 2 & 8 \end{bmatrix}$

$$\begin{bmatrix} 3 & 6 \\ 2 & 8 \end{bmatrix} \rightarrow \begin{bmatrix} 1 & 2 \\ 2 & 8 \end{bmatrix} \rightarrow \begin{bmatrix} 1 & 2 \\ 0 & 4 \end{bmatrix} = U$$

$$\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} \rightarrow \begin{bmatrix} 3 & 0 \\ 0 & 1 \end{bmatrix} \rightarrow \begin{bmatrix} 3 & 0 \\ 2 & 1 \end{bmatrix} = L$$

Check: $LU = \begin{bmatrix} 3 & 0 \\ 2 & 1 \end{bmatrix} \begin{bmatrix} 1 & 2 \\ 0 & 4 \end{bmatrix} = \begin{bmatrix} 3 & 6 \\ 2 & 8 \end{bmatrix}$

Answer: $L = \underline{\begin{bmatrix} 3 & 0 \\ 2 & 1 \end{bmatrix}}$ $U = \underline{\begin{bmatrix} 1 & 2 \\ 0 & 4 \end{bmatrix}}$

[23] 10.) Let $W = span\{1+2x+x^2,\ \ 1+2x-5x^2\}$. Use the inner product $< f, g >= \int_{-1}^{1} fgdx$ and some of the following information to solve problems 10a - 10h.

$< 1+2x+x^2,\ \ 1+2x+x^2 >= \frac{32}{5}$ $\quad$ $< 1+2x-5x^2,\ \ 1+2x-5x^2 >= 8$ $\quad$ $< 3, 3 >= 18$

$< 30x^2, 30x^2 >= 360$ $\quad$ $< 3, 30x^2 >= 60$ $\quad$ $< 30x^2,\ \ 1+2x+x^2 >= 32$

$< 1+2x-5x^2,\ \ 30x^2 >= -40$ $\quad$ $< 1+2x+x^2,\ \ 1+2x-5x^2 >= 0$

[2] 10a.) $||\ 1+2x+x^2\ || = \underline{\sqrt{\frac{32}{5}}}$

[6] 10b.) $proj_W 3 = \underline{\frac{3}{4} + \frac{6}{4}x + \frac{15}{4}x^2}$

$< 3,\ \ 1+2x+x^2 >= \int_{-1}^{1} 3+6x+3x^2 = 3x+3x^2+x^3|_{-1}^{1} = 6+2 = 8$

$< 1+2x-5x^2,\ \ 3 >= \int_{-1}^{1} 3+6x-15x^2 = 3x+3x^2-5x^3|_{-1}^{1} = 6-10 = -4$

$\frac{8}{\frac{32}{5}}(1+2x+x^2) - \frac{4}{8}(1+2x-5x^2) = \frac{5}{4}(1+2x+x^2) - \frac{2}{4}(1+2x-5x^2) = \frac{3}{4} + \frac{6}{4}x + \frac{15}{4}x^2$

[2] 10c.) $proj_W 30x^2 = \underline{30x^2}$

$\frac{32}{\frac{32}{5}}(1+2x+x^2) - \frac{40}{8}(1+2x-5x^2) = 5(1+2x+x^2) - 5(1+2x-5x^2) = 30x^2$

[2] 10d.) If possible, write the 3 as a linear combination of $\{1+2x+x^2,\ \ 1+2x-5x^2\}$. If not possible, state not possible.

Not possible

[4] 10e.) If possible, write the $30x^2$ as a linear combination of $\{1+2x+x^2,\ \ 1+2x-5x^2\}$. If not possible, state not possible.

$5(1+2x+x^2) - 5(1+2x-5x^2)$

[2] 10f.) If $W = span\{1+2x+x^2,\ \ 1+2x-5x^2\}$, find a basis for $W^\perp$.

$3 - (\frac{3}{4} + \frac{6}{4}x + \frac{15}{4}x^2) = \frac{9}{4} - \frac{6}{4}x - \frac{15}{4}x^2$

Thus a basis for $W^\perp$ is $\{\frac{9}{4} - \frac{6}{4}x - \frac{15}{4}x^2\}$

[2] 10g.) Find an orthogonal basis (according to the given inner product) for P_2 which contains the polynomials $1+2x+x^2$ and $1+2x-5x^2$

$\{1+2x+x^2, 1+2x-5x^2, \frac{9}{4} - \frac{6}{4}x - \frac{15}{4}x\}$

[1] 10h.) Is your answer to 9g an orthogonal basis for P_2 according to the inner product $< a_0, a_1x + a_2x^2, b_0 + b_1x + b_2x^2 > \ = \ a_0b_0 + a_1b_1 + a_2b_2$.

NO since $< 1+2x+x^2,\ \ \frac{9}{4} - \frac{6}{4}x - \frac{15}{4}x^2 >= \frac{9}{4} - \frac{12}{4} - \frac{15}{4} \neq 0$