Math 2418 Linear Algebra Exam #1 Name: ______________________________

Oct 1, 2001 SHOW ALL WORK Circle one: Wednesday/Thursday

[14] 1.) $det \begin{bmatrix} 4 & 8 & 3 & 4 \\ 5 & 0 & 10 & 4 \\ 2 & 4 & 3 & 1 \\ 2 & 0 & 2 & 1 \end{bmatrix} =$ _______________

[6] 2a.) The orthogonal projection of the vector (4, 5) onto the vector (1, 2) is ____________

[6] 2b.) The orthogonal component of the vector (4, 5) orthogonal to (1, 2) is ____________

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[12] 3.) Solve each of the following system of linear equations by using Gauss-Jordan elimination.

3a.)
$$\begin{aligned} x_2 + 3x_3 &= 1 \\ 3x_1 + 2x_2 \qquad &= 0 \\ 6x_1 + 5x_2 + 3x_3 &= 1 \end{aligned}$$

3b.)
$$\begin{aligned} x_2 + 3x_3 &= 1 \\ 3x_1 + 2x_2 \qquad &= 0 \\ 6x_1 + 5x_2 + 3x_3 &= 0 \end{aligned}$$

Answer 3a.) ______________________________ 3b.) ______________________________

[2] 3c.) If A = coefficient matrix in 1a, does A^{-1} exist? _______

[2] 3d.) If A = coefficient matrix in 1a, *det* $A =$ _______

[1] 3d.) The answer to 1a is a hyperplane that lives in R^m where $m =$ _______.

[1] 3e.) The dimension of the hyperplane in 1a is _______.

[5] 3f.) An equation of the hyperplane in 1a in point-parallel vector form is

[3] 3g.) Using different numbers, an equivalent equation of the hyperplane in 1a in point-parallel vector form is

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[20] 4.) Find and use an LU factorization to solve:

$$\begin{bmatrix} 4 & 8 \\ 3 & 7 \end{bmatrix} \mathbf{x} = \begin{bmatrix} 4 \\ 0 \end{bmatrix}$$

Answer: $L =$ ____________________ $U =$ ____________________ $\mathbf{x} =$ ____________________

5.) Circle T for True or F for False.

[3] a.) Suppose a homogeneous system of 3 linear equations with 2 unknowns has exactly one solution, then any system with the same coefficients will also have exactly one solution. T F

[3] b.) Suppose a homogeneous system of 3 linear equations with 3 unknowns has exactly one solution, then any system with the same coefficients will also have exactly one solution. T F

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[10] 6.) Suppose $\begin{bmatrix} 1 & 5 & 2 \\ 0 & 1 & 0 \\ 1 & 0 & 3 \end{bmatrix} \begin{bmatrix} 3 & -15 & -2 \\ 0 & 1 & 0 \\ -1 & 5 & 1 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$.

Solve the following system of equations using the method of inverses:

$$\begin{aligned} 3x_1 - 15x_2 - 2x_3 &= 10 \\ x_2 \qquad &= 0 \\ -x_1 + 5x_2 + x_3 &= 2 \end{aligned}$$

Answer 6.) __________________________

[5] 7a.) Given the line $x_1 = 3 + 2t, x_2 = 1 + t, x_3 = 5 + 4t$, then

a point on the line is ______________

and a vector describing the direction of the line is ______________.

[5] 7b.) A vector perpendicular to (1, 4, 0) and (5, 1, 2) is ______________.

[5] 7c.) Find an equation for the plane in point-parallel form that contains the line $x_1 = 3 + 2t, x_2 = 1 + t, x_3 = 5 + 4t$ and is parallel to the line of intersection of the planes $x_1 + 4x_2 + 1 = 0$ and $5x_1 + x_2 + 2x_3 = 0$ (Hint: use the point in 7a and the vectors in 7a and 7b.

Answer 7c.) ____.