Math 2418 Linear Algebra Exam #1
September 25, 2002 SHOW ALL WORK

1.) Solve the following system of equations by finding the reduced echelon form of an augmented matrix.

$$-x_2 + x_3 + 2x_4 = b_1$$
$$2x_1 + 6x_3 = b_2$$
$$4x_1 + 3x_2 + 8x_3 + x_5 = b_3$$
$$x_3 - 6x_4 - x_5 = b_4$$

[7] 1a.) when $(b_1, b_2, b_3, b_4) = (1, 0, 1, -4)$.

Answer 1a.) ______________________________

[3] 1b.) when $(b_1, b_2, b_3, b_4) = (2, 0, 2, -8)$.

Answer 1b.) ______________________________

[2] 1c.) Find two vectors (b_1, b_2, b_3, b_4) for which there does not exist a solution to the above system of equations.

Answer 1c.) ______________________________

[2] 1d.) If A is the coefficient matrix for the above system of equations, find two vectors, $\mathbf{x}$, such that $A\mathbf{x} = \begin{bmatrix} 1 \\ 0 \\ 1 \\ -4 \end{bmatrix}$

Answer 1c.) ______________________________

(see next page for room for scratch work)

Scratch work page for solving

$$\begin{gathered}-x_2 + x_3 + 2x_4 = b_1 \\ 2x_1 + 6x_3 = b_2 \\ 4x_1 + 3x_2 + 8x_3 + x_5 = b_3 \\ x_3 - 6x_4 - x_5 = b_4\end{gathered}$$

when $(b_1, b_2, b_3, b_4) = (1, 0, 1, -4)$. or $(b_1, b_2, b_3, b_4) = (2, 0, 2, -8)$.

[12] 2.) Given that $\begin{bmatrix} 3 & -6 & -3 \\ 2 & 0 & 6 \\ -4 & 7 & 4 \end{bmatrix} = \begin{bmatrix} 3 & 0 & 0 \\ 2 & 4 & 0 \\ -4 & -1 & 2 \end{bmatrix} \begin{bmatrix} 1 & -2 & -1 \\ 0 & 1 & 2 \\ 0 & 0 & 1 \end{bmatrix}$,

solve the following system of equations by LU factorization

$$3x_1 - 6x_2 - 3x_3 = -3$$
$$2x_1 + 6x_3 = 22$$
$$-4x_1 + 7x_2 + 4x_3 = 2$$

Answer: ______________________________

[12] 3.) Find an LU factorization of $A = \begin{bmatrix} 3 & 3 & 0 & 9 \\ 1 & 2 & 6 & 0 \\ 0 & 0 & 2 & 0 \\ 2 & 2 & 6 & 1 \end{bmatrix}$

$L =$ ______________________________ $U =$ ______________________________

4.) Find the determinant of $A = \begin{bmatrix} 3 & 3 & 0 & 3 \\ 1 & 2 & 6 & 0 \\ 0 & 5 & 2 & 0 \\ 2 & 0 & 2 & 3 \end{bmatrix}$

[10] 4a.) $det(A) =$ ____________________________

[2] 4b.) Is A invertible? ____________________________

[3] 4c.) Solve $A\mathbf{x} = \mathbf{0}$. Answer 4c.): ____________________________

[7] 5.) Prove by giving a specific example that $AB = AC$ does not imply $B = C$.

6.) Let $D = \begin{bmatrix} 2 & 1 & 60 & 4 & 7 & 6 & 3 \\ 0 & 5 & 89 & 8 & 9 & 5 & 5 \\ 0 & 0 & 10 & 9 & 2 & 4 & 7 \\ 0 & 0 & 0 & 1 & 4 & 7 & 8 \\ 0 & 0 & 0 & 0 & 3 & 3 & 7 \\ 0 & 0 & 0 & 0 & 0 & 1 & 4 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 \end{bmatrix}$ and $E = \begin{bmatrix} 1 & 0 & 3 \\ 2 & 0 & 6 \\ 0 & 0 & 1 \end{bmatrix}$

[2] 6a.) $det\ D =$ ________________.

[2] 6b.) How many solutions does $D\mathbf{x} = \mathbf{0}$ have? ________________.

[2] 6c.) Is D invertible? ________________.

[2] 6d.) $det\ E =$ ________________.

[2] 6e.) How many solutions does $E\mathbf{x} = \mathbf{0}$ have? ________________.

[2] 6f.) Is E invertible? ________________.

[5] 7.) If $A = \begin{bmatrix} 4 & 0 & 1 \\ 0 & 0 & 1 \end{bmatrix}$ and $B = \begin{bmatrix} 23 & 13 & 20 & 44 & 73 & 65 & 34 & 83 & 70 & 46 & 53 & 49 \\ 93 & 54 & 82 & 83 & 94 & 54 & 85 & 93 & 28 & 94 & 72 & 45 \\ 59 & 74 & 10 & 92 & 20 & 49 & 67 & 46 & 26 & 34 & 79 & 35 \end{bmatrix}$,

then the third column of $AB =$ ________________.

[2] 8.) If A is a 2×2 matrix and $det\ A = 10$, then $det\ [3A] =$ ________________.

[2] 9.) If A is a 4×4 matrix and $det\ A = 10$, then $det\ [3A] =$ ________________.

[2] 10.) If A is a 4×4 matrix, $det\ A = 10$ and B is a matrix obtained from A by multiplying the 2nd row of A by 3, then $det\ B =$ ________________.

[5] 11.) A unit vector in the same direction as the vector (2, 10, 5) is ________________.

12.) Circle T for true and F for False.

[3] 12a.) The LU method for solving equations can only be applied to square matrices.
T F

[3] 12b.) A system of equations with more variables than equations cannot have a unique solution.
T F

[3] 12c.) A system of equations with more variables than equations can have no solution.
T F

[3] 12d.) If $A\mathbf{x} = \mathbf{b}$ has a unique solution, then A$\mathbf{x} = \mathbf{0}$ has a unique solution. T F

[3] 12e.) If $A\mathbf{x} = \mathbf{0}$ has an infinite number of solutions, then $A\mathbf{x} = \mathbf{b}$ has an infinite number of solutions.
T F