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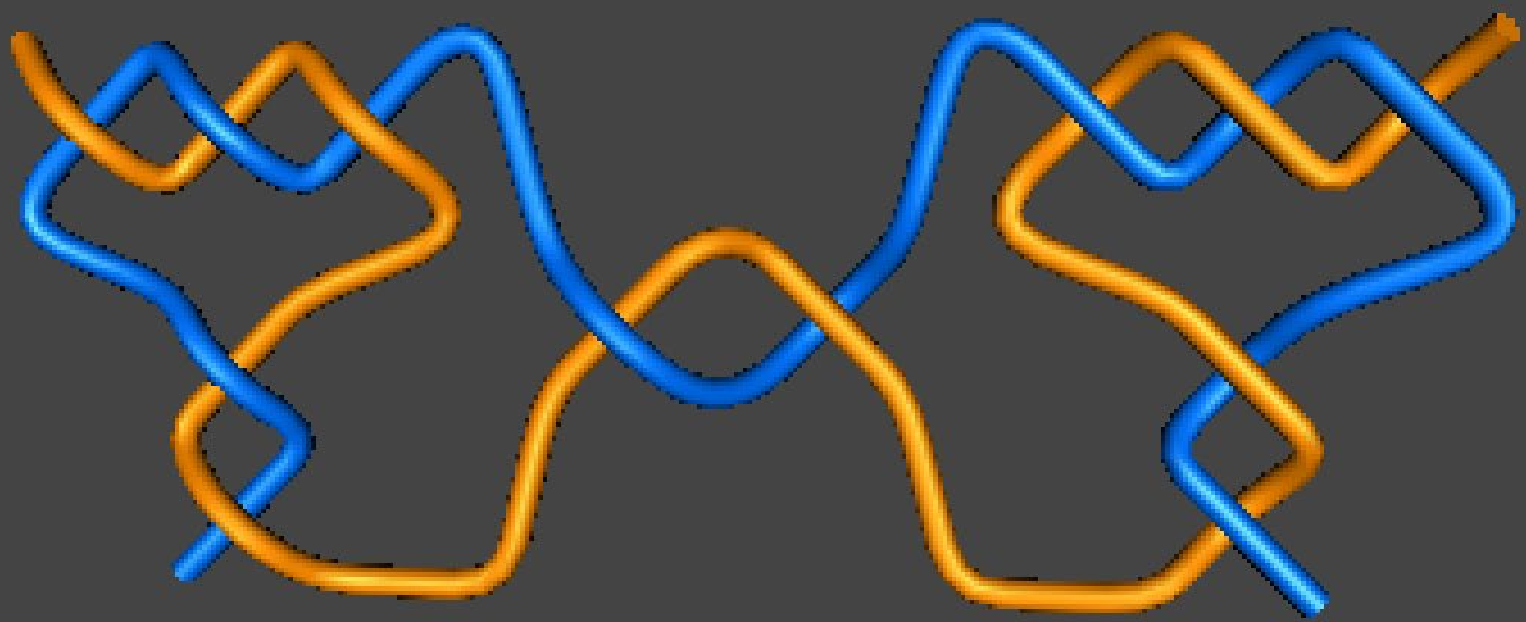
$$(c_1, \ldots, c_{n-1}, c_n + d_k, d_{k-1}, \ldots, d_1)$$

(3, 2, 1 + 1, 2, 3)



KnotPlot> tangle 321o32*1*xz#.

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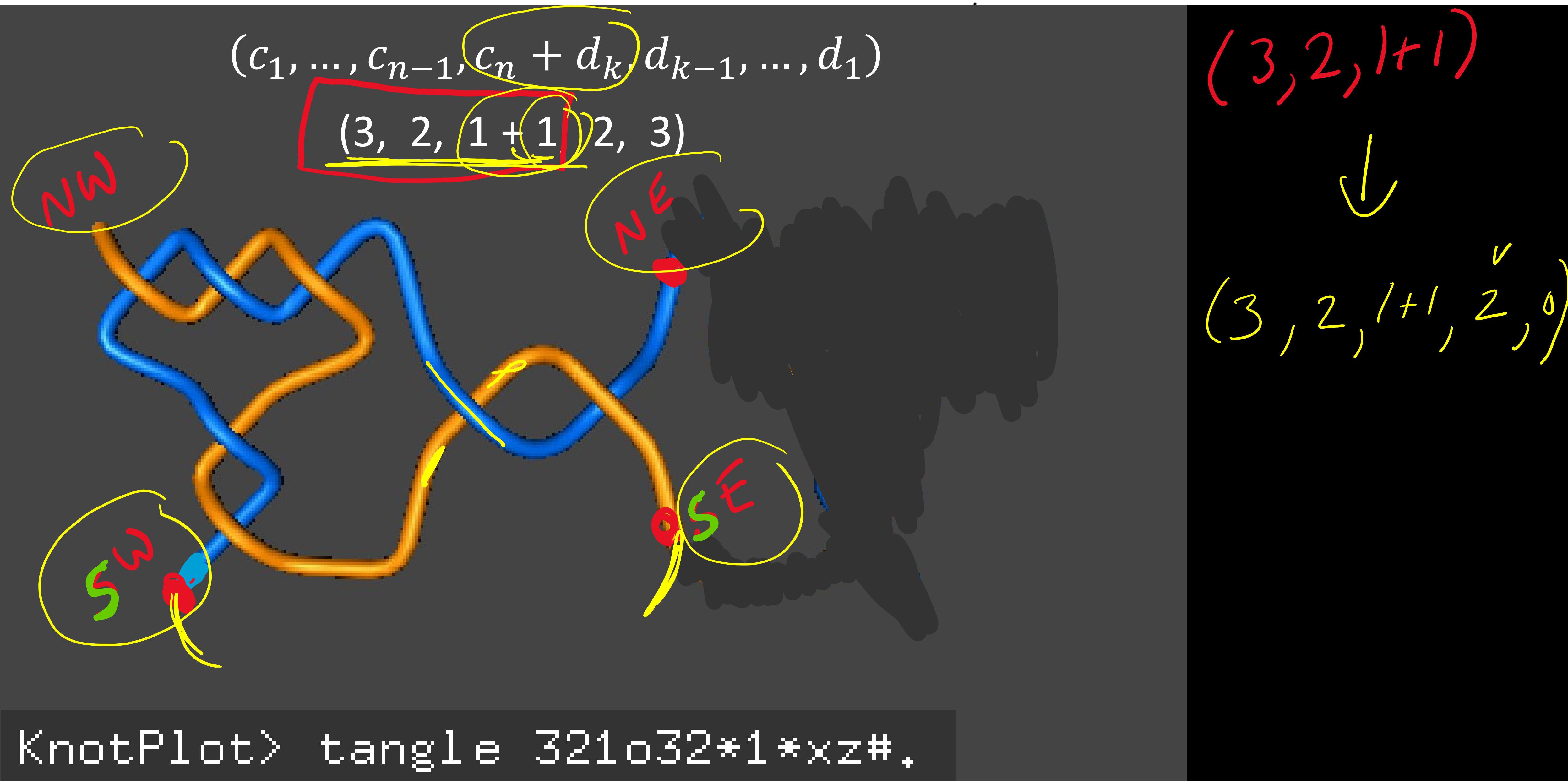

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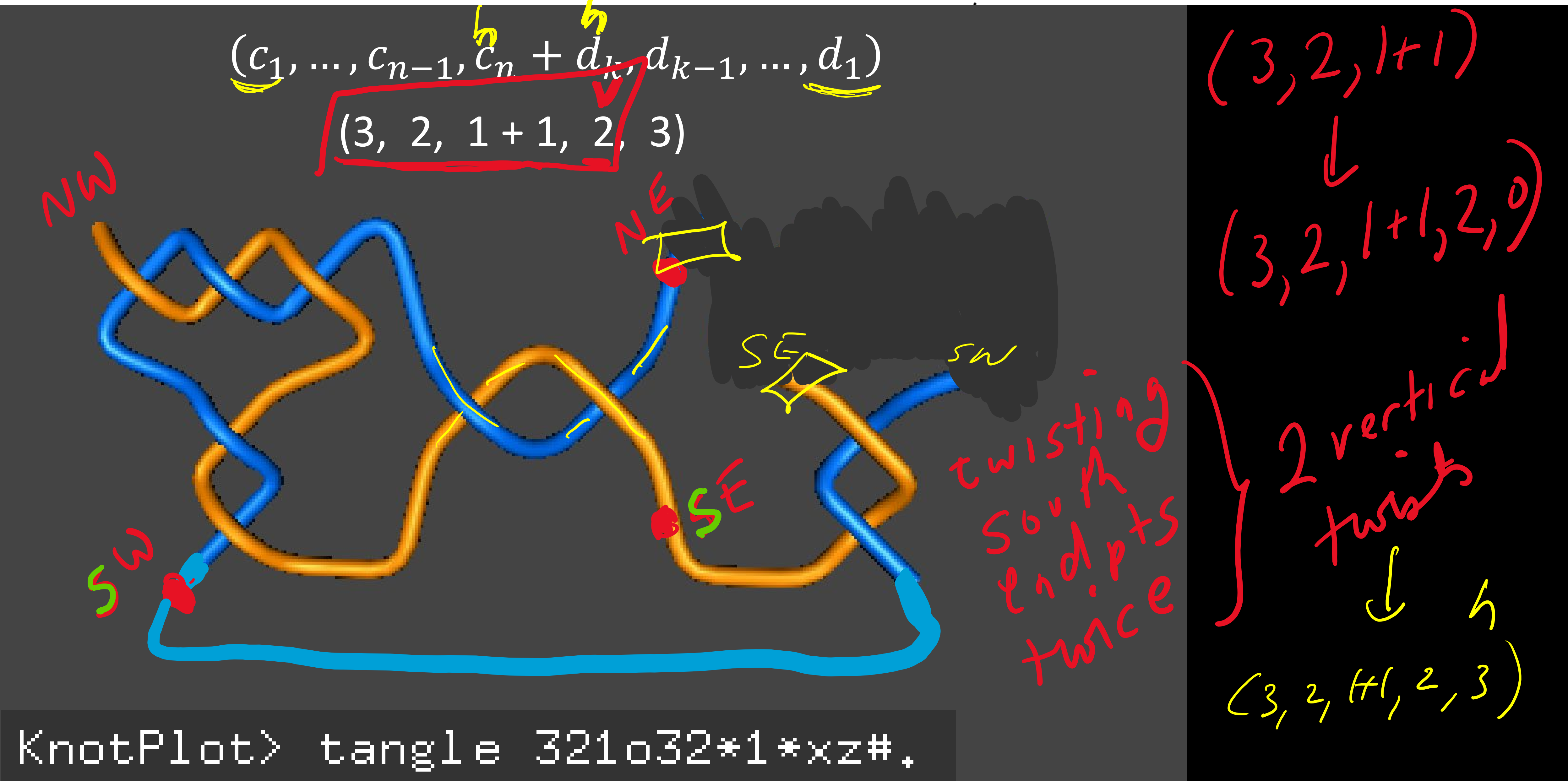

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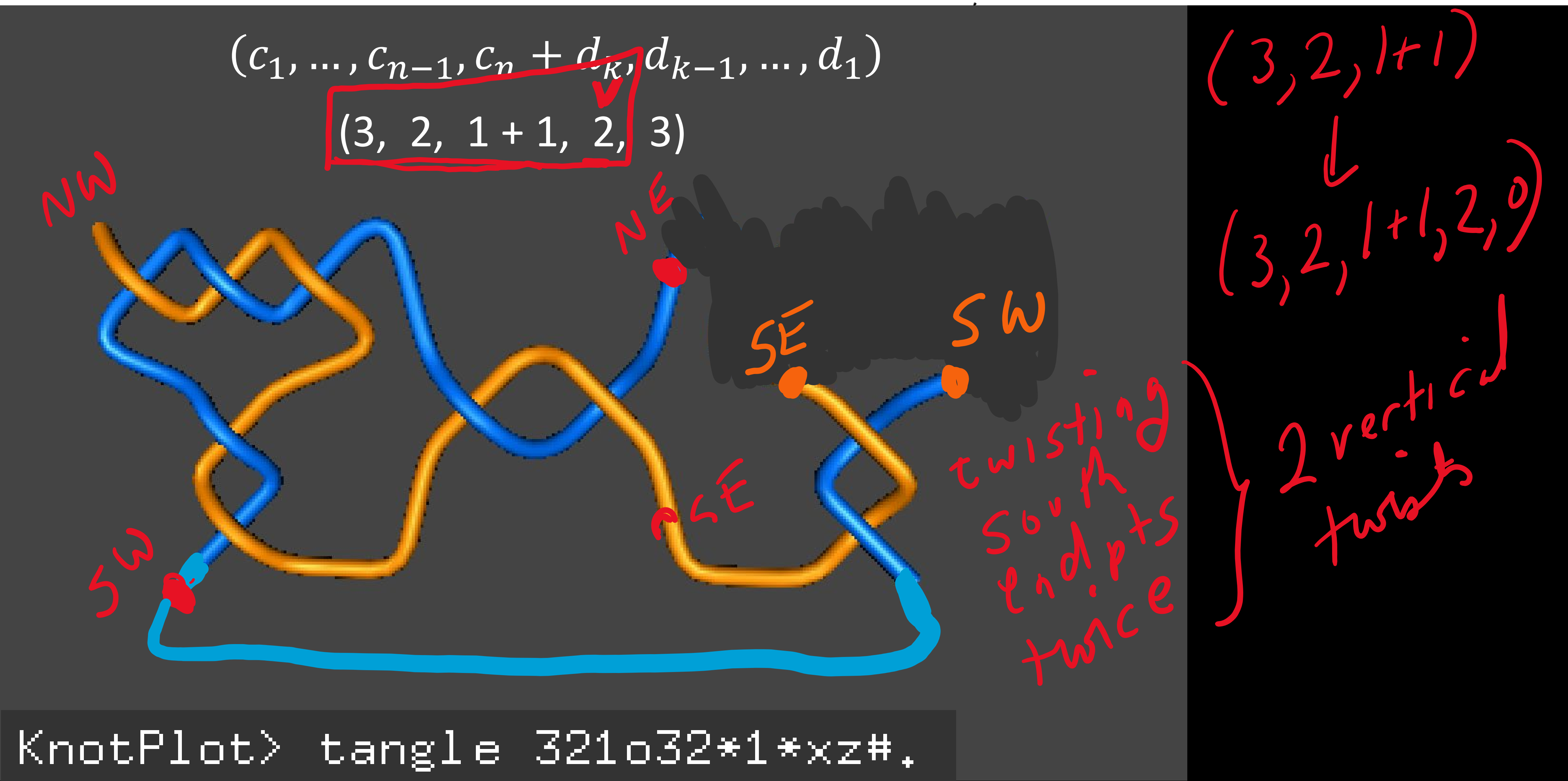

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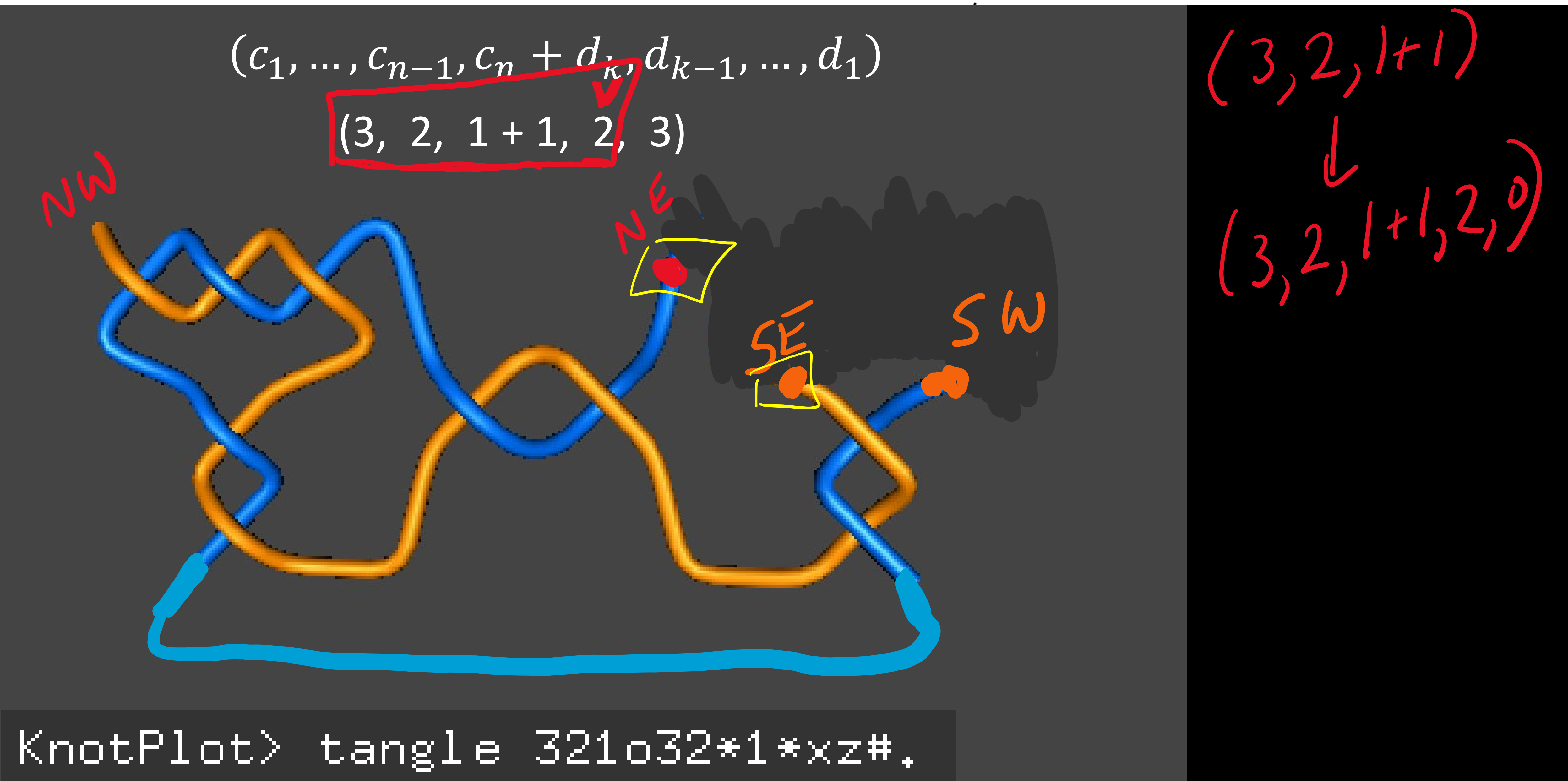

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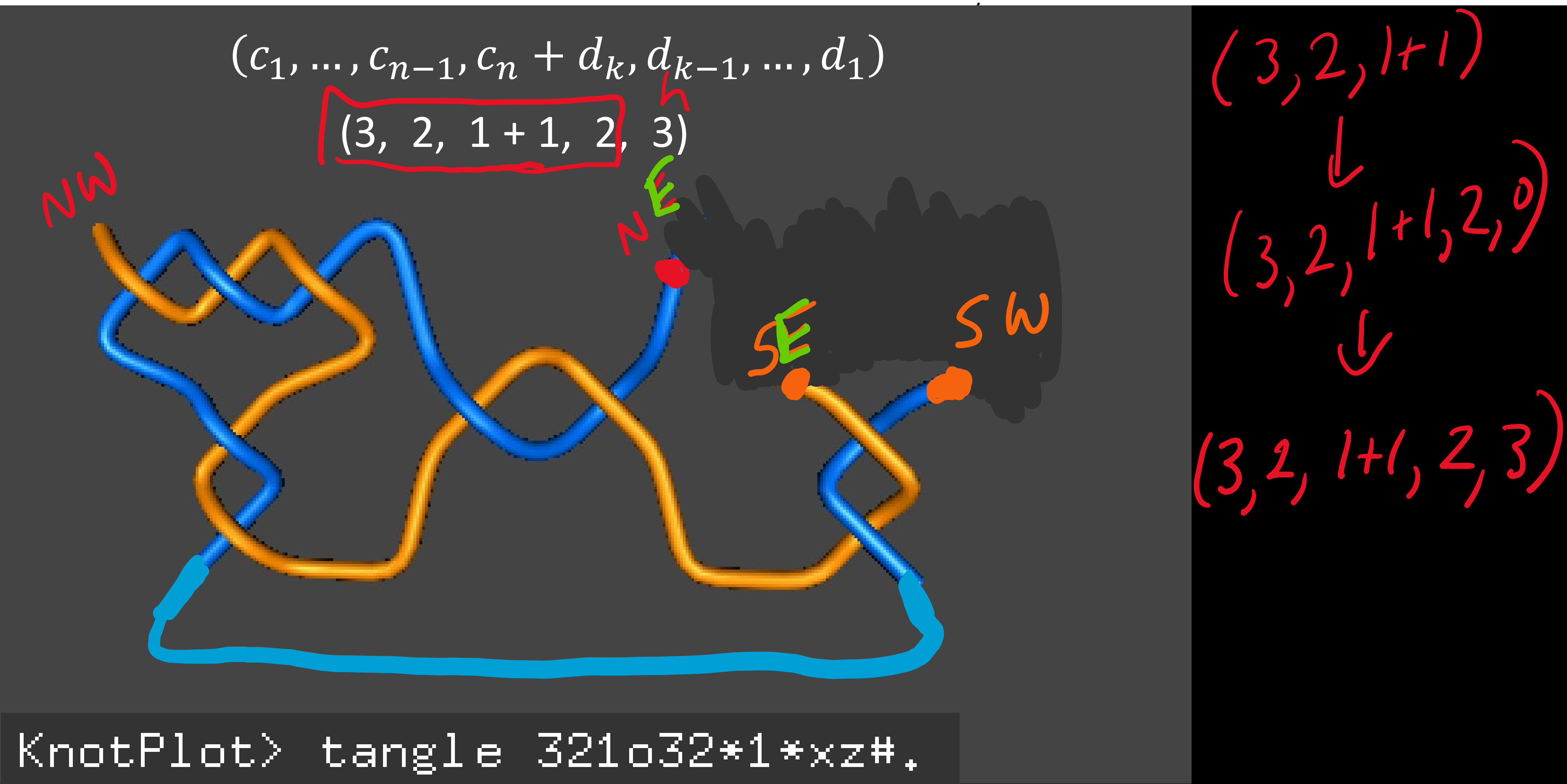

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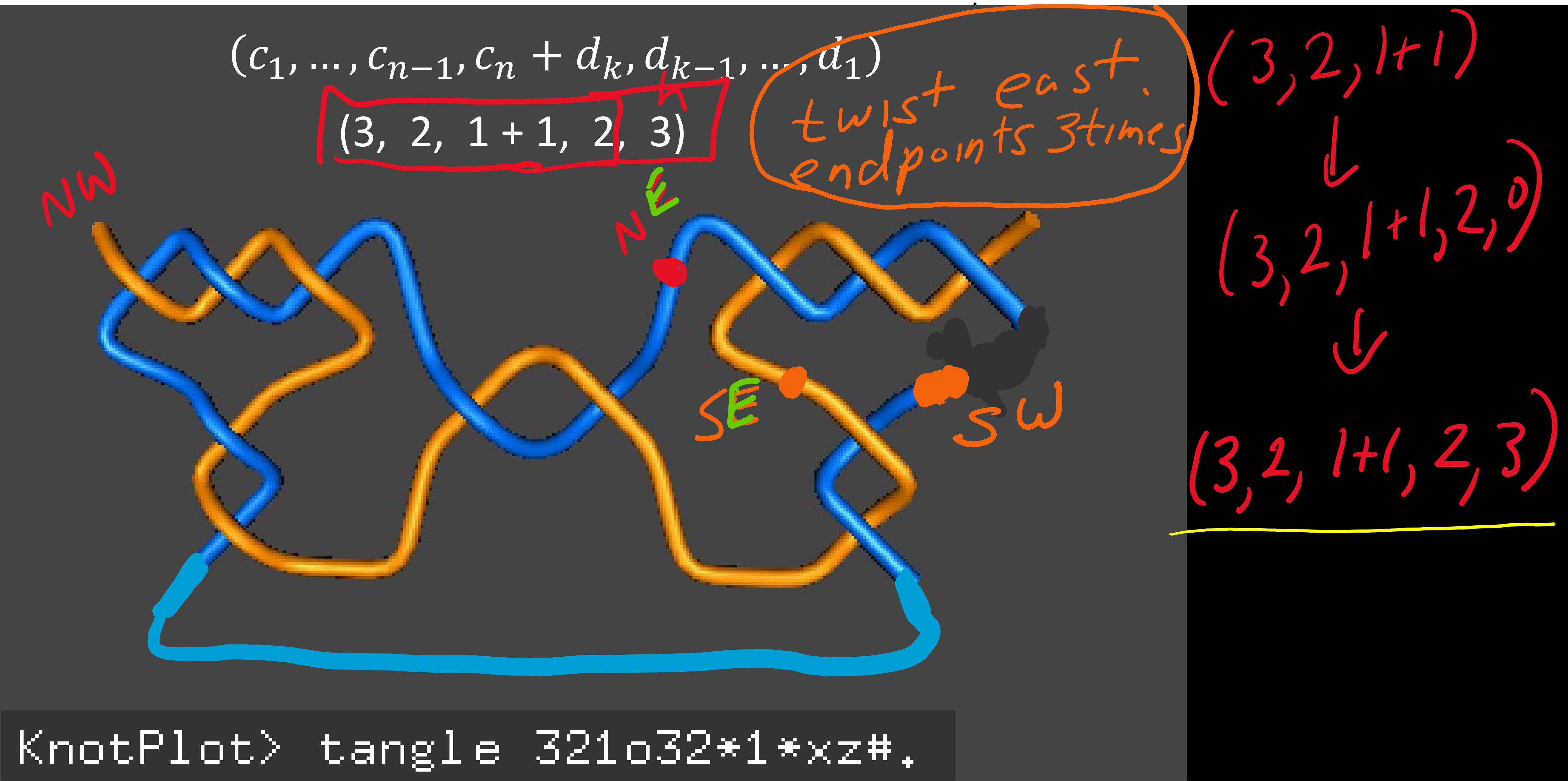

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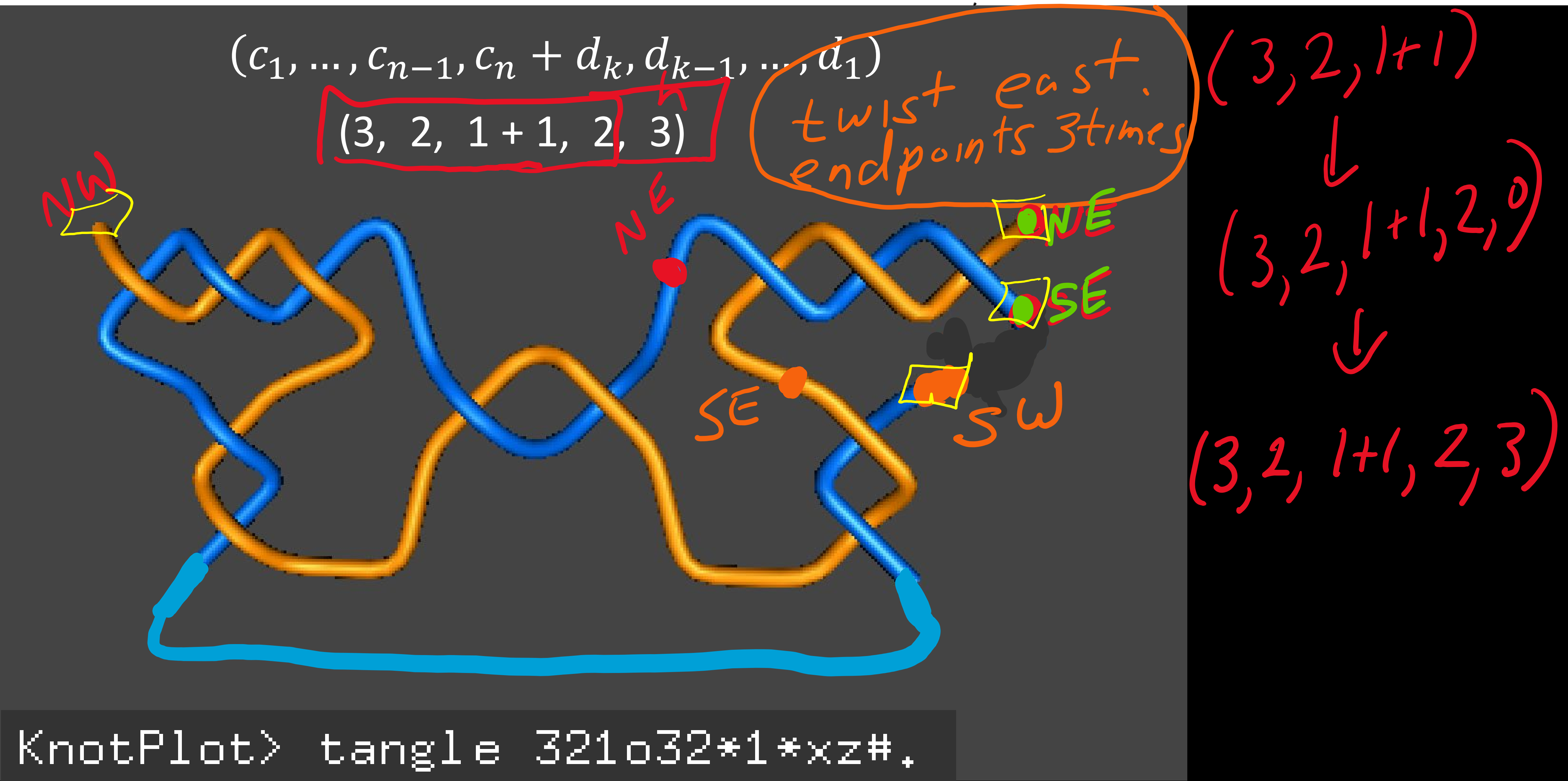

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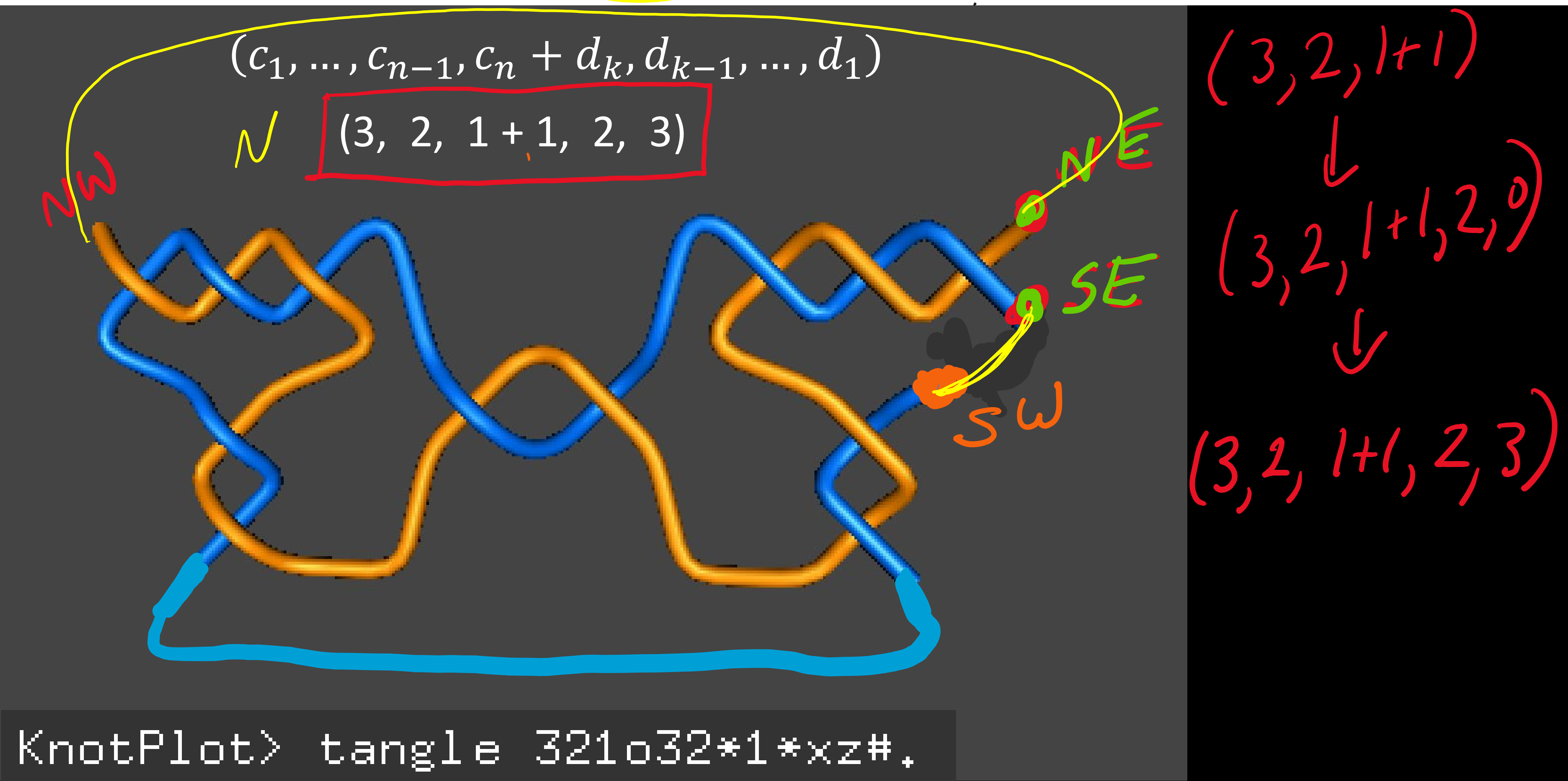

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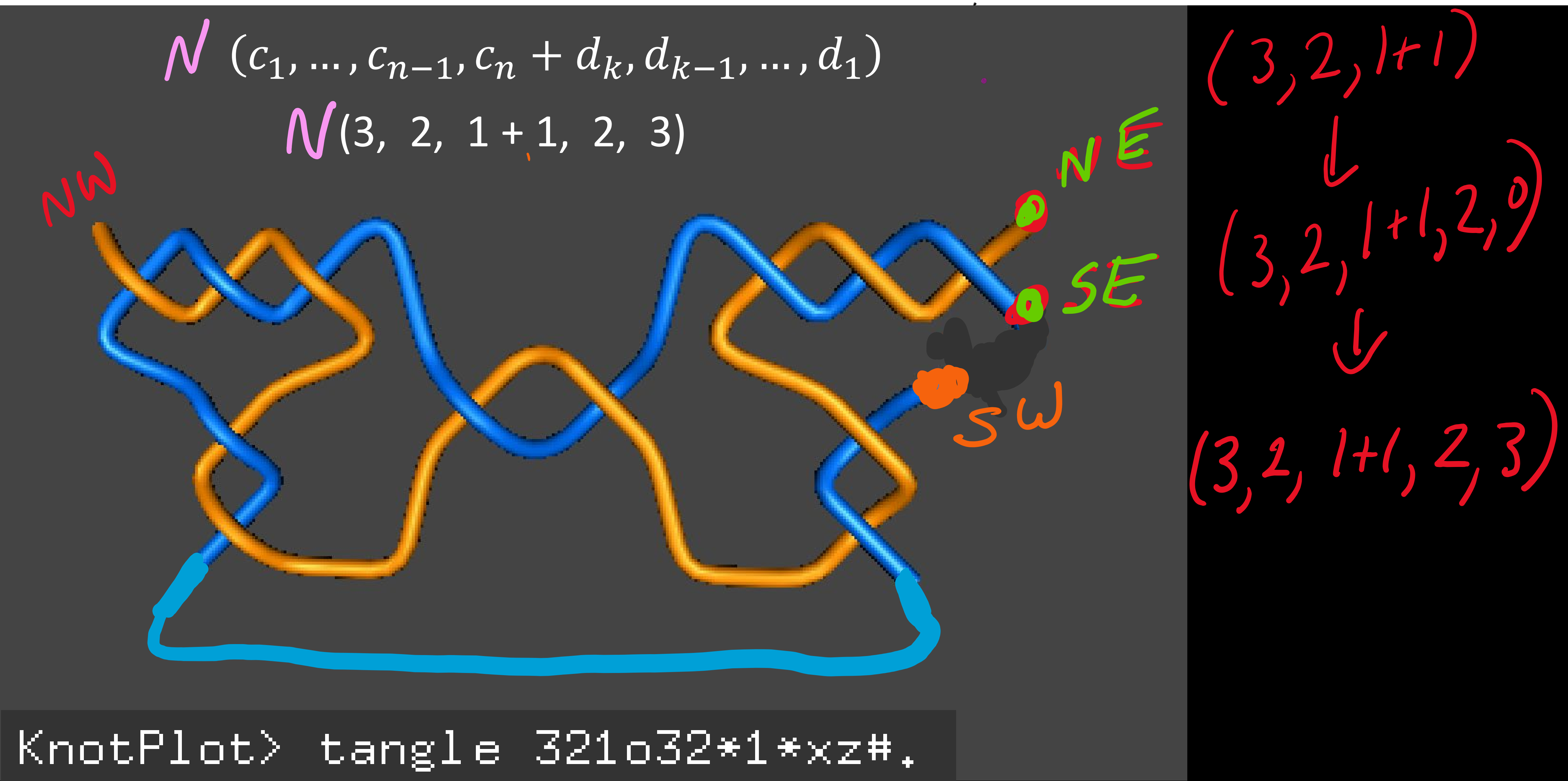

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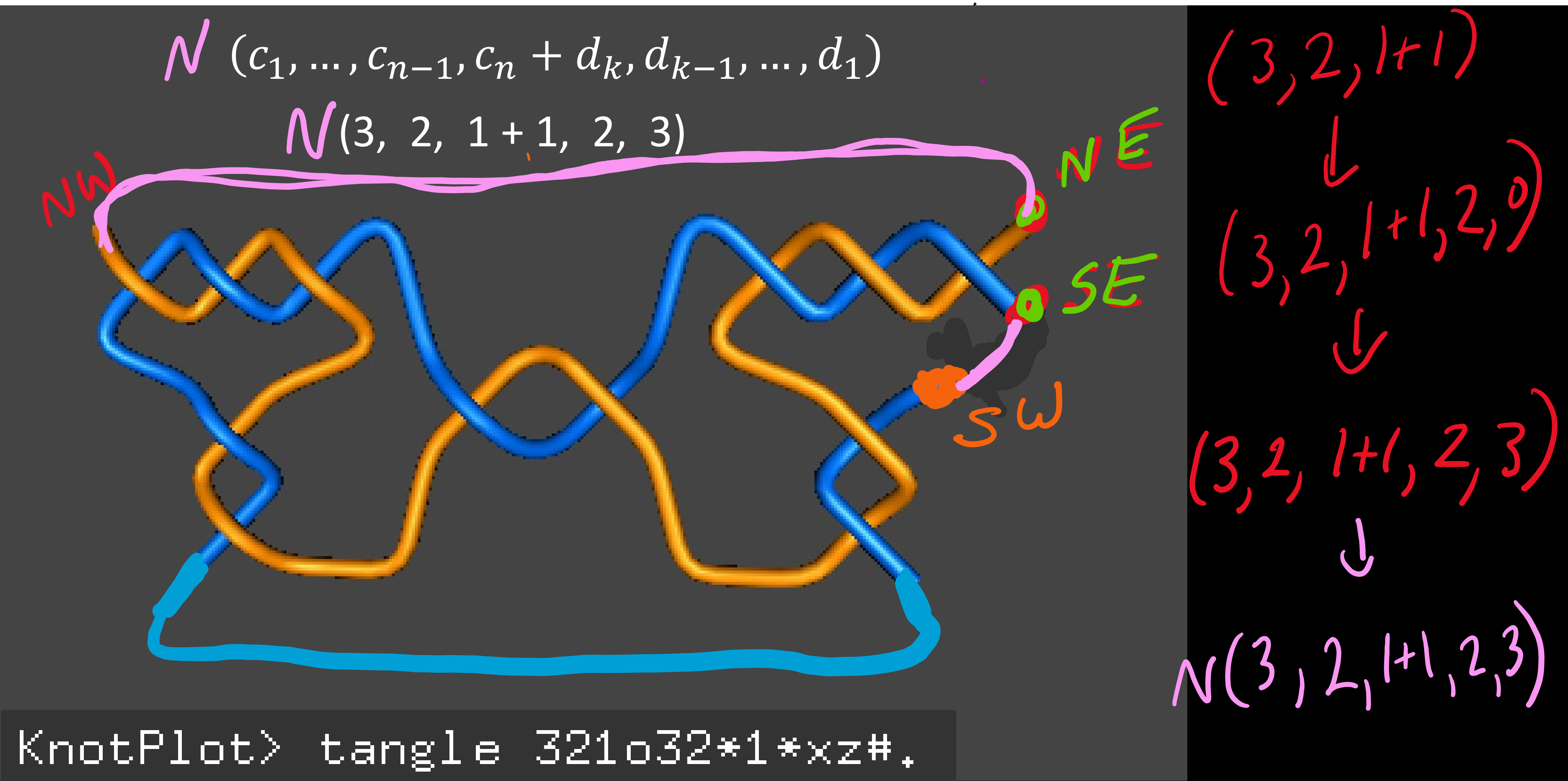

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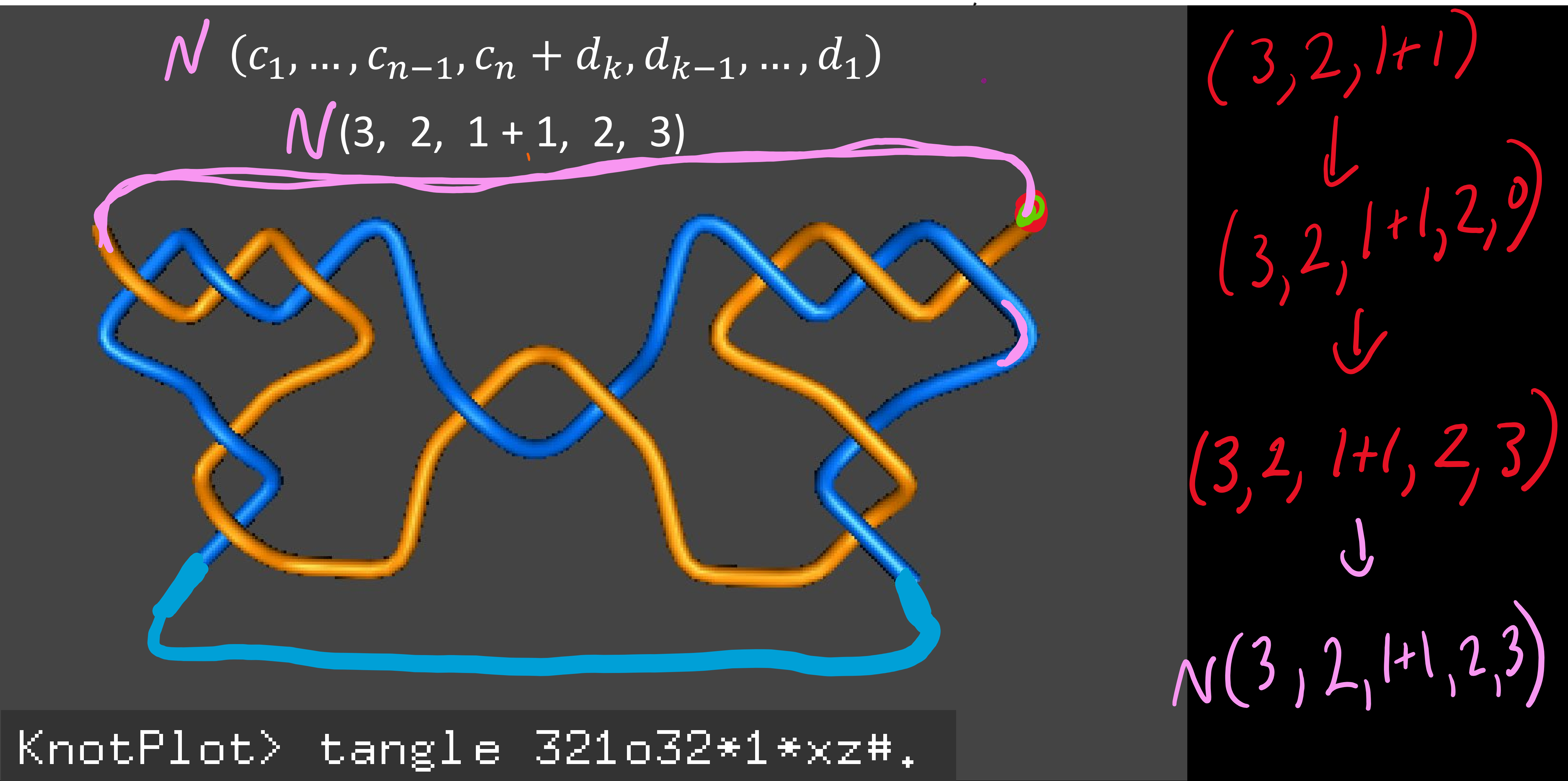

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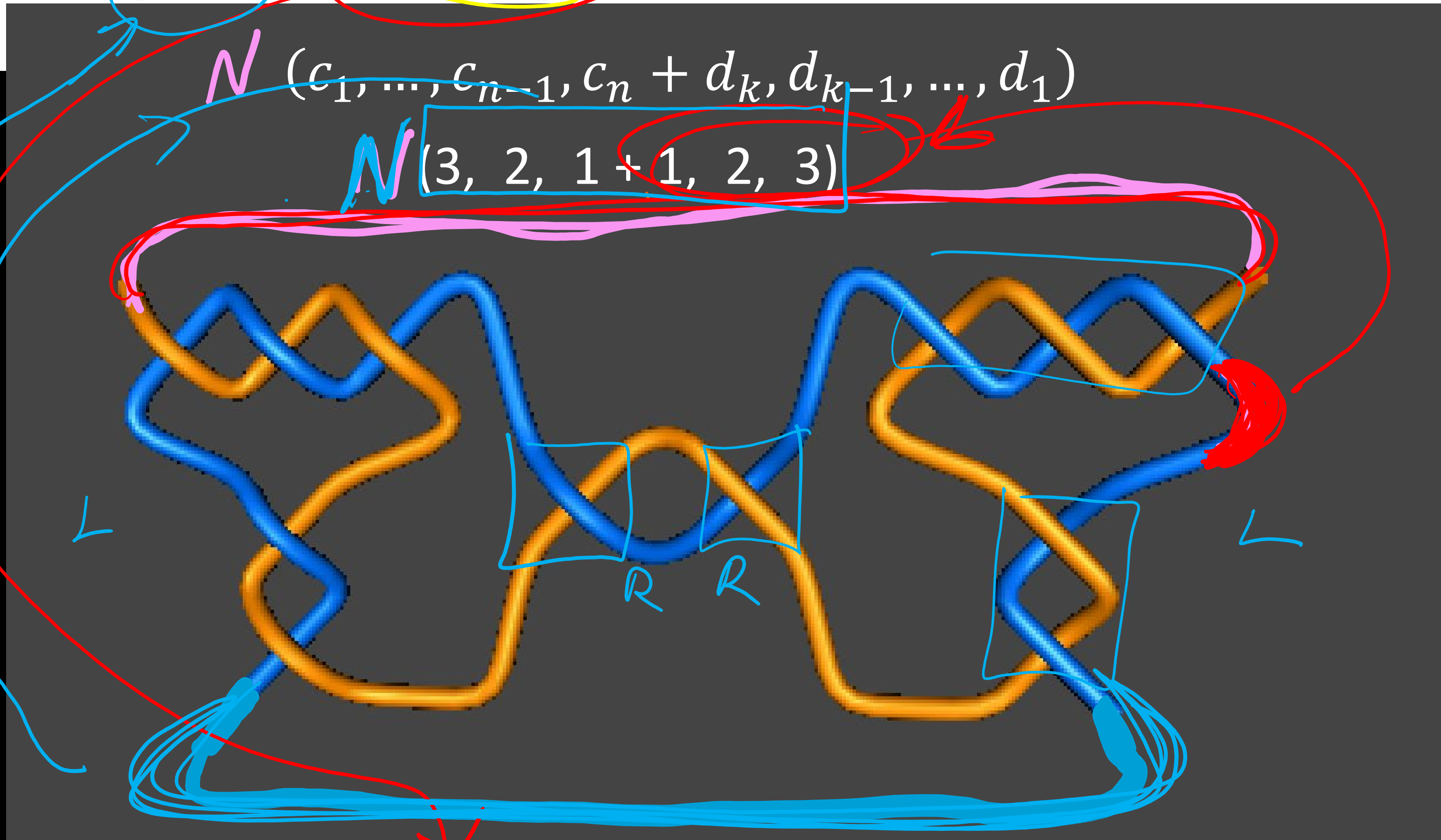
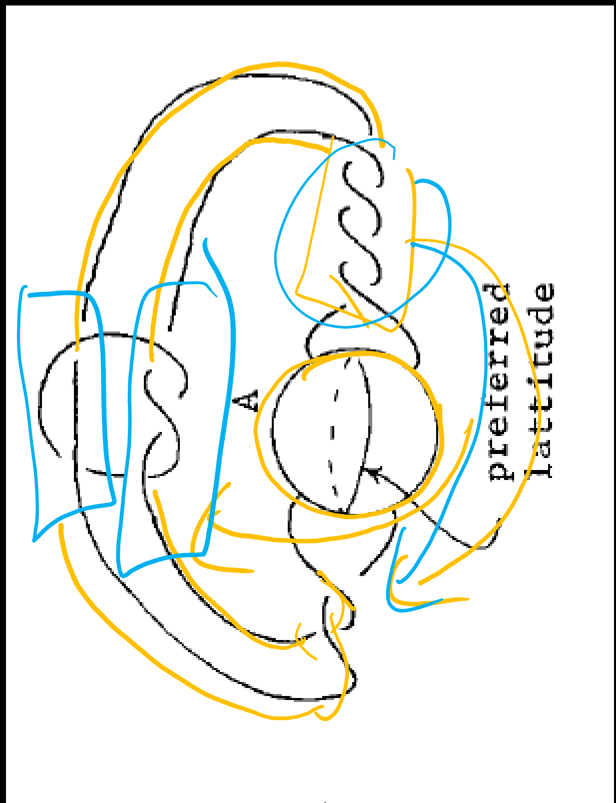
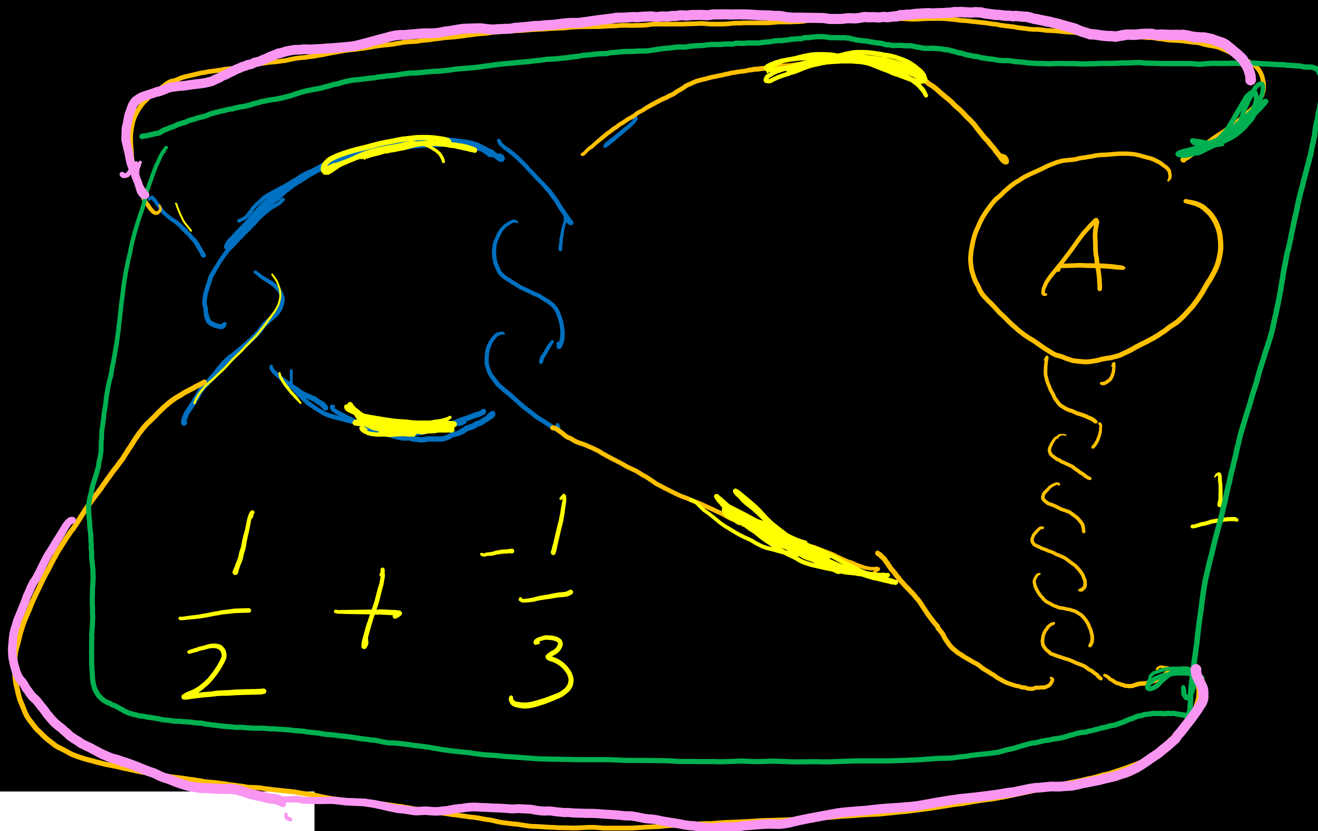


$$[c_1, ..., c_n + d_m, ..., d_1] = \frac{E[c_1, ..., c_n]E[d_1, ... d_{m-1}] + E[c_1, ..., c_{n-1}]E[d_1, ... d_m]}{E[c_2, ..., c_n]E[d_1, ... d_{m-1}] + E[c_2, ..., c_{n-1}]E[d_1, ... d_m]}$$





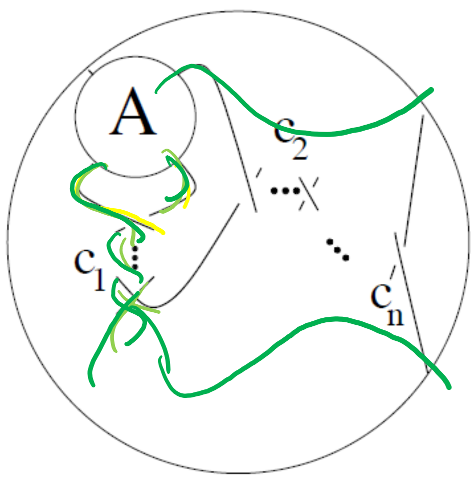
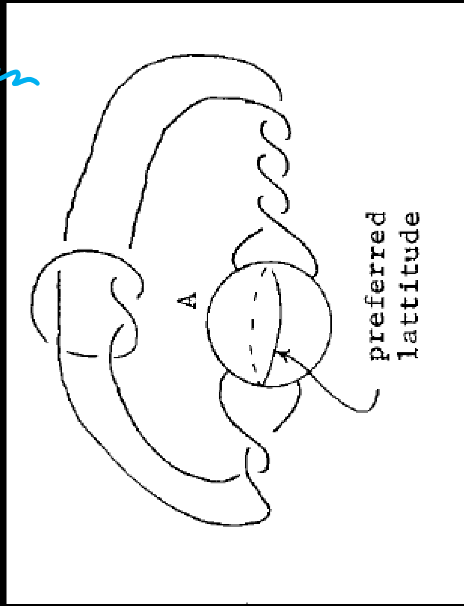


Fig. 5. $A \circ (c_1, ..., c_n)$, n even

$$N\left(\frac{1}{2} + \frac{-1}{3} + A \circ (6,0)\right)$$

Montesinos tangle = sum not 1/0 rational sum



If $A \circ (b, 0) = \frac{c}{d}$

$N(\frac{1}{2} + \frac{-1}{3} + \frac{c}{d})$

Let $d \geq 0$ since $\frac{c}{d} = \frac{-c}{-d}$

$$N(\tfrac{1}{2} + \tfrac{-1}{3} + \tfrac{c}{d}) = \begin{cases} D(\frac{1}{2})\#D(\frac{-1}{3}) & \text{if } d = 0 \\ \textit{Montensinos tangle} & \text{if } d > 1 \\ N(\frac{1+6c}{2+3c}) & \text{if } d = 1 \end{cases}$$

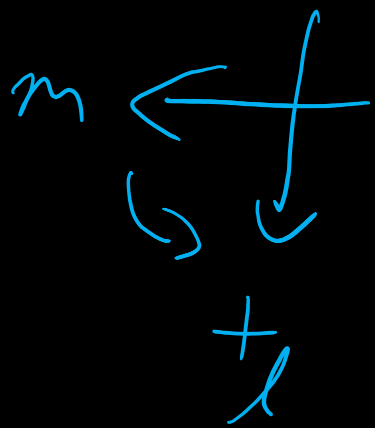
NOT rational knot/link

$$N(\tfrac{1}{2} + \tfrac{-1}{3} + c) = N(\tfrac{1}{2} + \tfrac{-1+3c}{3}) = N(\tfrac{3-2+6c}{3-1+3c}) = N(\tfrac{1+6c}{2+3c})$$

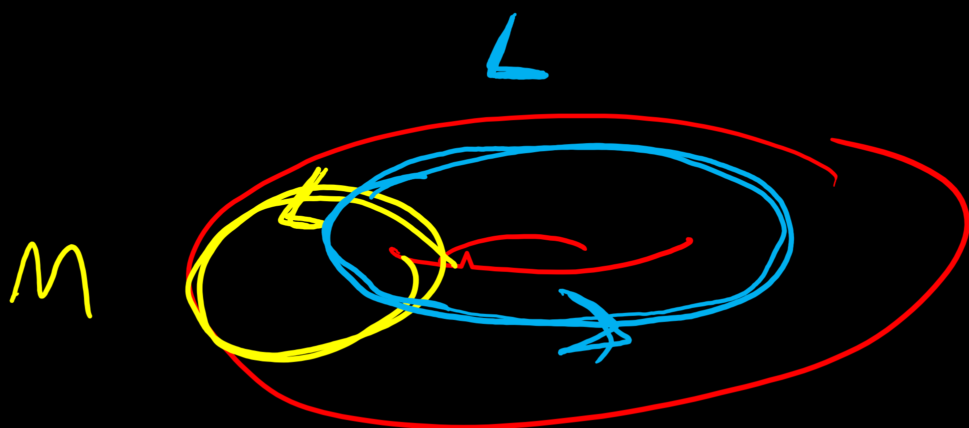
Lemma 3. [8] $N(\frac{j}{p} + \frac{t}{w}) = N(\frac{jw+pt}{dw+qt})$ *where d and q are any integers such that $pd - qj = 1$.*

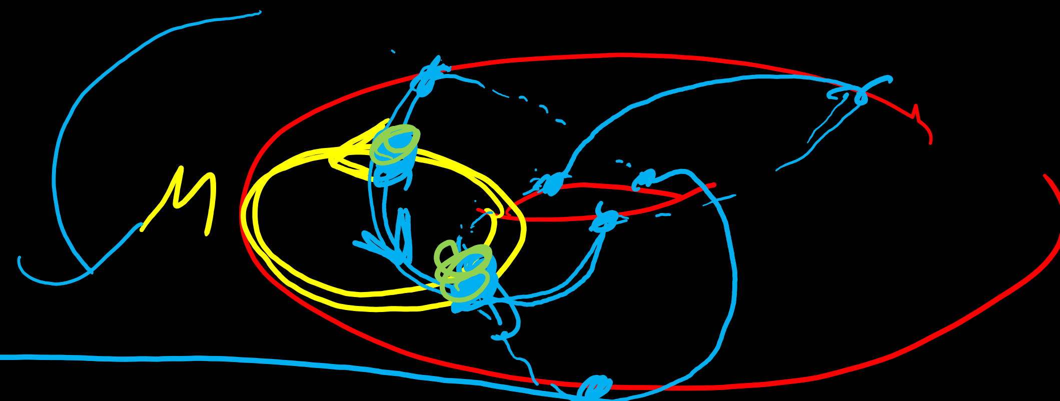
Intersection Number

Figure 3.3: Intersection number.



$i(2l + 3m, m) = +2$





$i(l, m) = +1$

$\det A = \det A^T \quad \begin{vmatrix} 1 & 0 \\ 3 & 2 \end{vmatrix} = \begin{matrix} M \to \\ L \to \end{matrix} \begin{vmatrix} 1 & 3 \\ 0 & 2 \end{vmatrix} = +2$

$$\begin{pmatrix} 1 & 3 \\ 0 & 1 \end{pmatrix}\begin{pmatrix} 1 & 0 \\ 2 & 1 \end{pmatrix}\begin{pmatrix} 1 & 5 \\ 0 & 1 \end{pmatrix}\begin{pmatrix} 1 & 0 \\ 4 & 1 \end{pmatrix} = \begin{pmatrix} 159 & 38 \\ 46 & 11 \end{pmatrix}$$

$$\begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} \xrightarrow{V^4} \begin{pmatrix} 1 & 0 \\ 4 & 1 \end{pmatrix} \xrightarrow{H^5} \begin{pmatrix} 21 & 5 \\ 4 & 1 \end{pmatrix} \xrightarrow{V^2} \begin{pmatrix} 21 & 5 \\ 46 & 11 \end{pmatrix} \xrightarrow{H^3} \begin{pmatrix} 159 & 38 \\ 46 & 11 \end{pmatrix}$$

Let $A = \begin{pmatrix} 159 & 38 \\ 46 & 11 \end{pmatrix}$

Then A corresponds to an orientation preserving homeomorphism sending

$$M \to 159M + 46L \text{ and } L \to 38M + 11L$$

since $A\begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} = \begin{pmatrix} 159 & 38 \\ 46 & 11 \end{pmatrix}$

Thus $1 = det\begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} = i(L, M) = i(38M + 11L, 159M + 46L) = det\begin{pmatrix} 159 & 38 \\ 46 & 11 \end{pmatrix}$

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Thus $1 = det \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} = i(L, M) = i(38M + 11L, 159M + 46L) = det \begin{pmatrix} 159 & 38 \\ 46 & 11 \end{pmatrix}$

Since A^{-1} is an orientation preserving homeomorphism, $det(A^{-1}) = 1$

$$A^{-1} \begin{pmatrix} 159 & b \\ 46 & a \end{pmatrix} = \begin{pmatrix} 1 & ? \\ 0 & 159a - 46b \end{pmatrix}$$

$i(bM + aL, 159M + 46L) = i(?M + (159a - 46b)L, M) = 159a - 46b = det \begin{pmatrix} 159 & b \\ 46 & a \end{pmatrix}$

Definition $L(a, b) = V_1 \cup_h V_2$ where $h : \partial V_2 \to \partial V_1$ is an orientation preserving homeomorphism and $h(M_2) = aL_1 + bM_1$.

$\pi_1(L(a,b)) = \mathbb{Z}_a$

by

SvK

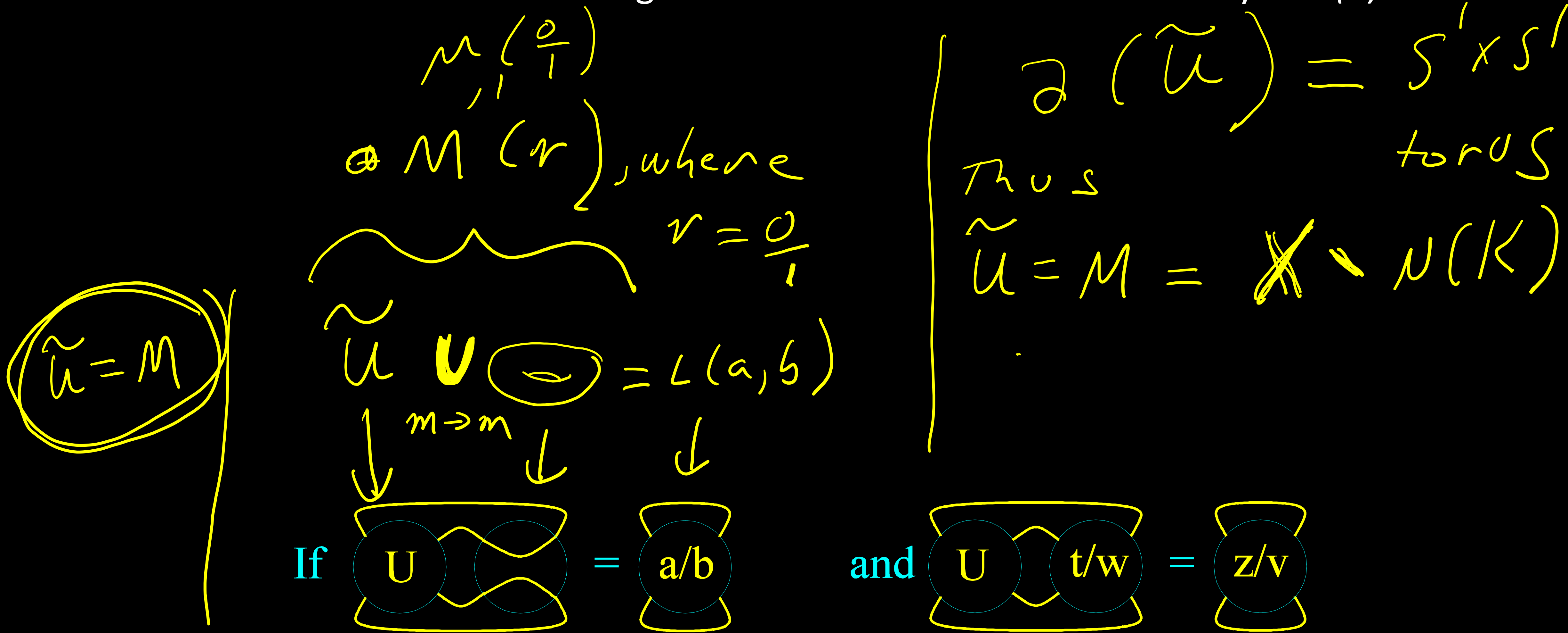
π_1 is cyclic

M. Culler, C. Gordon, J. Luecke, P. Shalen (1987). Dehn surgery on knots. The Annals of Mathematics 125 (2): 237-300. https://marc-culler.info/static/home/papers/CyclicSurgery.pdf

CYCLIC SURGERY THEOREM. *Suppose that M is not a Seifert fibered space. If $\pi_1(M(r))$ and $\pi_1(M(s))$ are cyclic, then $\Delta(r, s) \leq 1$. Hence there are at most three slopes r such that $\pi_1(M(r))$ is cyclic.*

f(m') = curve of slope *r* ~ *pl* + *qm*; *r* = *q*/*p* ,

in terms of a bases: *l* = longitude and *m* = meridian of boundary of *N(K)*



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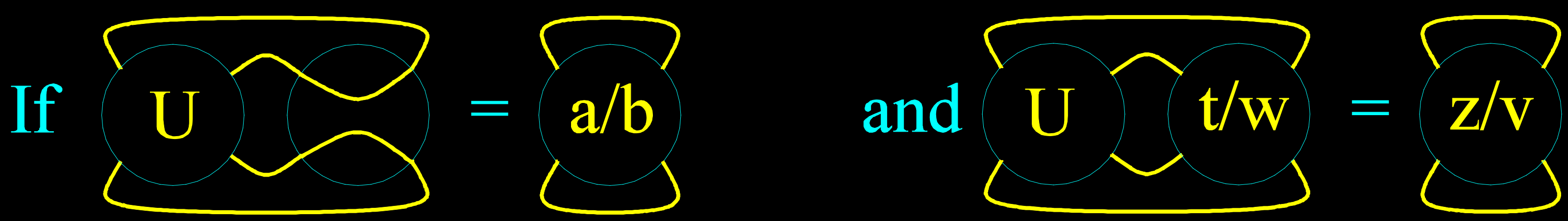
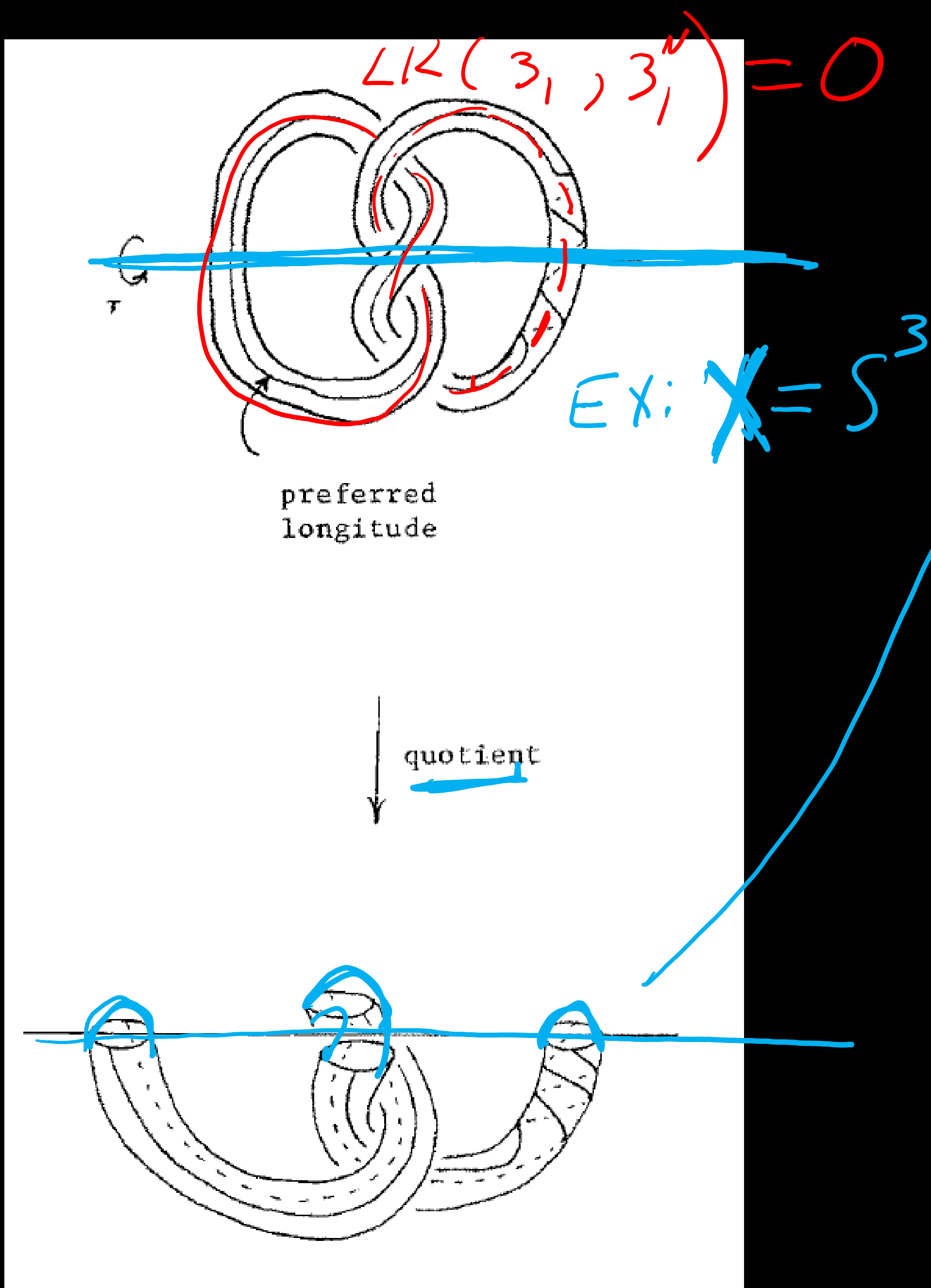
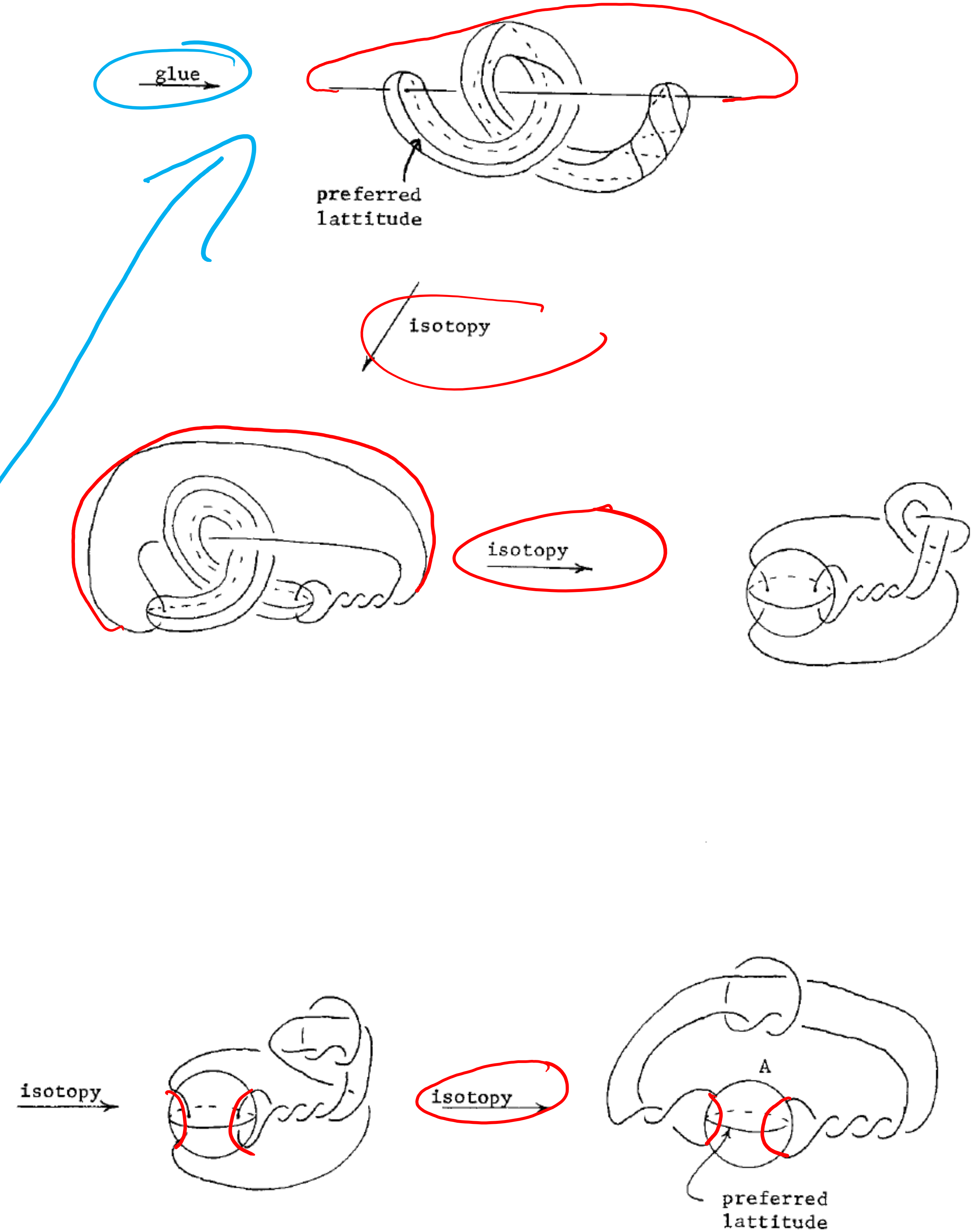
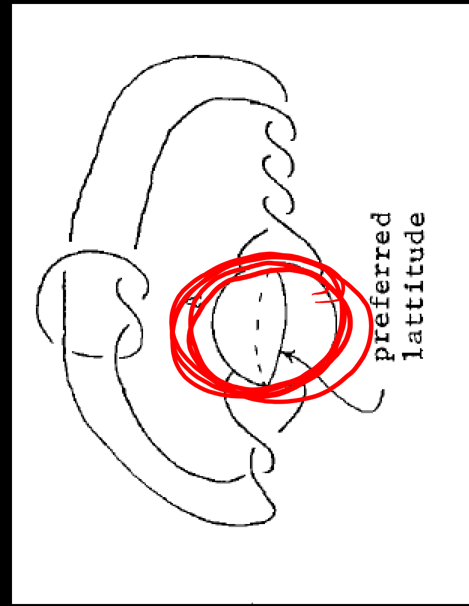


PRIME TANGLES AND COMPOSITE KNOTS

Steven A. Bleiler







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$$N\left(\frac{1}{2} + \frac{-1}{3} + c\right) = N\left(\frac{1}{2} + \frac{-1+3c}{3}\right) = N\left(\frac{3-2+6c}{3-1+3c}\right) = N\left(\frac{1+6c}{2+3c}\right)$$

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$$N\left(\frac{1+6c}{2+3c}\right)$$

but not all choices ⇒ $\pi_1(M \cup V^3)$ is cyclic for many choices

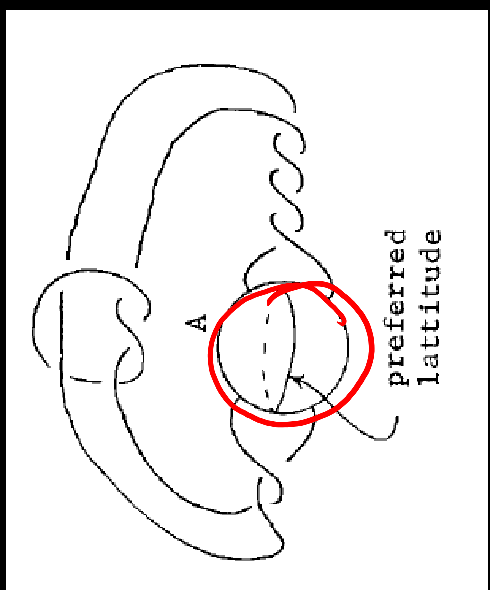
π_1 is cyclic | π_1 cy | cycl | $M(r) = M \cup V^3$

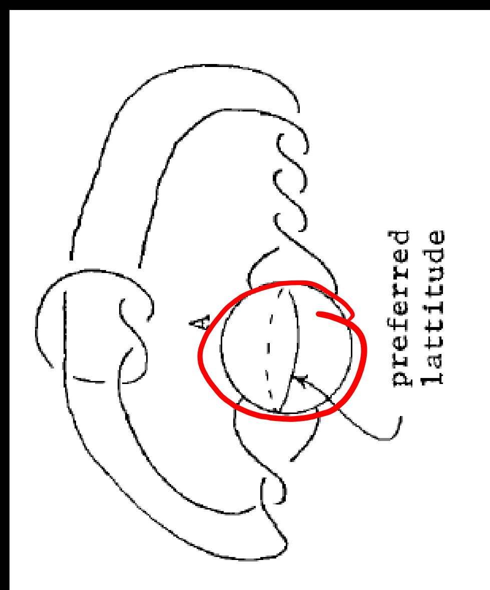
S^3 ↓ If $c=0$: $N(\frac{1}{2}) = 0$

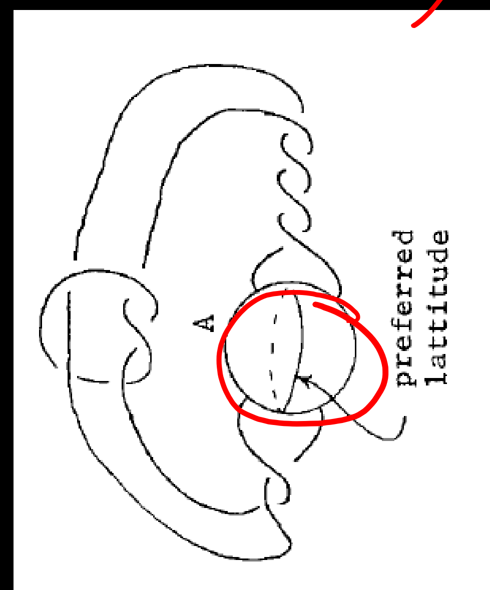
$L(7,5)$ ↓ $c=1$: $N(\frac{7}{5})$

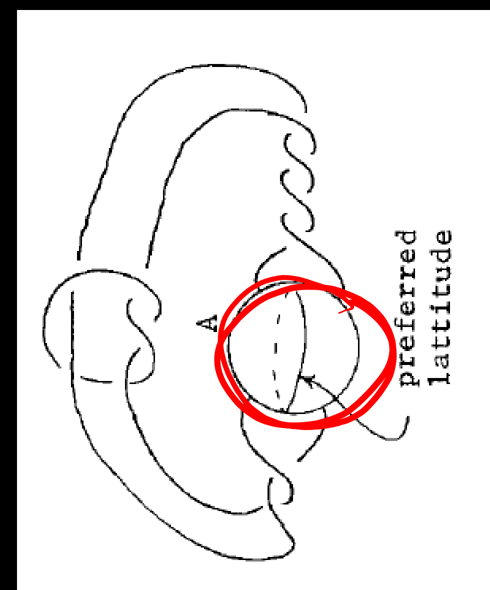
$L(12,8)$ ↓ $N(\frac{12}{8})$

$\tilde{U} \cup$ (solid torus) ↓









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$f(m')$ = curve of slope $r \sim pl + qm$; $r = q/p$,

in terms of a bases: l = longitude and m = meridian of boundary of $N(K)$

We will focus on 2 slope

want $\Delta(r,s) > 1$

$\begin{vmatrix} 1 & ? \\ 0 & t \end{vmatrix} = t > 1$

$m \to m$

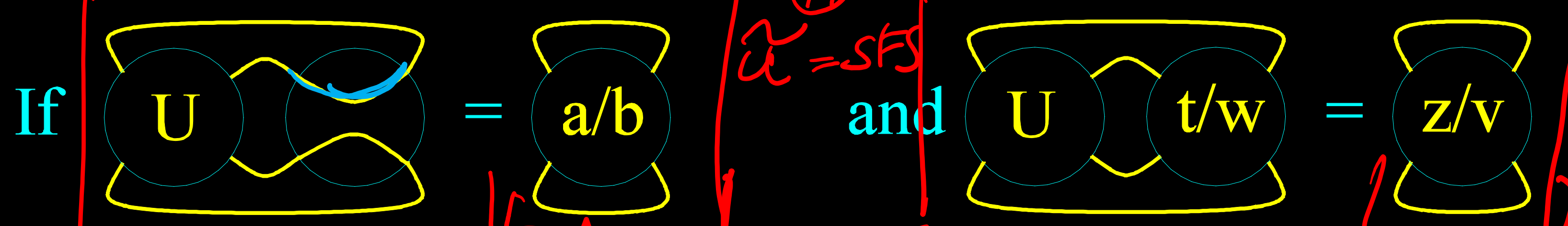
cyclic surgery thm applies

In previous example on previous slide ∞ # of slopes where $\pi_1(M(r))$ is cyclic

⇓

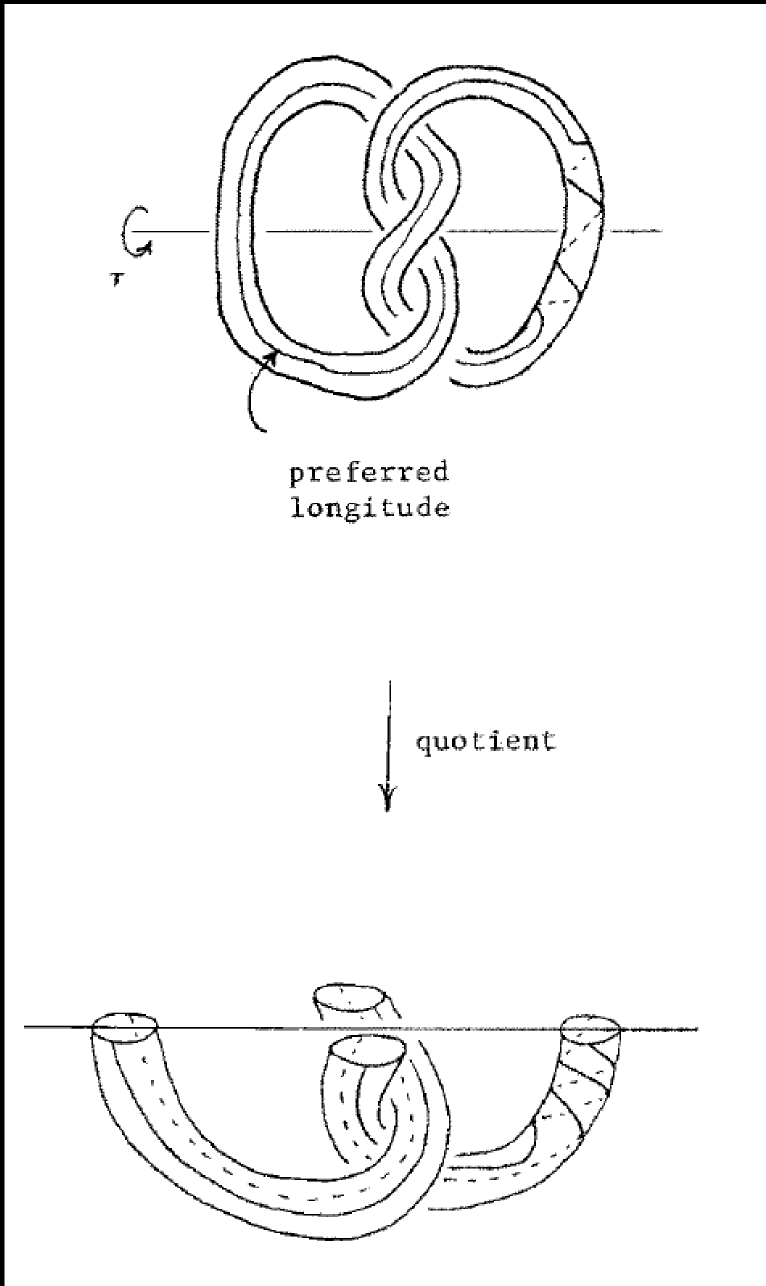
M is SFS

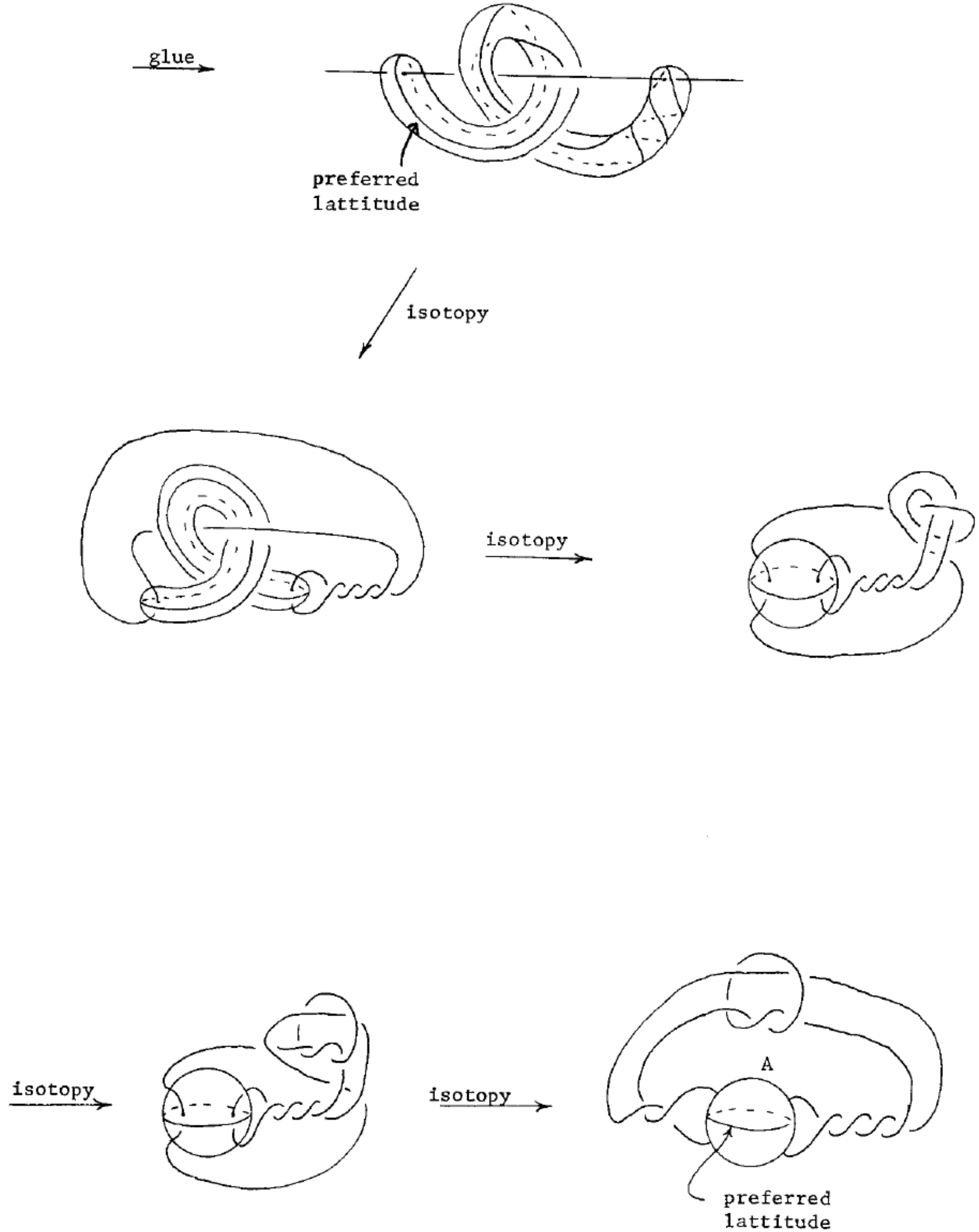
⇓ $\tilde{U}$ = SFS



PRIME TANGLES AND COMPOSITE KNOTS

Steven A. Bleiler





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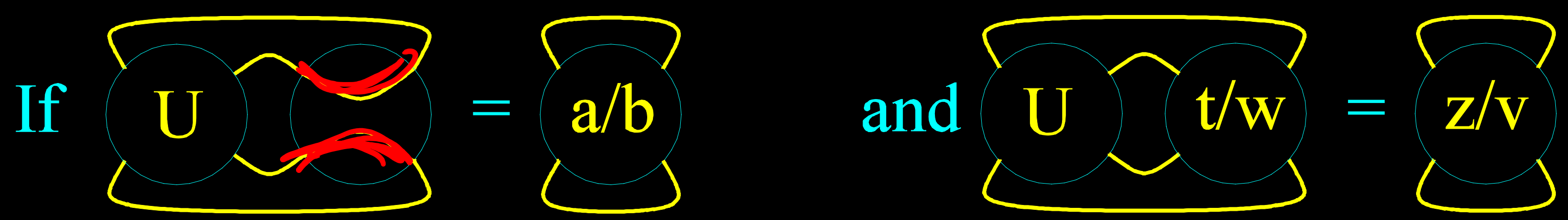
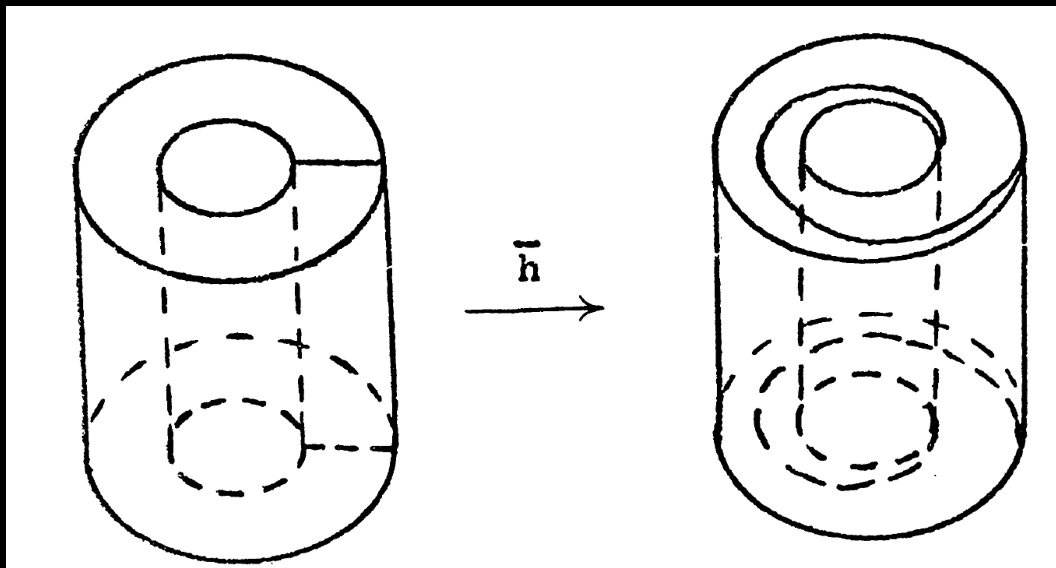


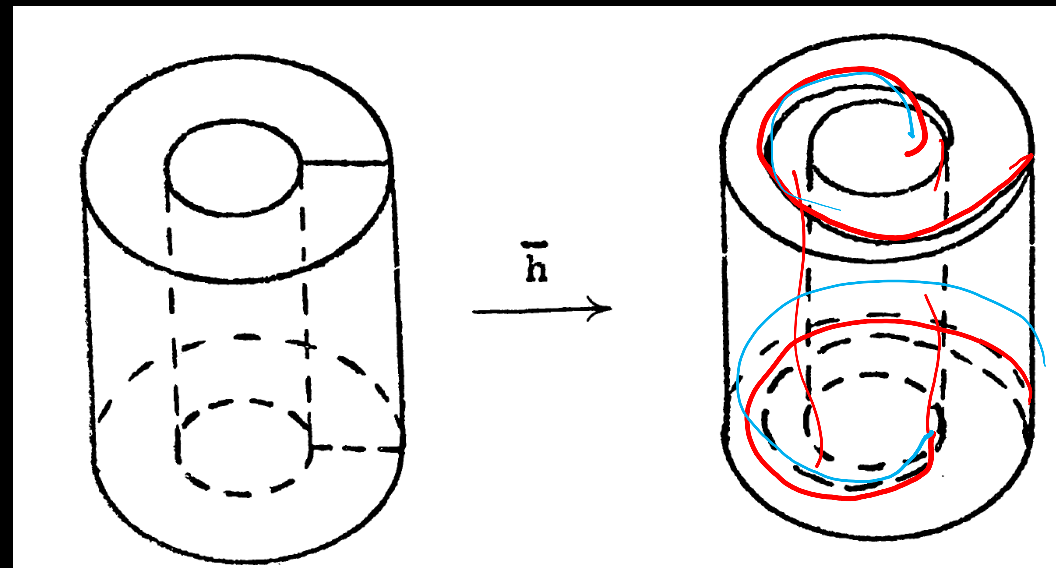
http://dx.doi.org/10.1090/conm/044/813107

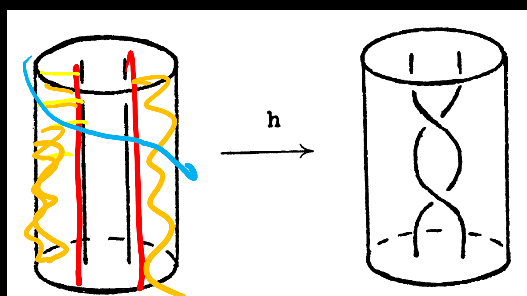
Contemporary Mathematics
Volume 44, 1985

THE UNKNOTTING NUMBER OF A CLASSICAL KNOT

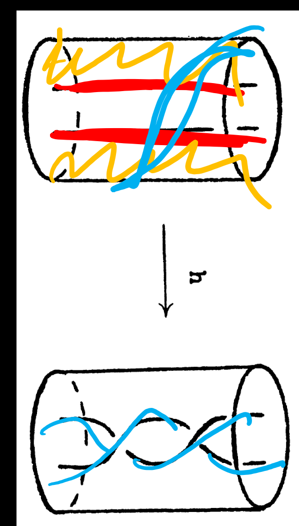
W. B. Raymond Lickorish



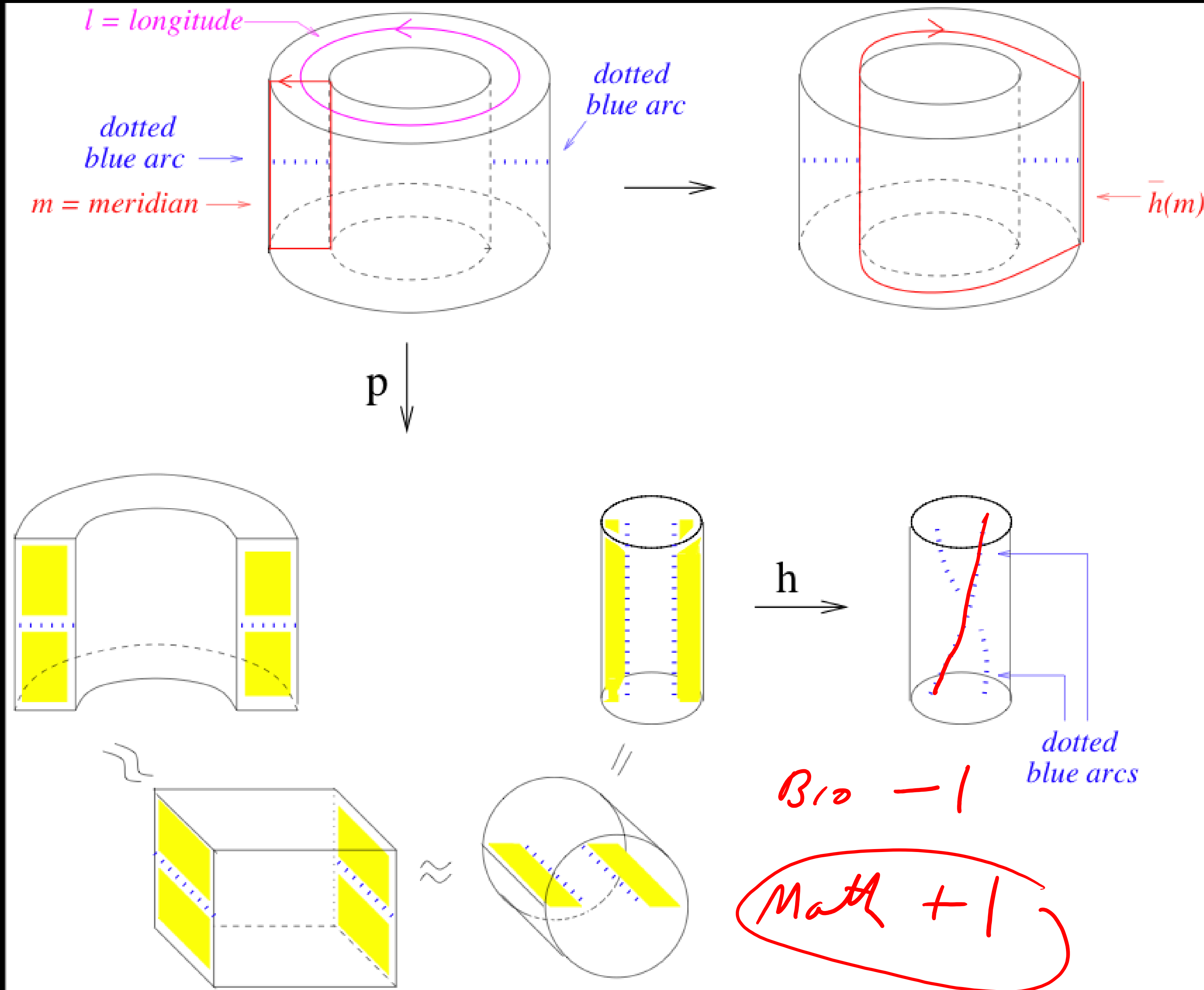


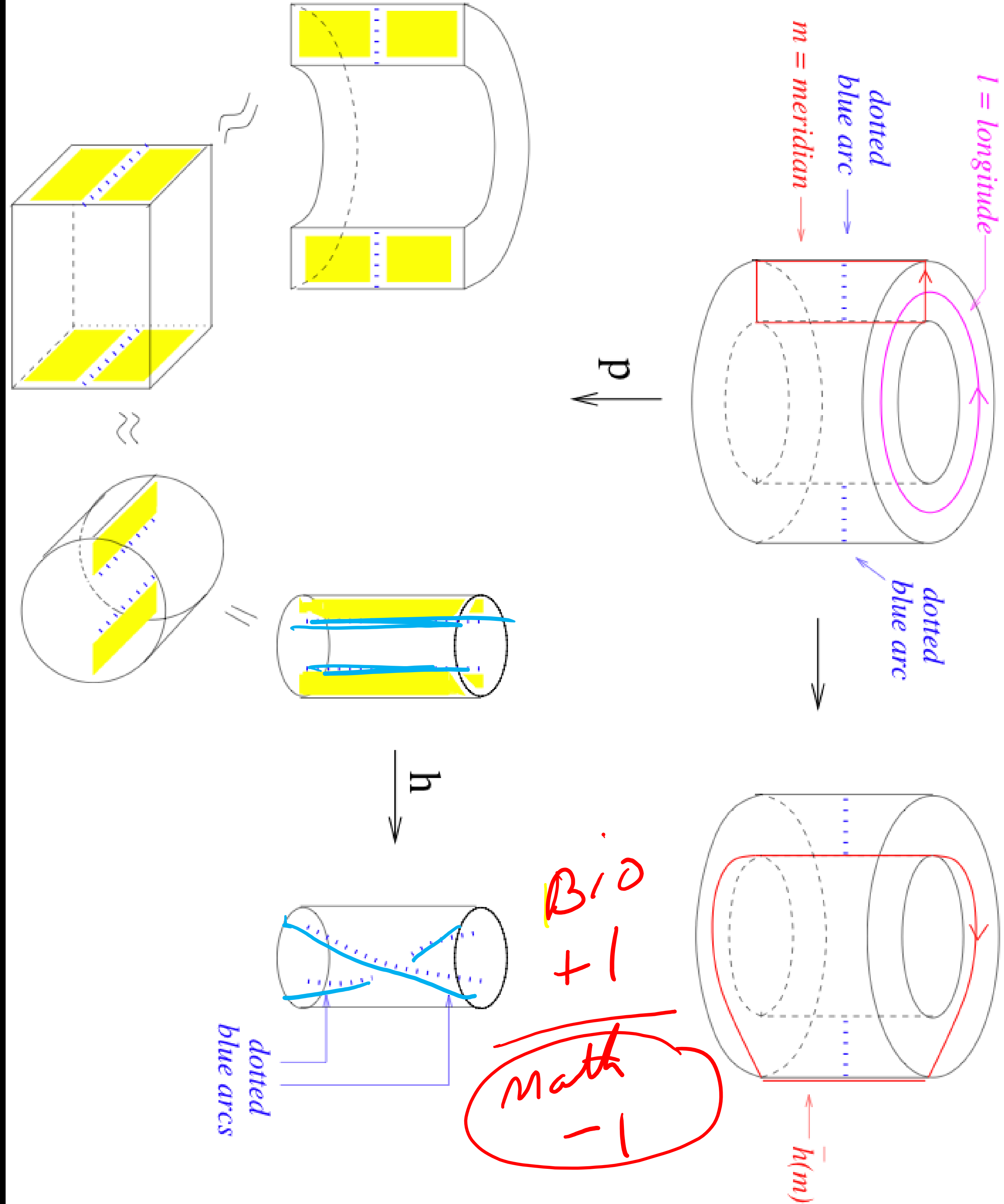


rotating by 90° does not change double branch cover



Lemma 1. *If k has unknotting number equal to one, then M_k is obtained by $n/2$-surgery on some knot in S^3, n being an odd integer.*


l = longitude
dotted blue arc
dotted blue arc
m = meridian
$\bar{h}(m)$
p
h
dotted blue arcs
Bio −1
Math +1



$m \to m + l \to m + Kl$

$\frac{0}{1} \to \frac{1}{1} \to \frac{K}{1}$

$l = L + x m$

$m \to m + K(L + xm)$

$m(1 + Kx) + KL$

$\Rightarrow \frac{1+Kx}{K}$ surgery

M. Culler, C. Gordon, J. Luecke, P. Shalen (1987). Dehn surgery on knots. The Annals of Mathematics 125 (2): 237-300. https://marc-culler.info/static/home/papers/CyclicSurgery.pdf

CYCLIC SURGERY THEOREM. *Suppose that M is not a Seifert fibered space. If $\pi_1(M(r))$ and $\pi_1(M(s))$ are cyclic, then $\Delta(r, s) \leq 1$. Hence there are at most three slopes r such that $\pi_1(M(r))$ is cyclic.*

f(m') = curve of slope $r \sim pl + qm$; $r = q/p$,

in terms of a bases: *l* = longitude and *m* = meridian of boundary of *N(K)*

$r = \frac{0}{1}$

$s = \frac{tx+w}{t}$

$m \to tl + wm$
$= t(L + xm) + wm$
$= tL + (tx+w)m$

$\frac{0}{1} \longrightarrow \frac{t}{w}$

M. Culler, C. Gordon, J. Luecke, P. Shalen (1987). Dehn surgery on knots. The Annals of Mathematics 125 (2): 237-300. https://marc-culler.info/static/home/papers/CyclicSurgery.pdf

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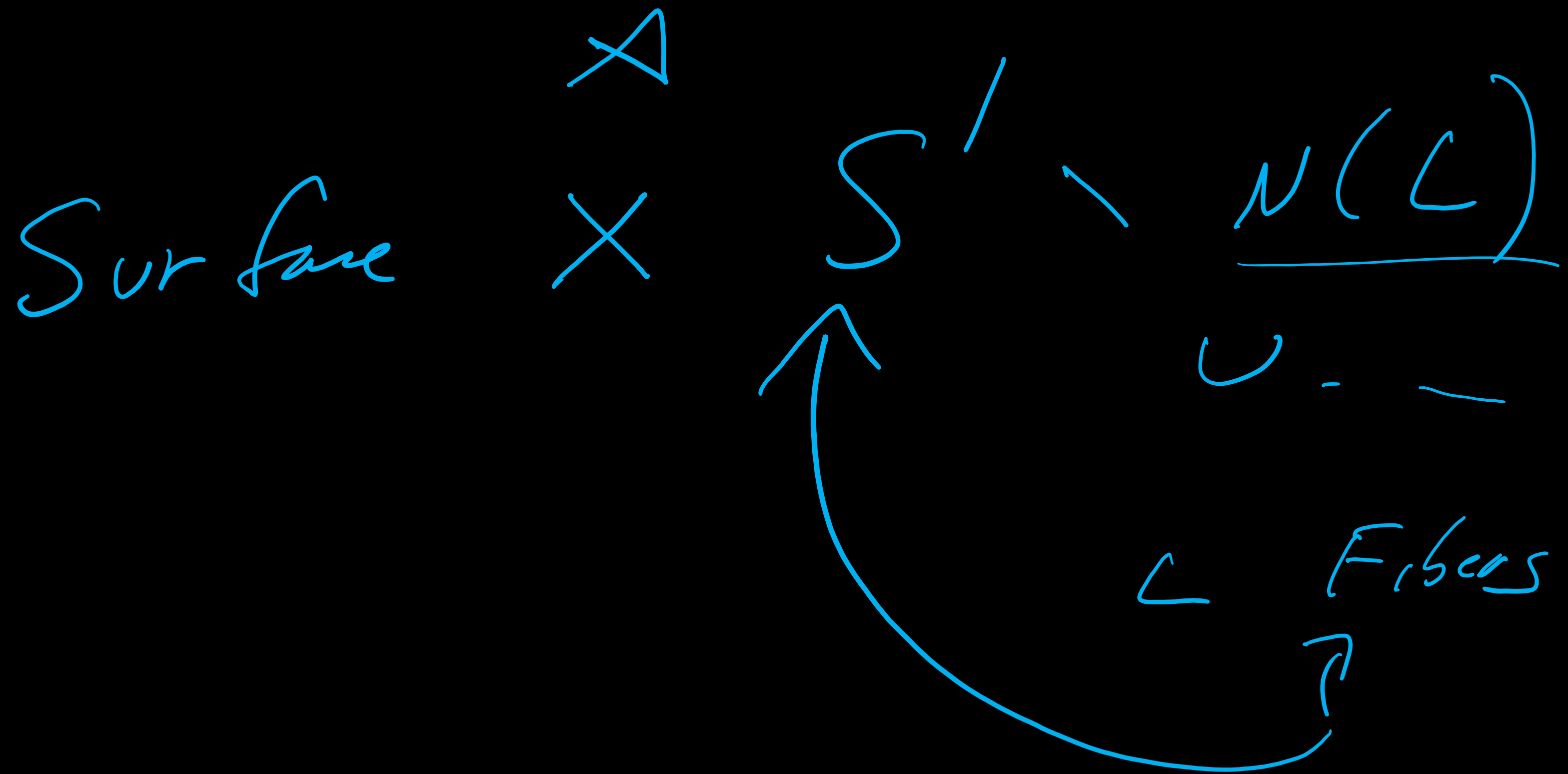

The proof of the Cyclic Surgery Theorem gives a rather stronger result. Let us define a closed 3-manifold L to be *small* if

(*) there exists no incompressible surface in L; and

(**) there exists no representation of $\pi_1(L)$ into $PSL_2(\mathbf{C})$ with non-cyclic image.

Then in both the statement and proof of the Cyclic Surgery Theorem, the hypothesis that $M(r)$ and $M(s)$ have cyclic fundamental groups may be replaced by the condition that they are small. (A connected sum of two non-trivial lens spaces violates (**) because a free product of two cyclic groups is Fuchsian and hence embeds in $PSL_2(\mathbf{R})$.)