Is $d(4^2_1, 0^2_1) = 1$?

Is it possible to change 4^2_1 to 0^2_1 by changing only one crossing?

Does $N(U + 1) = 4^2_1, N(U + -1) = 0^2_1$ have a solution.

Is it possible for topoisomerase to change 4^2_1 to 0^2_1 by acting exactly once?

To determine answer, check how crossing change affects a link invariant.

Example: Linking number of a 2 component oriented link

Let L be a two component link with components J and K. Then $Lk(L) = lk(J, K) = \frac{1}{2}\Sigma sign(c)$ where

sign(c) = +1 or sign(c) = -1

Example:

Thm: Lk is a link invariant of two component oriented links.

Pf: Check Reidemeister moves:

R1:

No change in linking number since only count signed crossings between different components. Thus R1 moves do not change linking number of a link.

R2:

case 1: R2 move involves only one component.
No change in linking number since only count signed crossings between different components.

case 2: R2 move involves both components.

Linking number changes by +1 - 1 = 0.

Other subcases where the R2 move involves both components are similar. Thus R2 moves do not change linking number of a link.

R3

Note sign of crossing i = sign of crossing i'.

crossing i involves same components as crossing i'.

Thus R3 moves do not change linking number of a link.

Side question: What are the oriented Reidemeister moves?

Note, we won't answer this question unless you need it.

How does a crossing change affect the linking number of an oriented 2-component link

Suppose $d(L, L') = 1$. Let $c =$ the crossing in L which is changed to convert L into L'. After performing the crossing change to obtain L', call this crossing c'.

Case 1: c involves only one component of the L. Then c' involves only one component of the L'.

In this case, we don't count c (or c') when calculating linking number, so $Lk(L) = Lk(L')$

Case 2: c involves both components of the L. Then c' involves both components of the L'.

Case 2a: $sign(c) = +1$. Then $sign(c') = -1$. Hence $Lk(L') = Lk(L) - 1$

Case 2b: $sign(c) = -1$. Then $sign(c') = +1$. Hence $Lk(L') = Lk(L) + 1$

Thus $|Lk(L) - Lk(L')| \leq 1$.

Since $|Lk(4_1^2) - Lk(0_1^2)| = 2, \quad d(4_1^2, 0_1^2) > 1$.

Project: Knot/Link Invariant Table

Definition(s): Let L be a two component link with components J and K. Then $Lk(L) = lk(J, K) = \frac{1}{2}\Sigma sign(c)$ where

sign(c) = +1 or sign(c) = -1

What are its properties?

$Lk(L^*) =$

$Lk(-L) =$

$Lk(-L^*) =$

$Lk(L\#L') =$

If $d(L, L') = 1$, then $|Lk(L) - Lk(L')| \leq 1$.

Other distances?

What software computes linking number? What are the commands for calculating this invariant?

How fast is it to compute this invariant?

How good of an invariant is it?

Ex: $Lk(0^2_1) = 0 = Lk(5^2_1)$

Livingston, C. and Cha, J.C., KnotInfo, http://www.indiana.edu/~knotinfo/

Bar-Natan, D. and Morrison, S, The Knot Atlas, http://katlas.math.toronto.edu/wiki/Main_Page

Jablan, S. and Sazdanovi, R. (2007) LinKnot. World Scientific Publishing Co. Pte. Ltd., Hackensack, NJ. http://katlas.math.toronto.edu/wiki/Using_the_LinKnot_packa

www.knotplot.com

Nakagawa, Y. (2003) An introduction of knotscape. Proceedings of the Winter Workshop of Topology/Workshop of Topology and Computer (Sendai, 2002/Nara, 2001), 9, 89–94. https://ir.library.tohoku.ac.jp/re/bitstream/10097/1721