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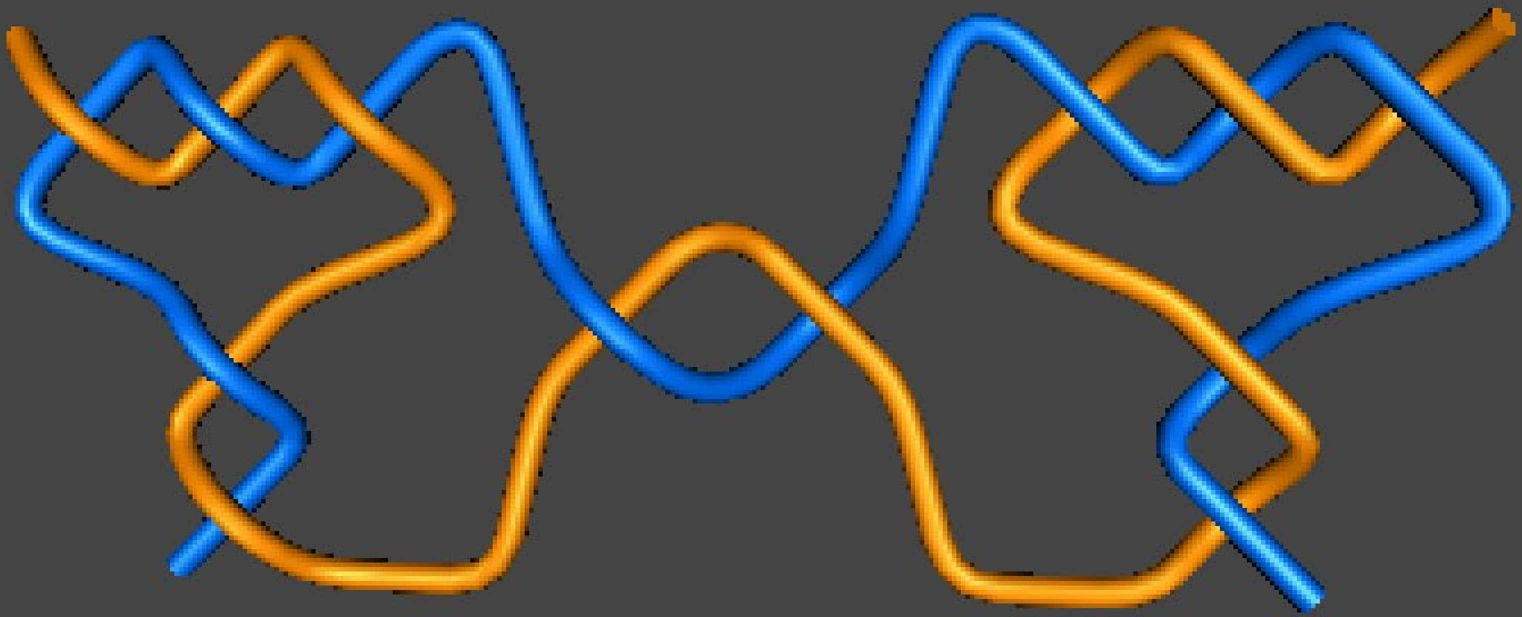
$$(c_1, \ldots, c_{n-1}, c_n + d_k, d_{k-1}, \ldots, d_1)$$

(3, 2, 1 + 1, 2, 3)



KnotPlot> tangle 321o32*1*xz#.

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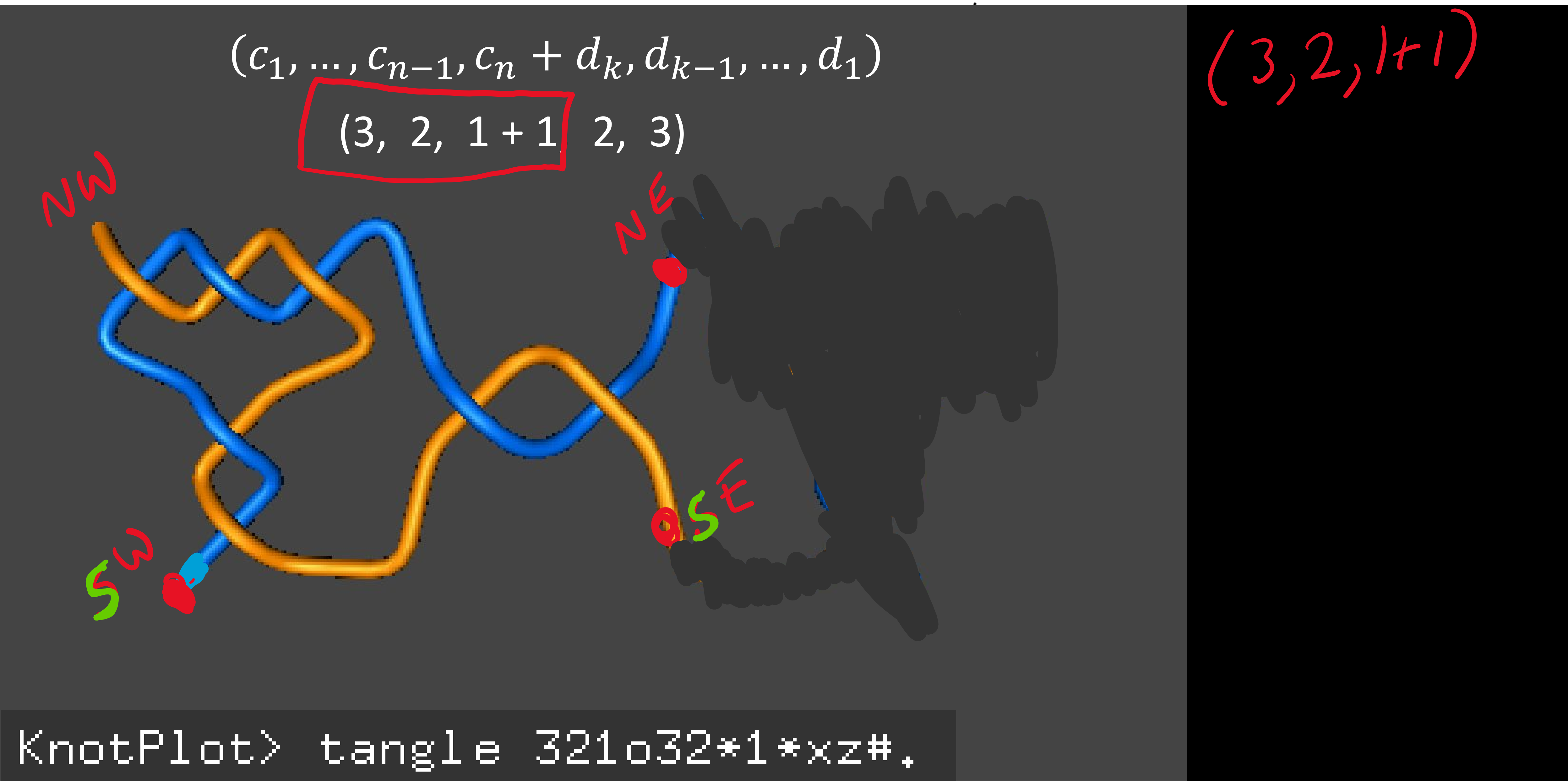

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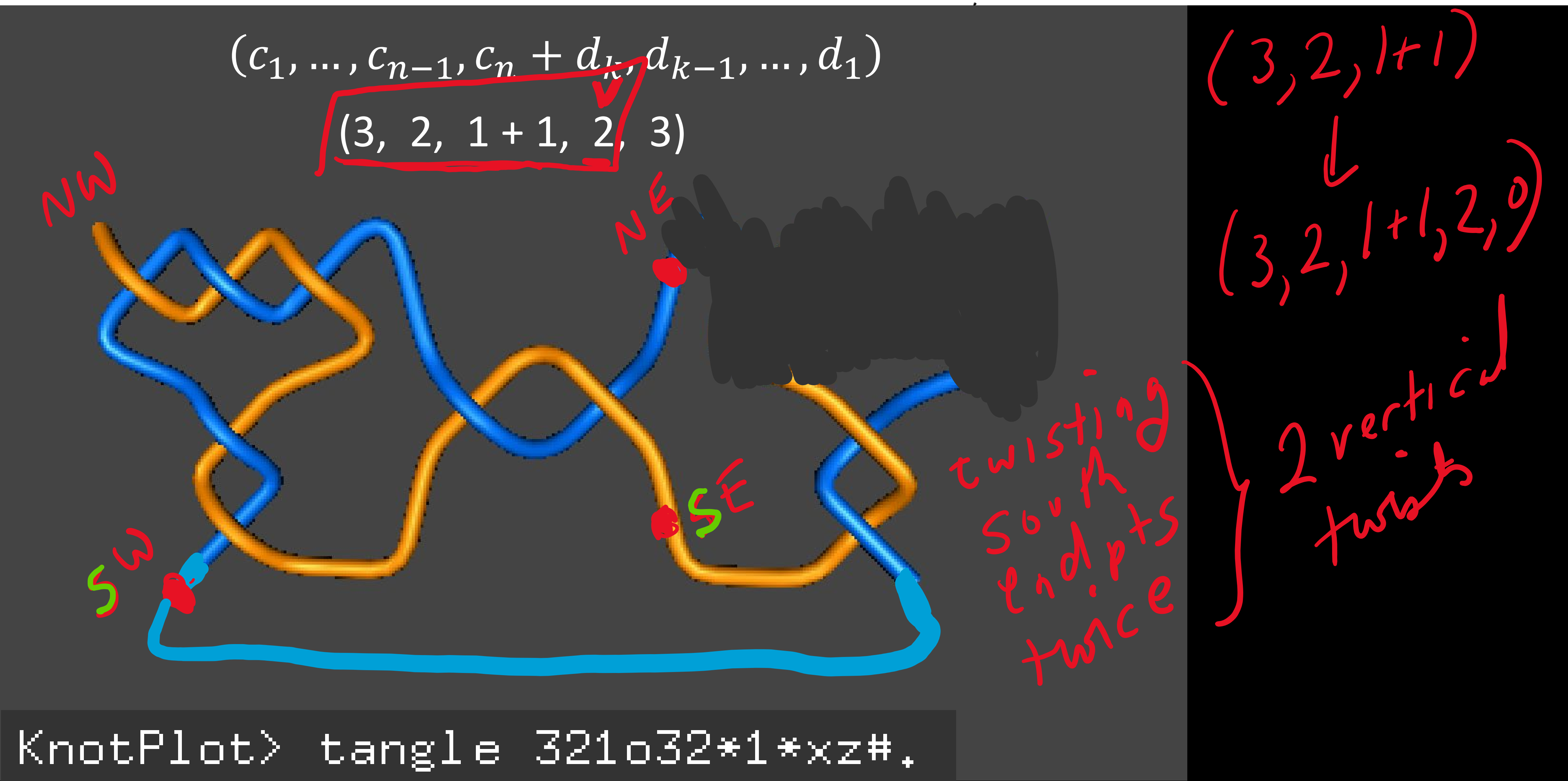

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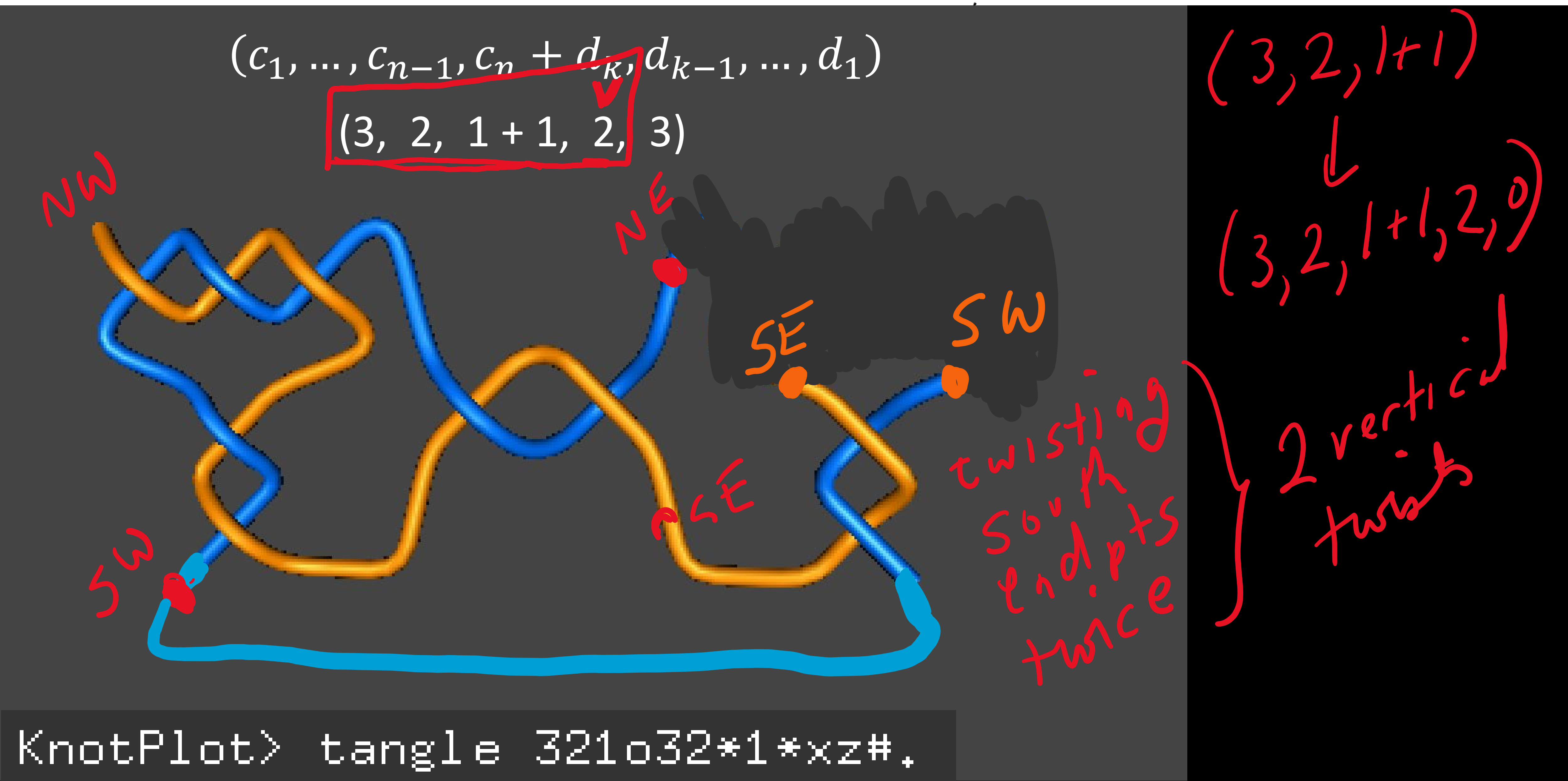

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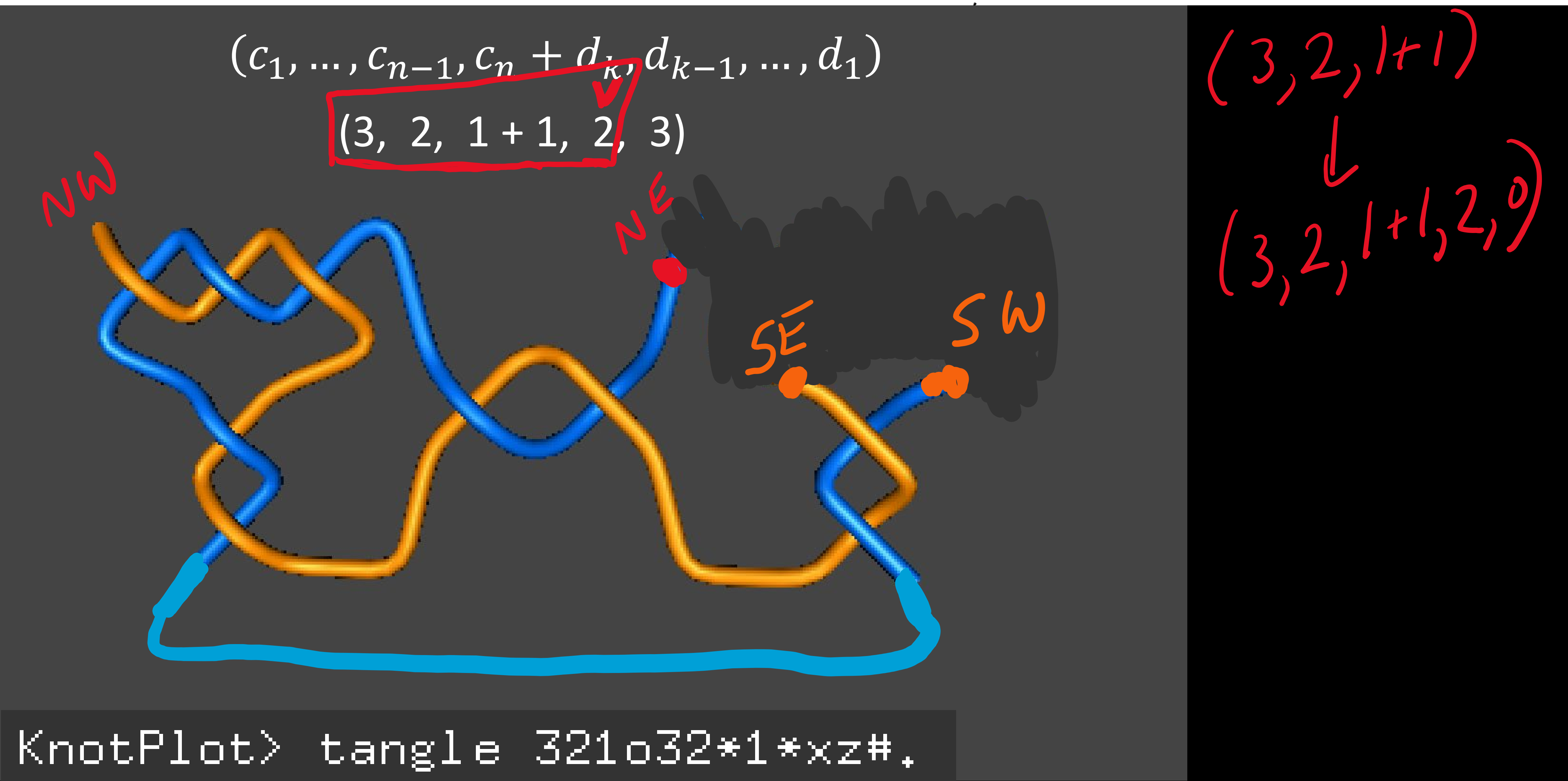

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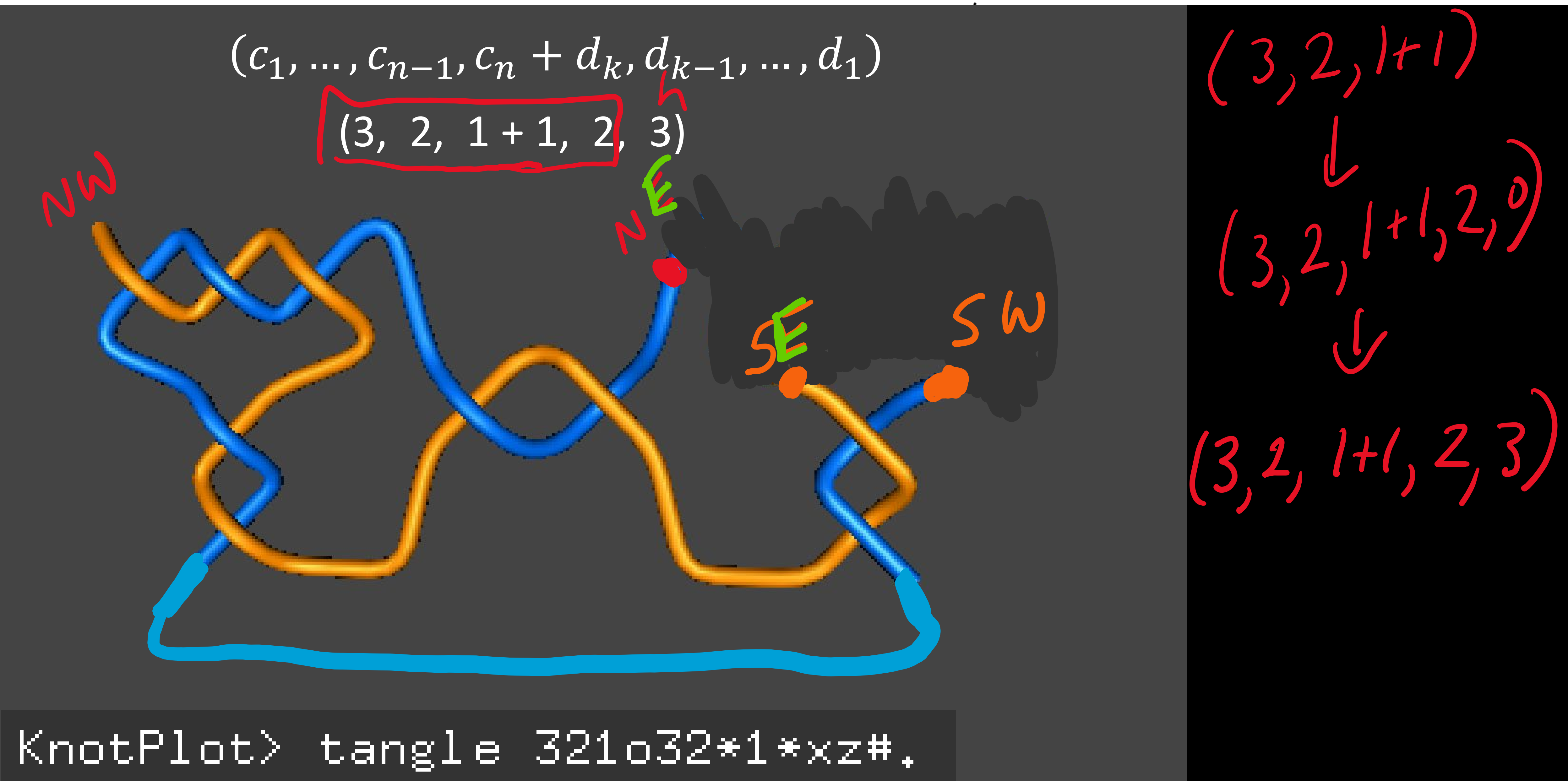

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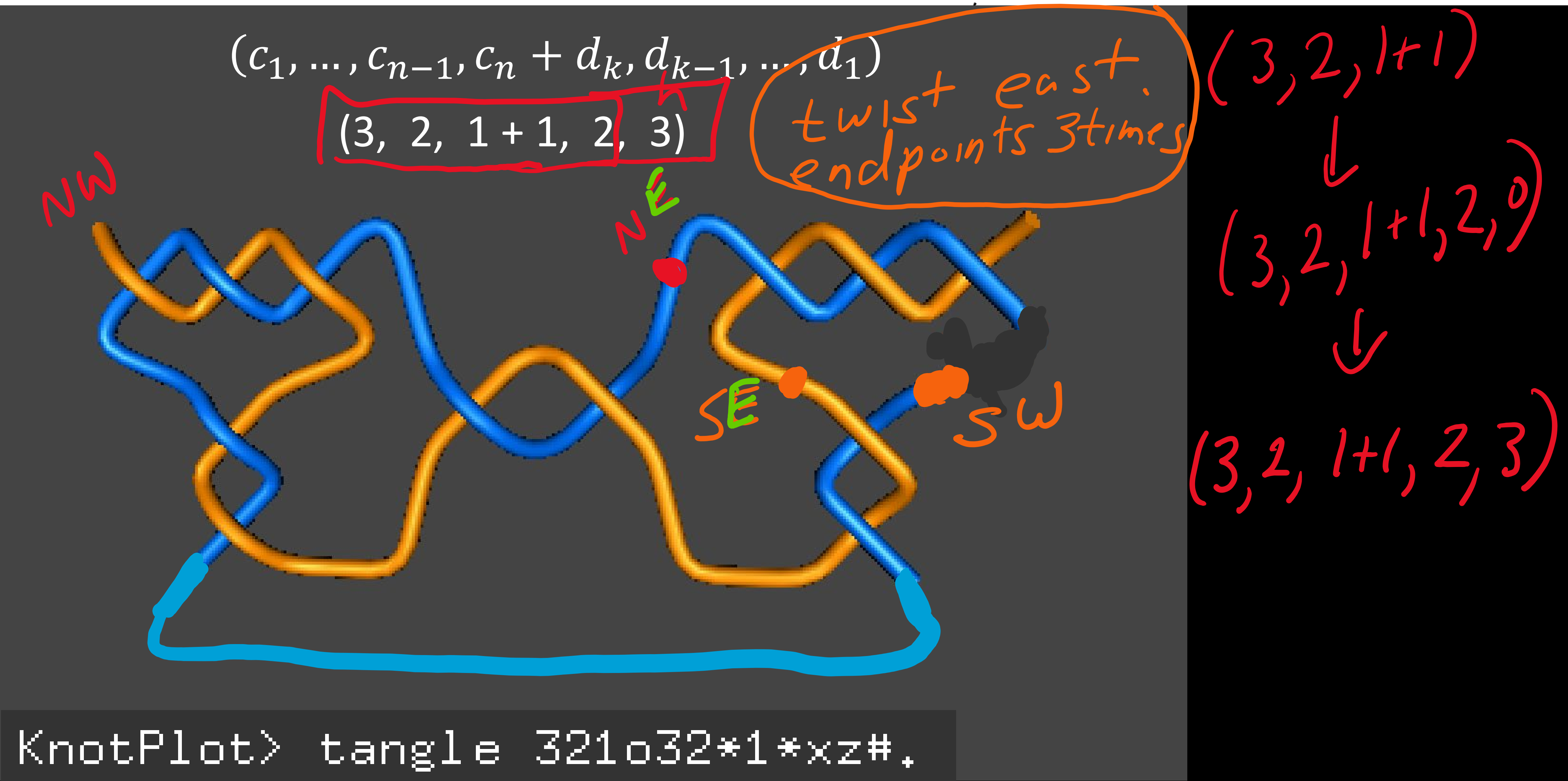

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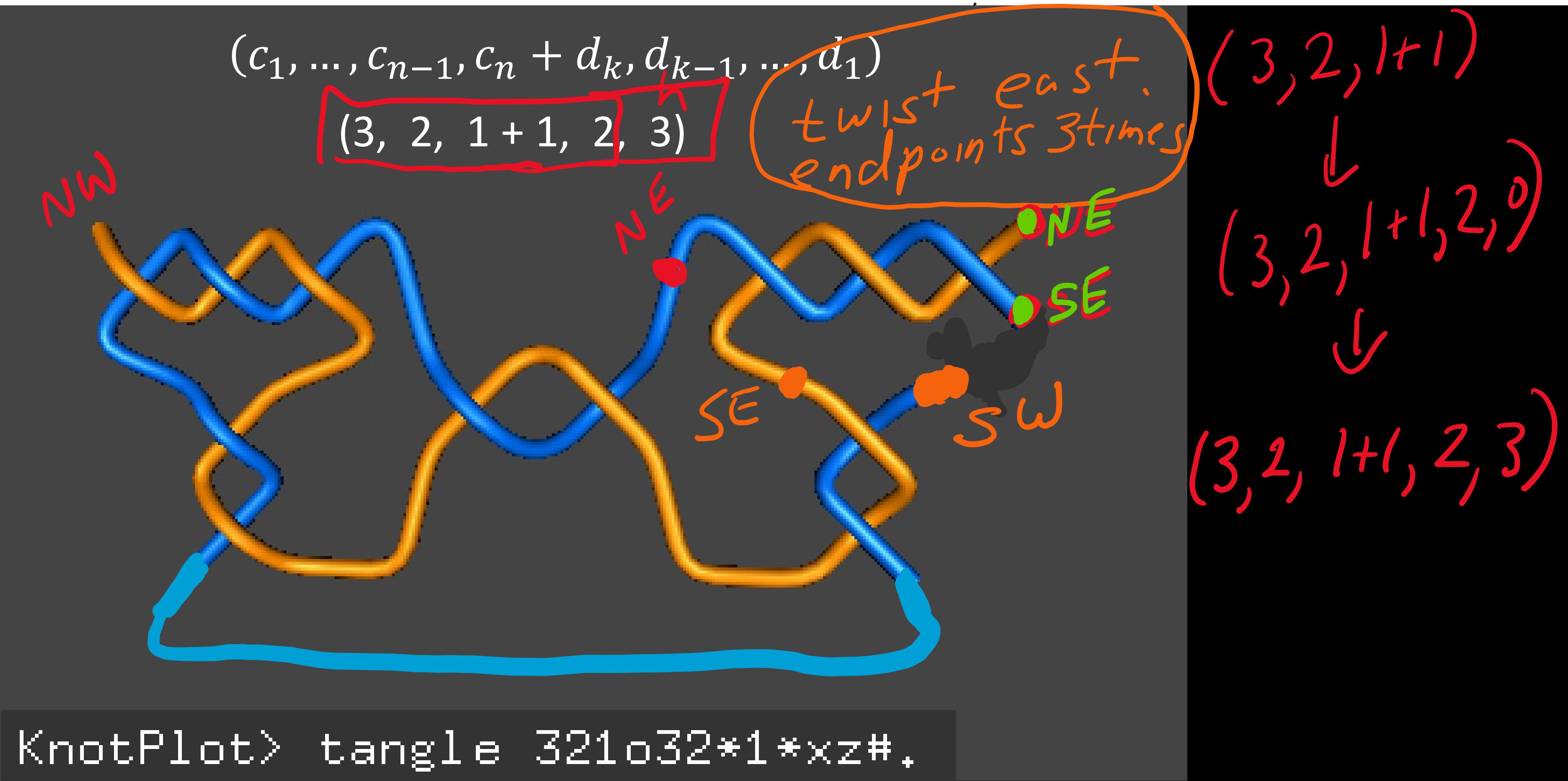

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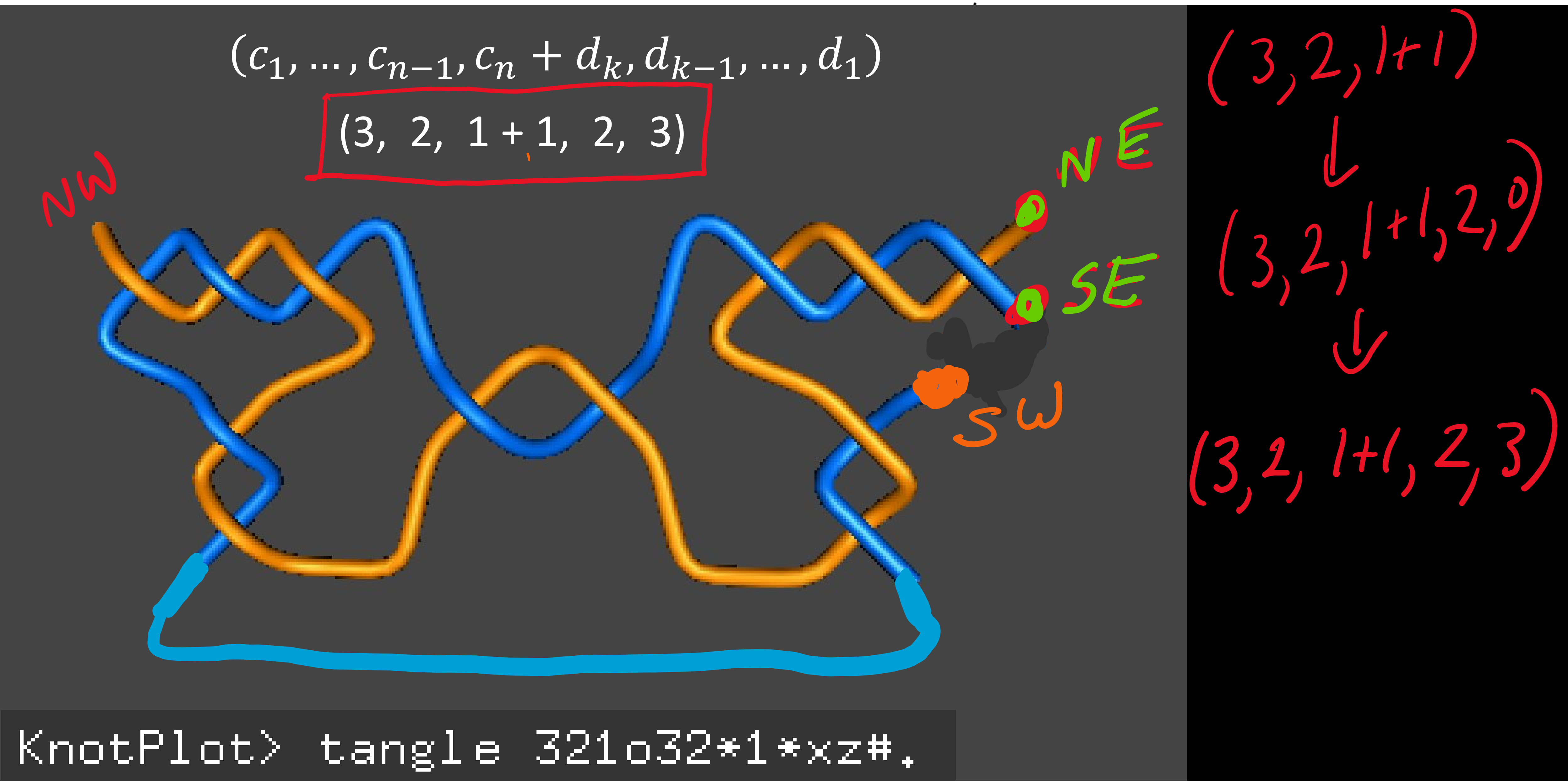

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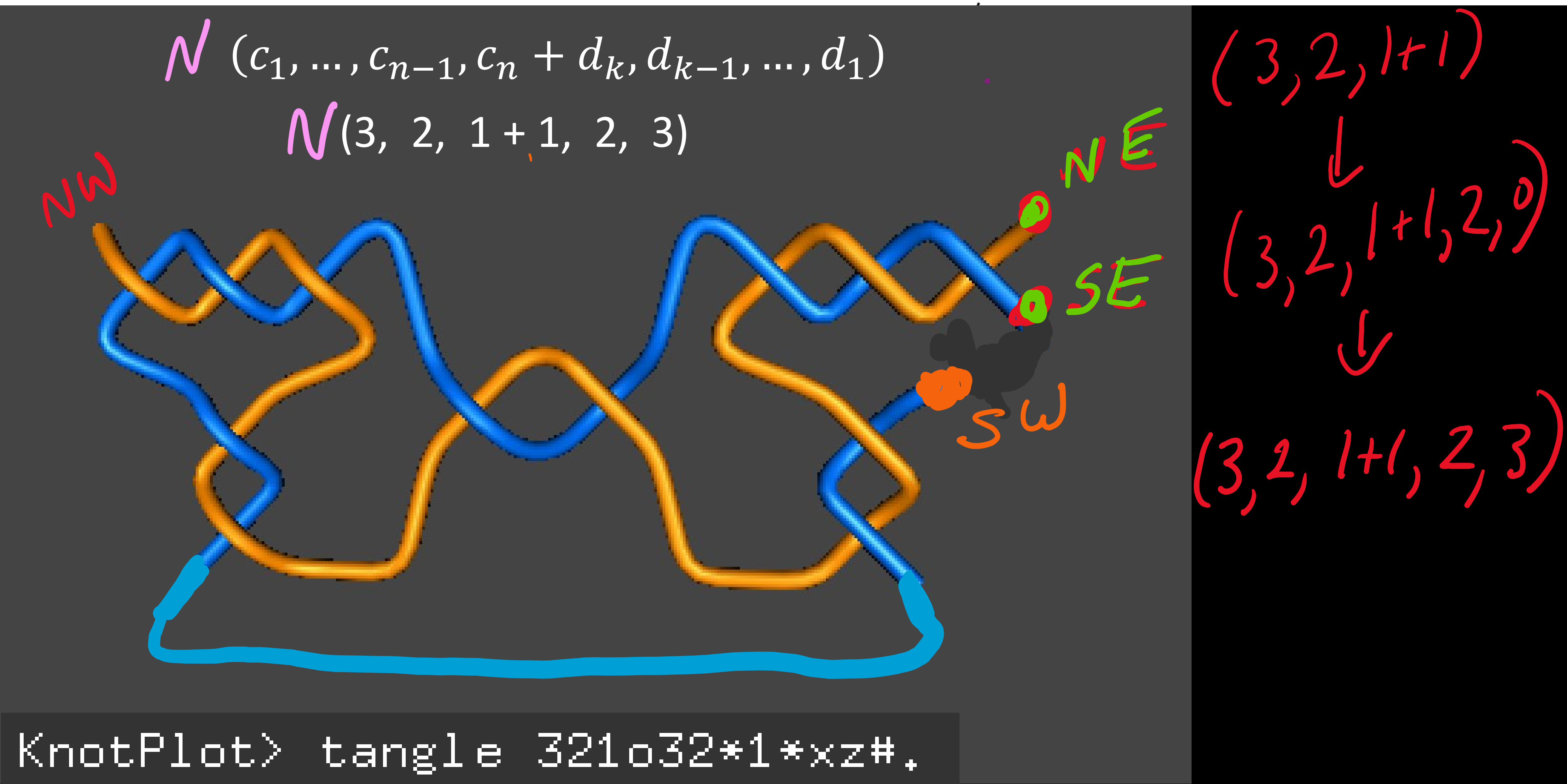

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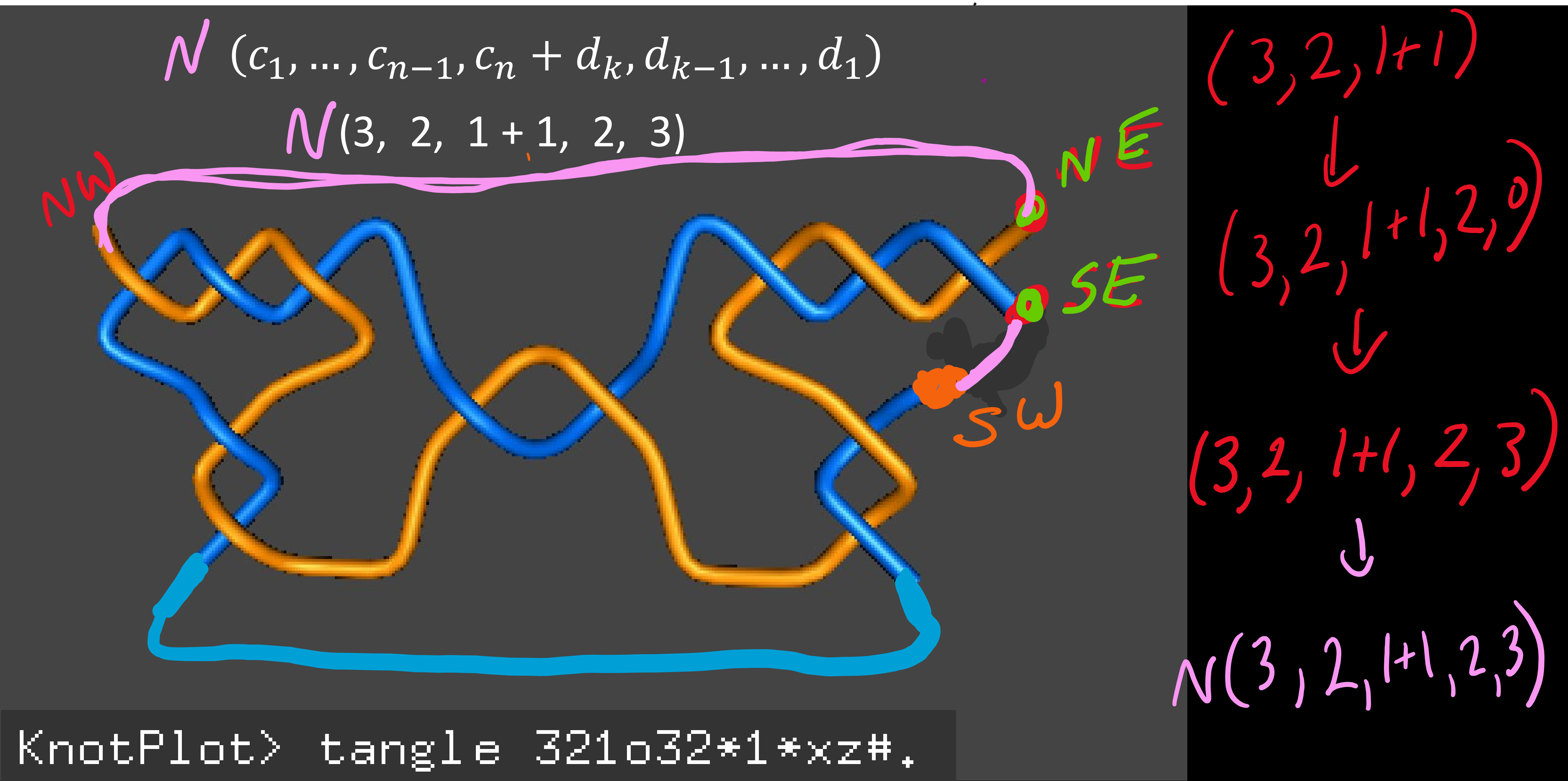

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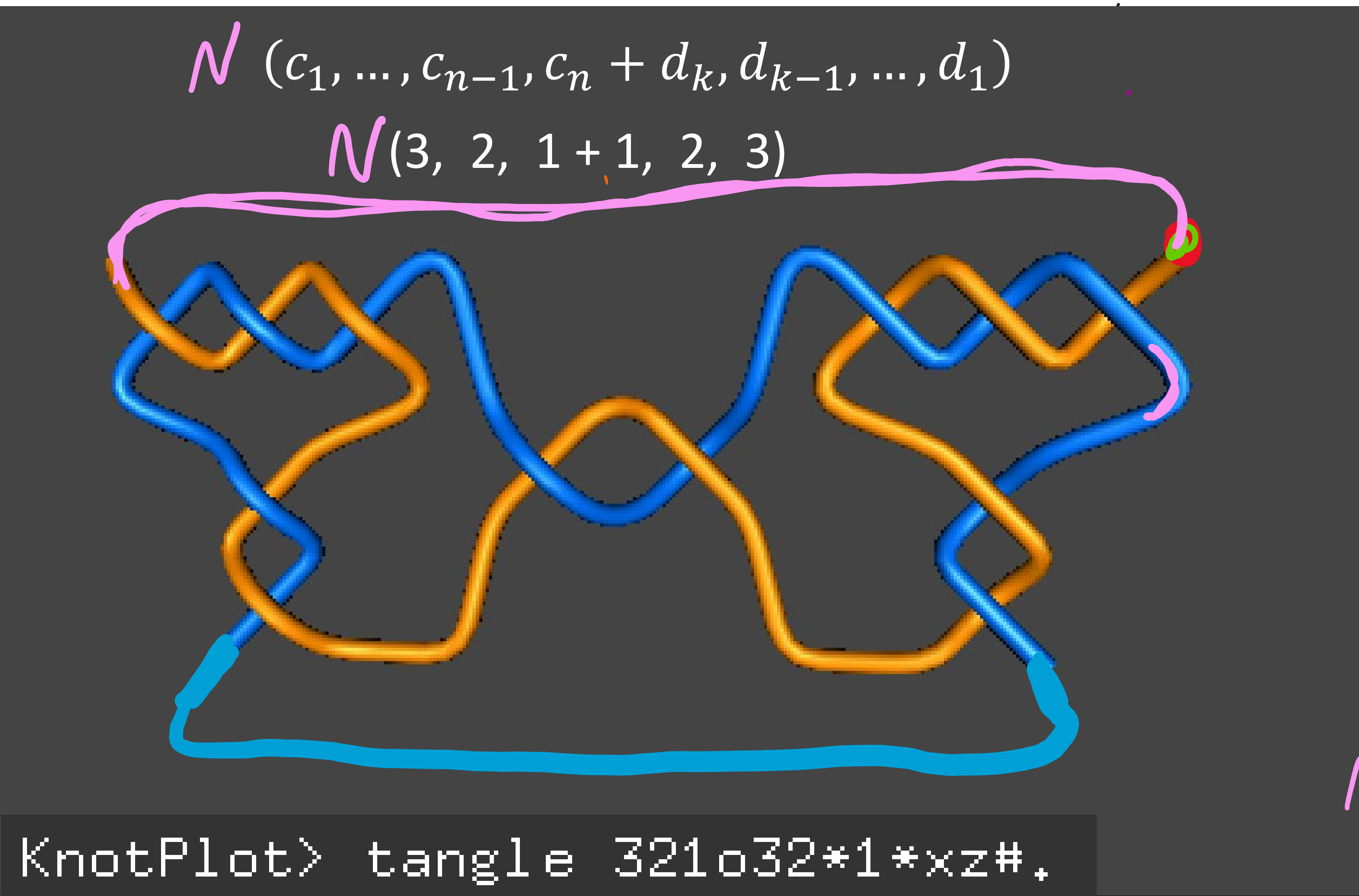

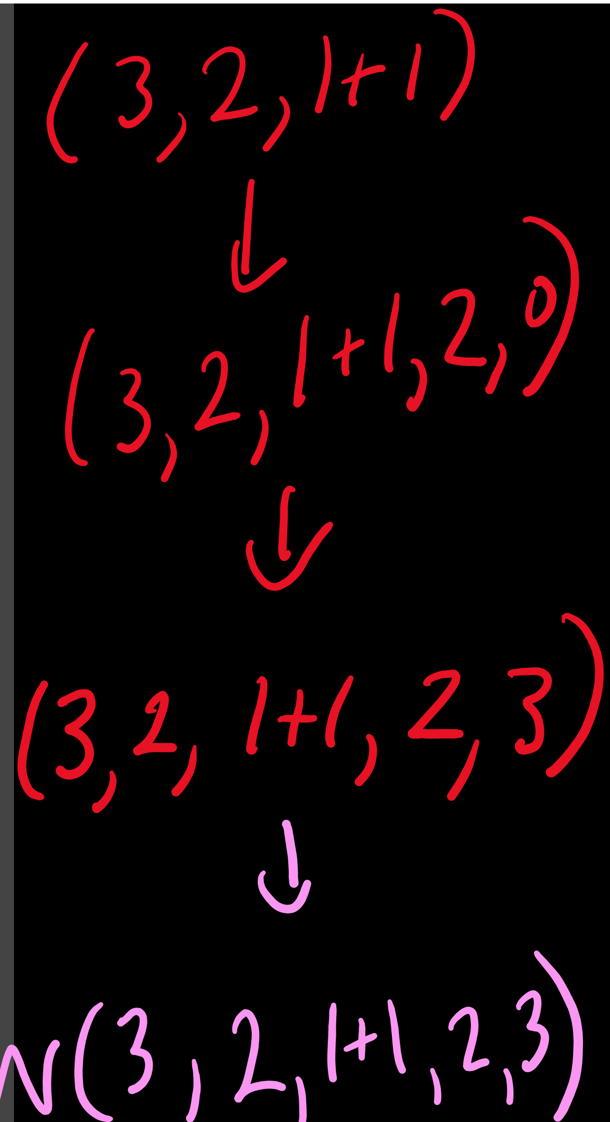

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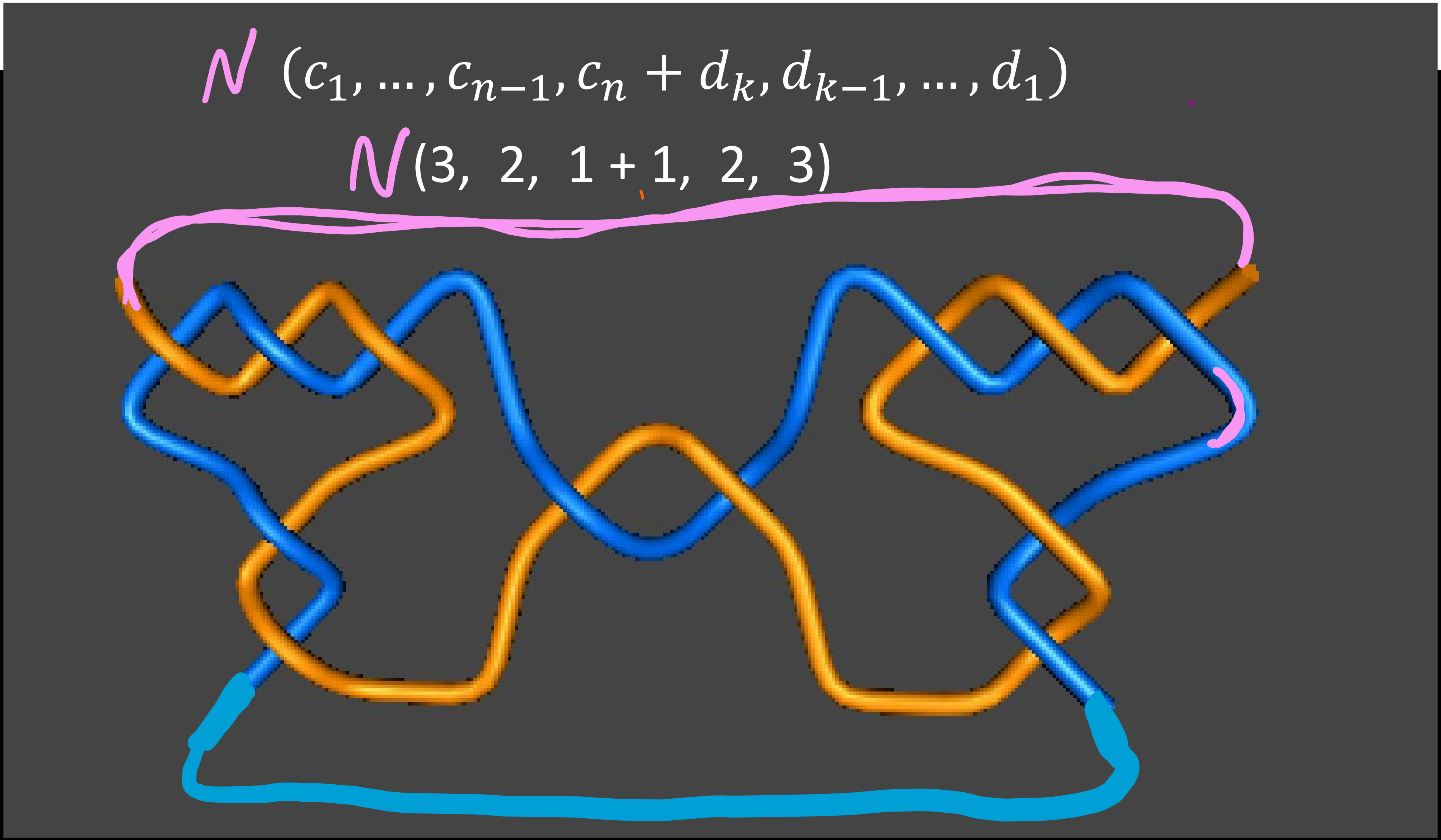
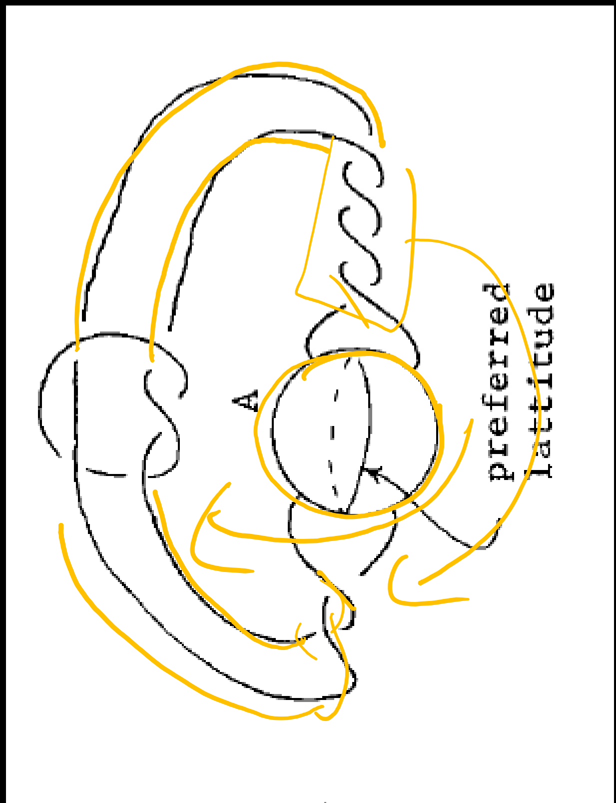
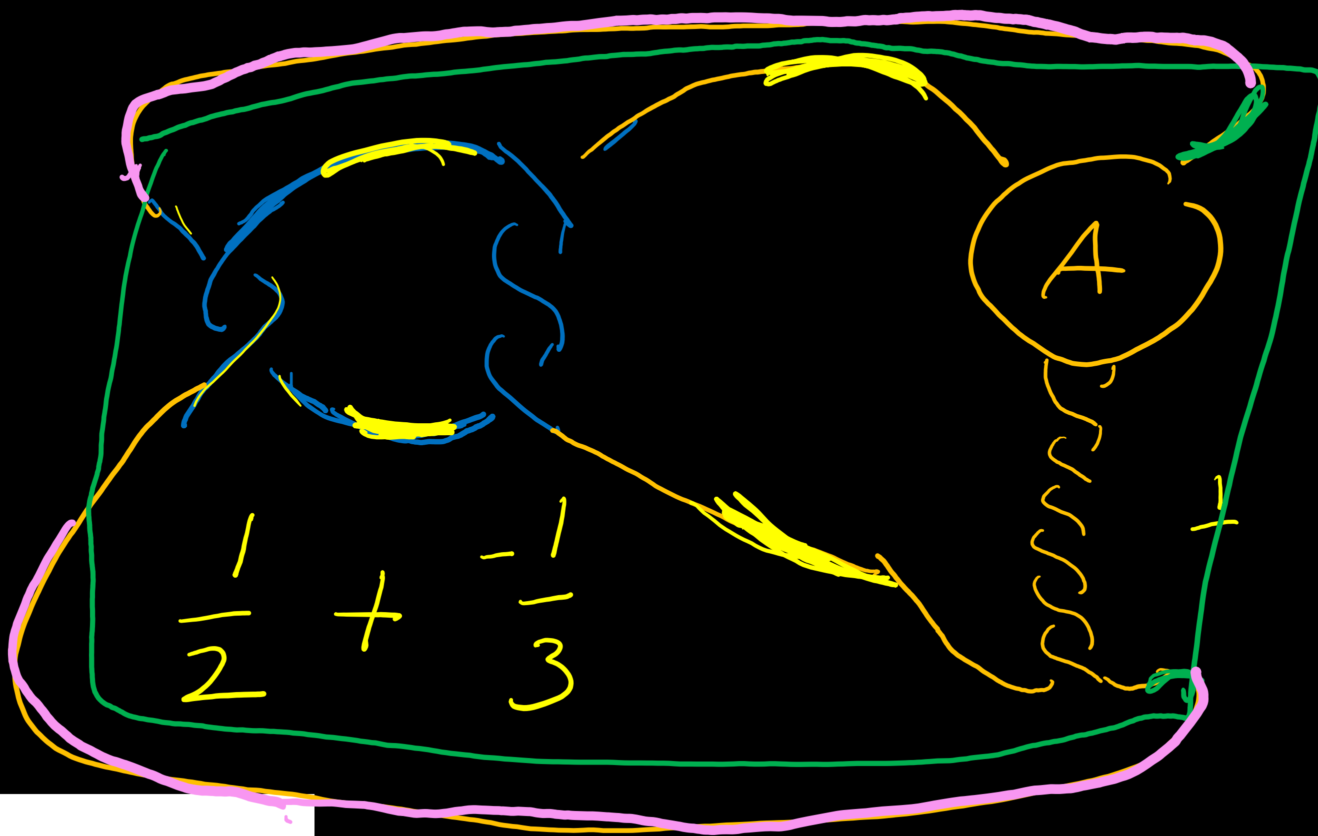


$$[c_1, \ldots, c_n + d_m, \ldots, d_1] = \frac{E[c_1, \ldots, c_n]E[d_1, \ldots d_{m-1}] + E[c_1, \ldots, c_{n-1}]E[d_1, \ldots d_m]}{E[c_2, \ldots, c_n]E[d_1, \ldots d_{m-1}] + E[c_2, \ldots, c_{n-1}]E[d_1, \ldots d_m]}$$





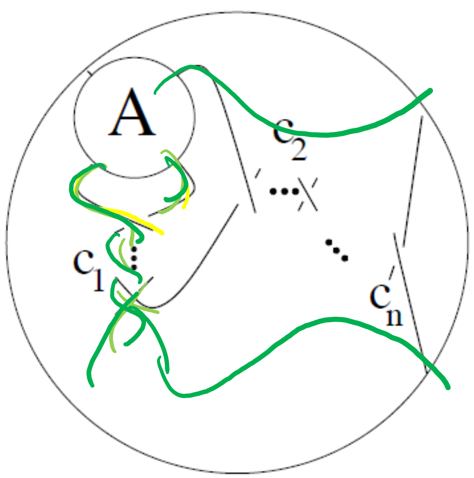
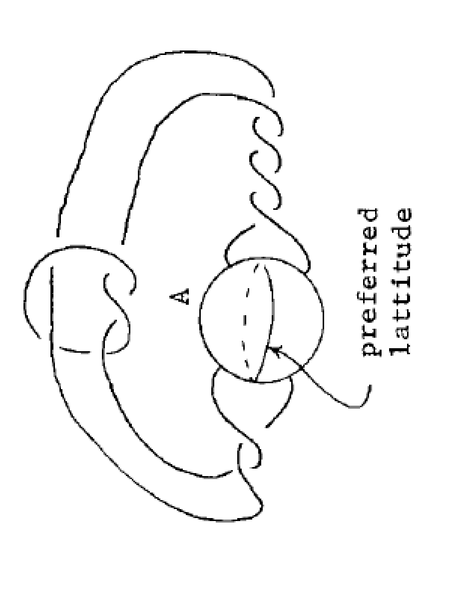


Fig. 5. $A \circ (c_1, ..., c_n)$, n even

$$N\left(\frac{1}{2} + \frac{-1}{3} + A \circ (h, 0)\right)$$

If $A \circ (h,0) = \frac{c}{d}$

Let $d \geq 0$ since $\frac{c}{d} = \frac{-c}{-d}$



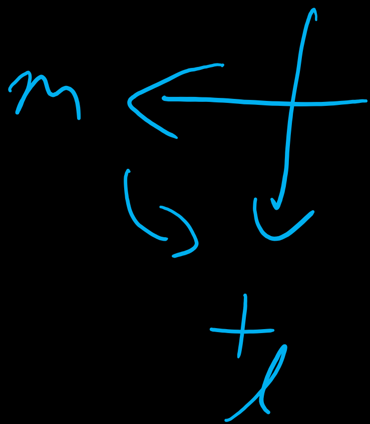
$$N(\tfrac{1}{2} + \tfrac{-1}{3} + \tfrac{c}{d}) = \begin{cases} D(\frac{1}{2})\#D(\frac{-1}{3}) & \text{if } d = 0 \\ Montensinos\ tangle & \text{if } d > 1 \\ N(\frac{1+6c}{2+3c}) & \text{if } d = 1 \end{cases}$$

$$N(\tfrac{1}{2} + \tfrac{-1}{3} + c) = N(\tfrac{1}{2} + \tfrac{-1+3c}{3}) = N(\tfrac{3-2+6c}{3-1+3c}) = N(\tfrac{1+6c}{2+3c})$$

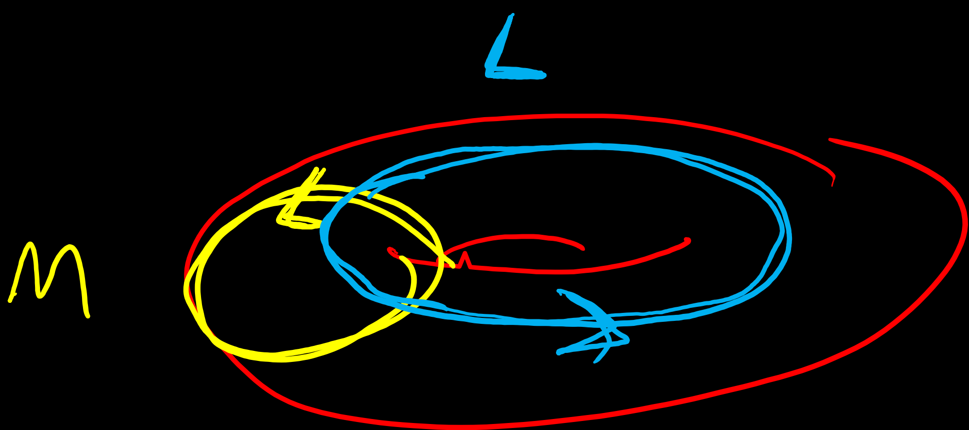
Lemma 3. [8] $N(\frac{j}{p} + \frac{t}{w}) = N(\frac{jw+pt}{dw+qt})$ *where* d *and* q *are any integers such that* pd - qj = *1.*

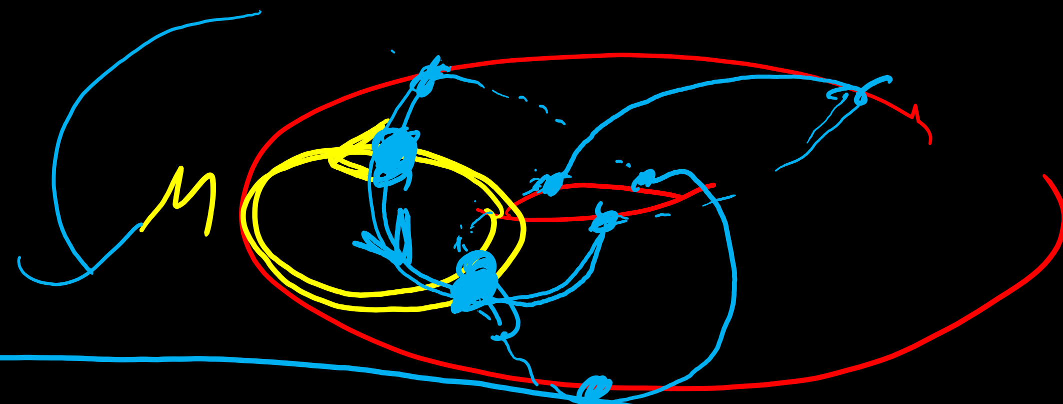
Intersection Number

Figure 3.3: Intersection number.



$$i(2L+3M,\ M) = +2$$





$$i(l,m) = +1$$

$$\det A = \det A^T \begin{vmatrix} 1 & 0 \\ 3 & 2 \end{vmatrix} = \begin{matrix} M \to \\ L \to \end{matrix} \begin{vmatrix} 1 & 3 \\ 0 & 2 \end{vmatrix} = +2$$

$$\begin{pmatrix} 1 & 3 \\ 0 & 1 \end{pmatrix}\begin{pmatrix} 1 & 0 \\ 2 & 1 \end{pmatrix}\begin{pmatrix} 1 & 5 \\ 0 & 1 \end{pmatrix}\begin{pmatrix} 1 & 0 \\ 4 & 1 \end{pmatrix} = \begin{pmatrix} 159 & 38 \\ 46 & 11 \end{pmatrix}$$

$$\begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} \xrightarrow{V^4} \begin{pmatrix} 1 & 0 \\ 4 & 1 \end{pmatrix} \xrightarrow{H^5} \begin{pmatrix} 21 & 5 \\ 4 & 1 \end{pmatrix} \xrightarrow{V^2} \begin{pmatrix} 21 & 5 \\ 46 & 11 \end{pmatrix} \xrightarrow{H^3} \begin{pmatrix} 159 & 38 \\ 46 & 11 \end{pmatrix}$$

Let $A = \begin{pmatrix} 159 & 38 \\ 46 & 11 \end{pmatrix}$

Then A corresponds to an orientation preserving homeomorphism sending

$$M \to 159M + 46L \text{ and } L \to 38M + 11L$$

since $A \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} = \begin{pmatrix} 159 & 38 \\ 46 & 11 \end{pmatrix}$

Thus $1 = det \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} = i(L, M) = i(38M + 11L, 159M + 46L) = det \begin{pmatrix} 159 & 38 \\ 46 & 11 \end{pmatrix}$

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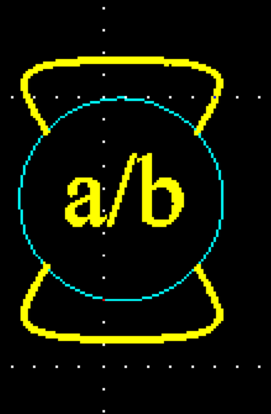
Since A^{-1} is an orientation preserving homeomorphism, $det(A^{-1}) = 1$

$$A^{-1} \begin{pmatrix} 159 & b \\ 46 & a \end{pmatrix} = \begin{pmatrix} 1 & ? \\ 0 & 159a - 46b \end{pmatrix}$$

$i(bM + aL, 159M + 46L) = i(?M + (159a - 46b)L, M) = 159a - 46b = det \begin{pmatrix} 159 & b \\ 46 & a \end{pmatrix}$

DEFINITION $L(a,b) = V_1 \cup_h V_2$ where $h : \partial V_2 \to \partial V_1$ is an orientation preserving homeomorphism and $h(M_2) = aL_1 + bM_1$.

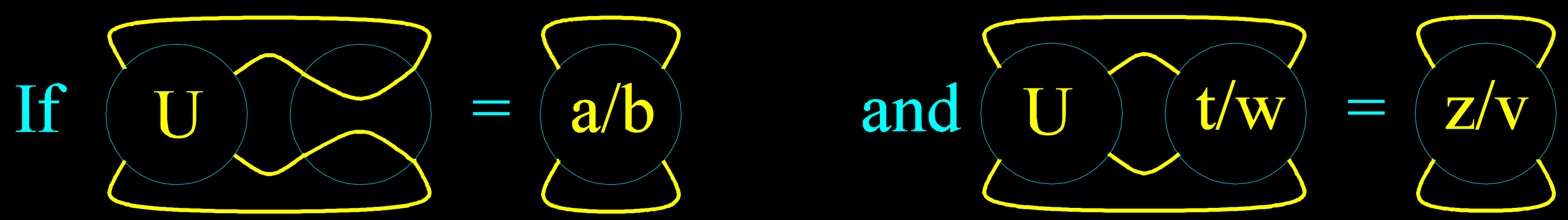


M. Culler, C. Gordon, J. Luecke, P. Shalen (1987). Dehn surgery on knots. The Annals of Mathematics 125 (2): 237-300. https://marc-culler.info/static/home/papers/CyclicSurgery.pdf

Cyclic Surgery Theorem. *Suppose that M is not a Seifert fibered space. If $\pi_1(M(r))$ and $\pi_1(M(s))$ are cyclic, then $\Delta(r, s) \leq 1$. Hence there are at most three slopes r such that $\pi_1(M(r))$ is cyclic.*

f(m') = curve of slope *r* ~ *pl + qm*; *r = q/p* ,

in terms of a bases: *l* = longitude and *m* = meridian of boundary of *N(K)*



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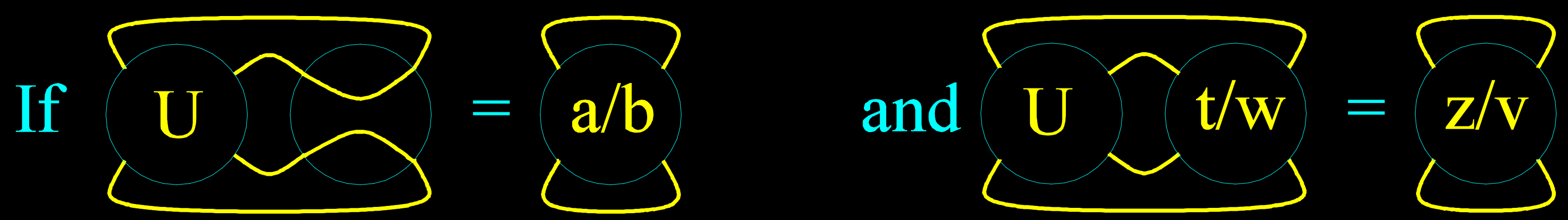

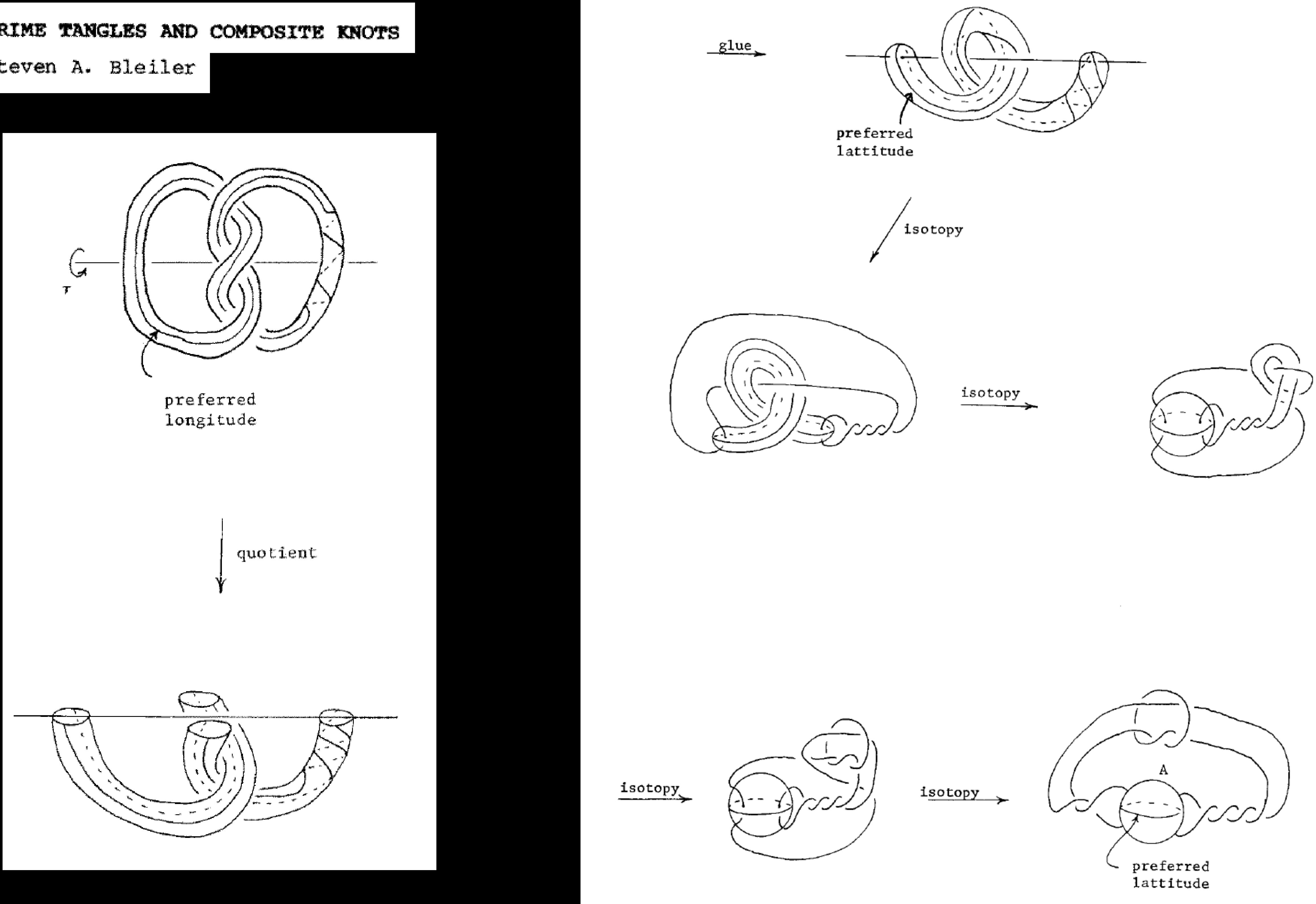
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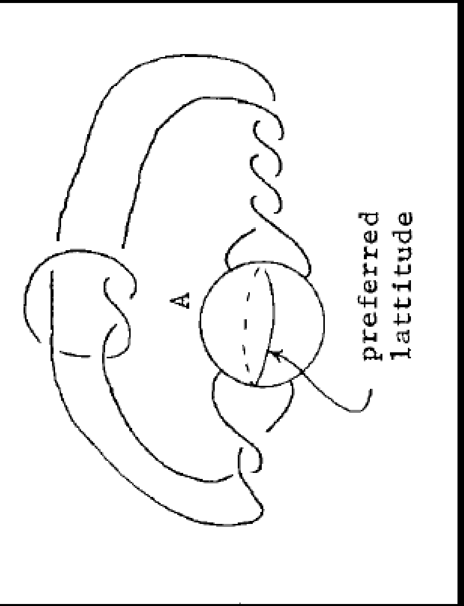

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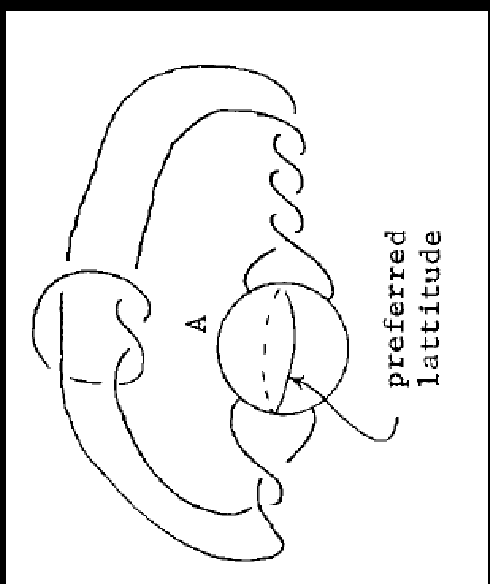
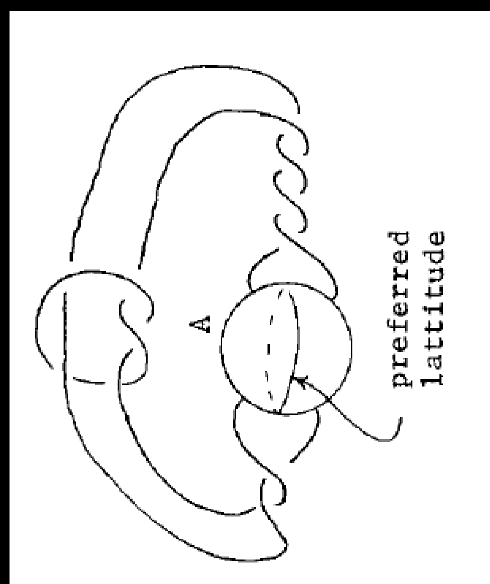
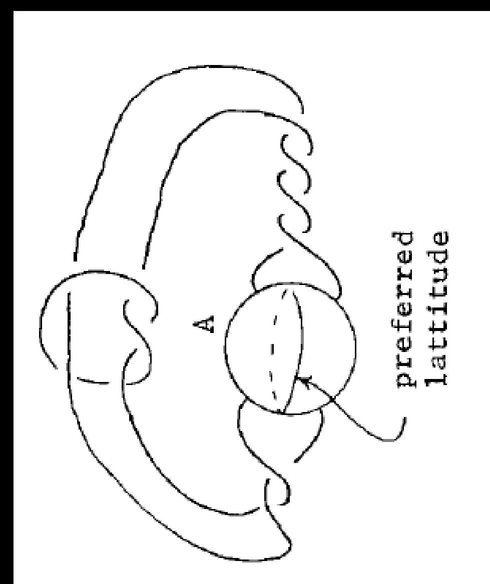
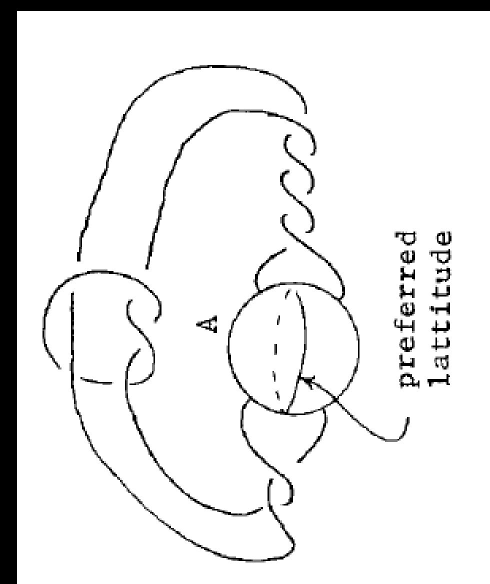
$$N\left(\frac{1+6c}{2+3c}\right)$$









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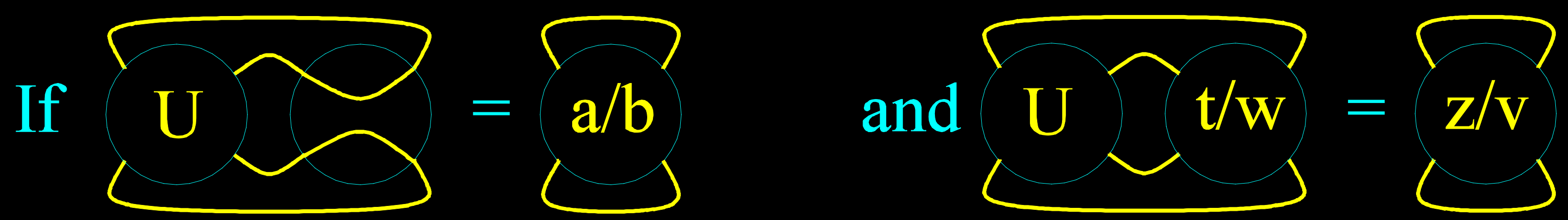
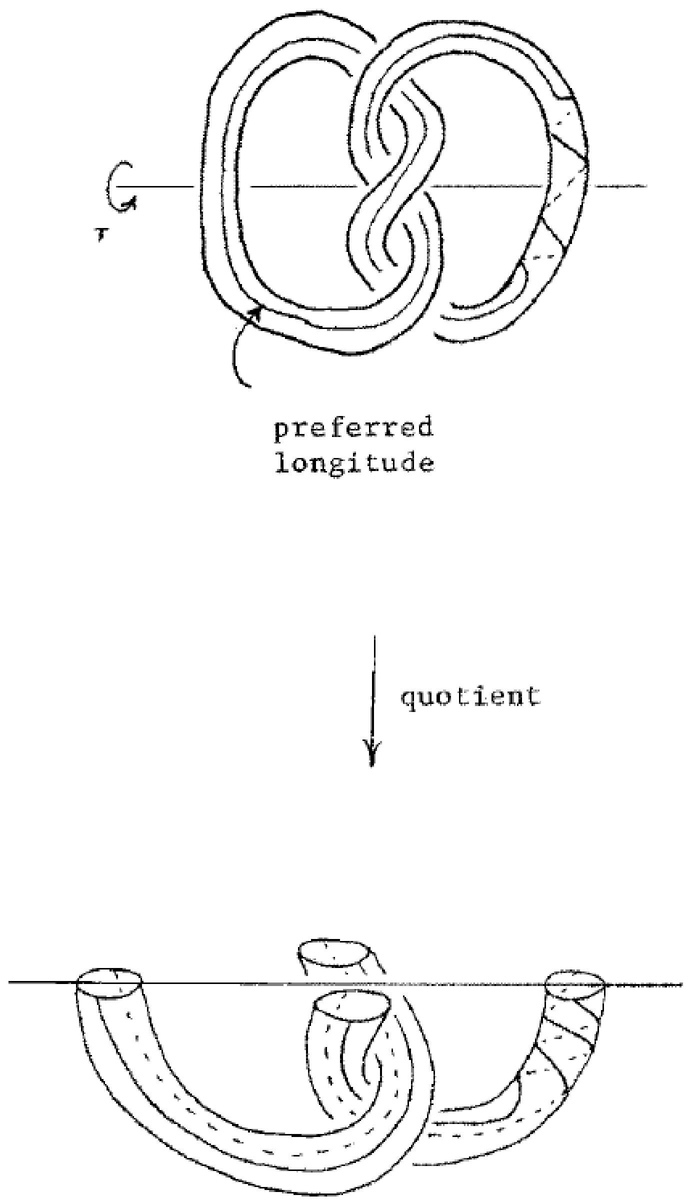
f(m') = curve of slope *r* ~ *pl* + *qm*; *r* = *q/p* ,
in terms of a bases: *l* = longitude and *m* = meridian of boundary of *N(K)*

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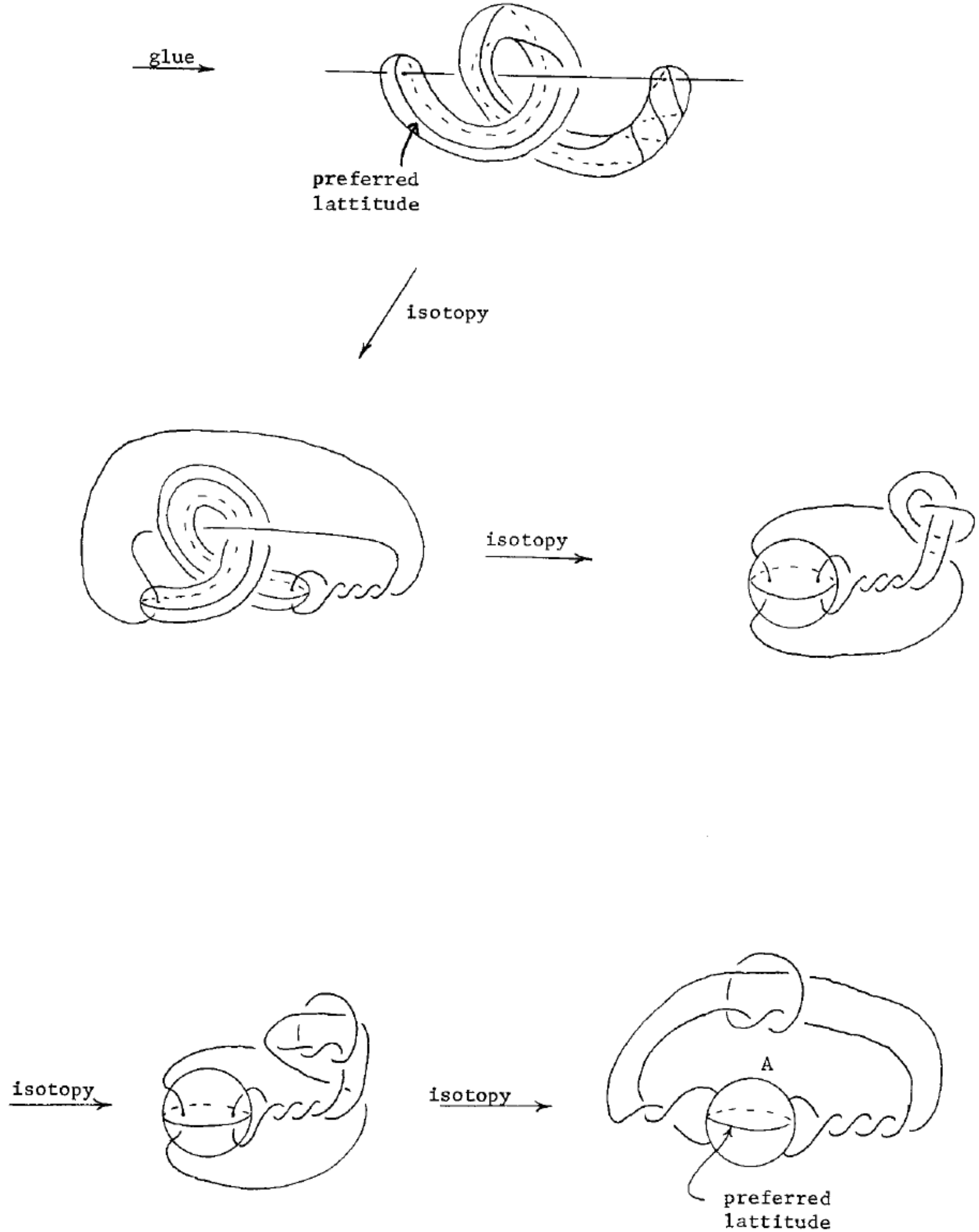
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http://dx.doi.org/10.1090/conm/044/813107

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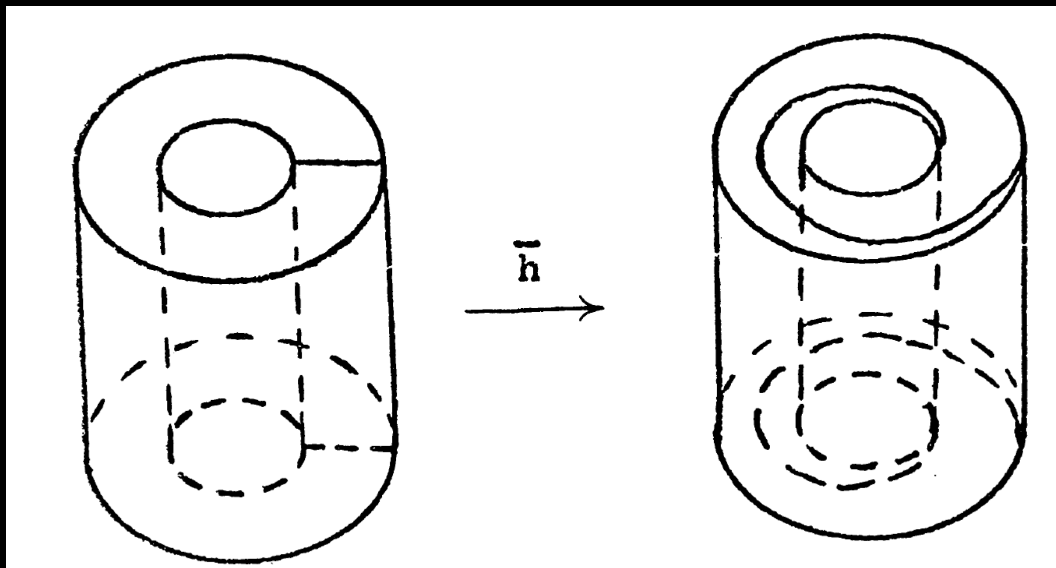
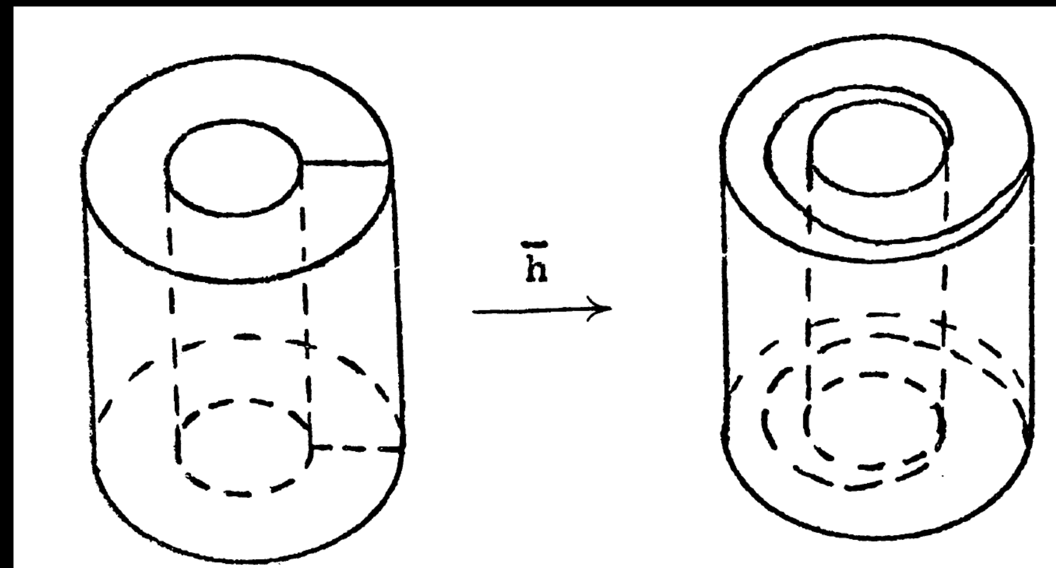
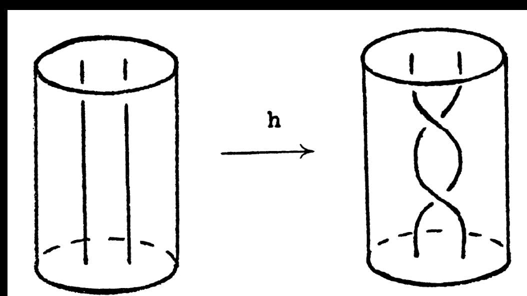
Volume **44**, 1985

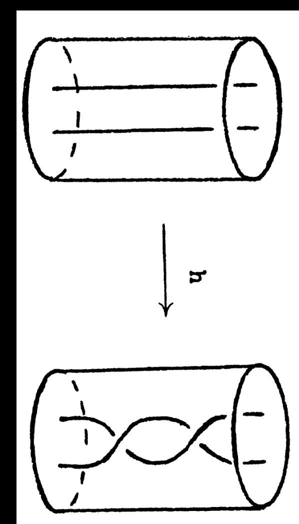
THE UNKNOTTING NUMBER OF A CLASSICAL KNOT

W. B. Raymond Lickorish

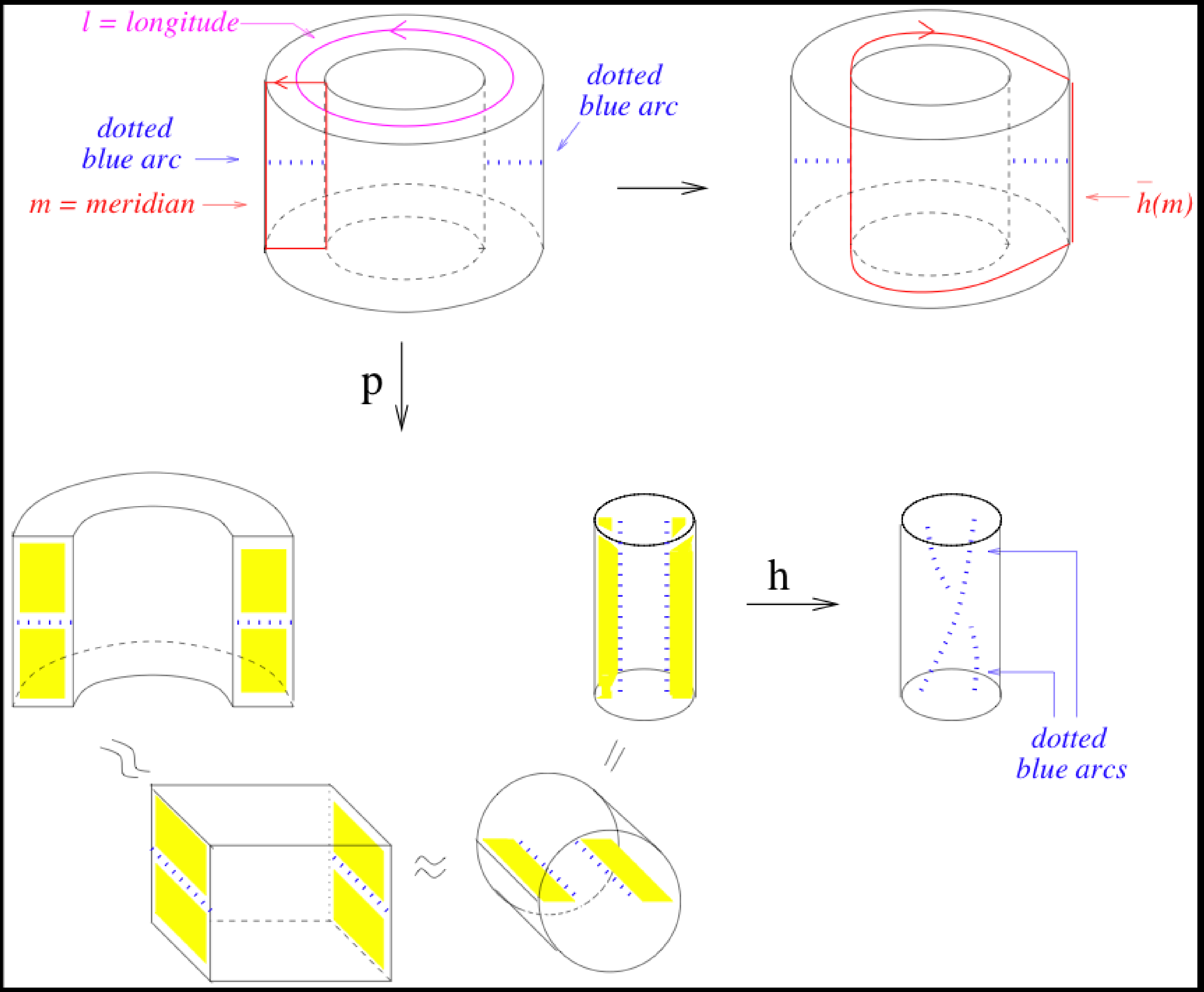








LEMMA 1. *If k has unknotting number equal to one, then M_k is obtained by $n/2$-surgery on some knot in S^3, n being an odd integer.*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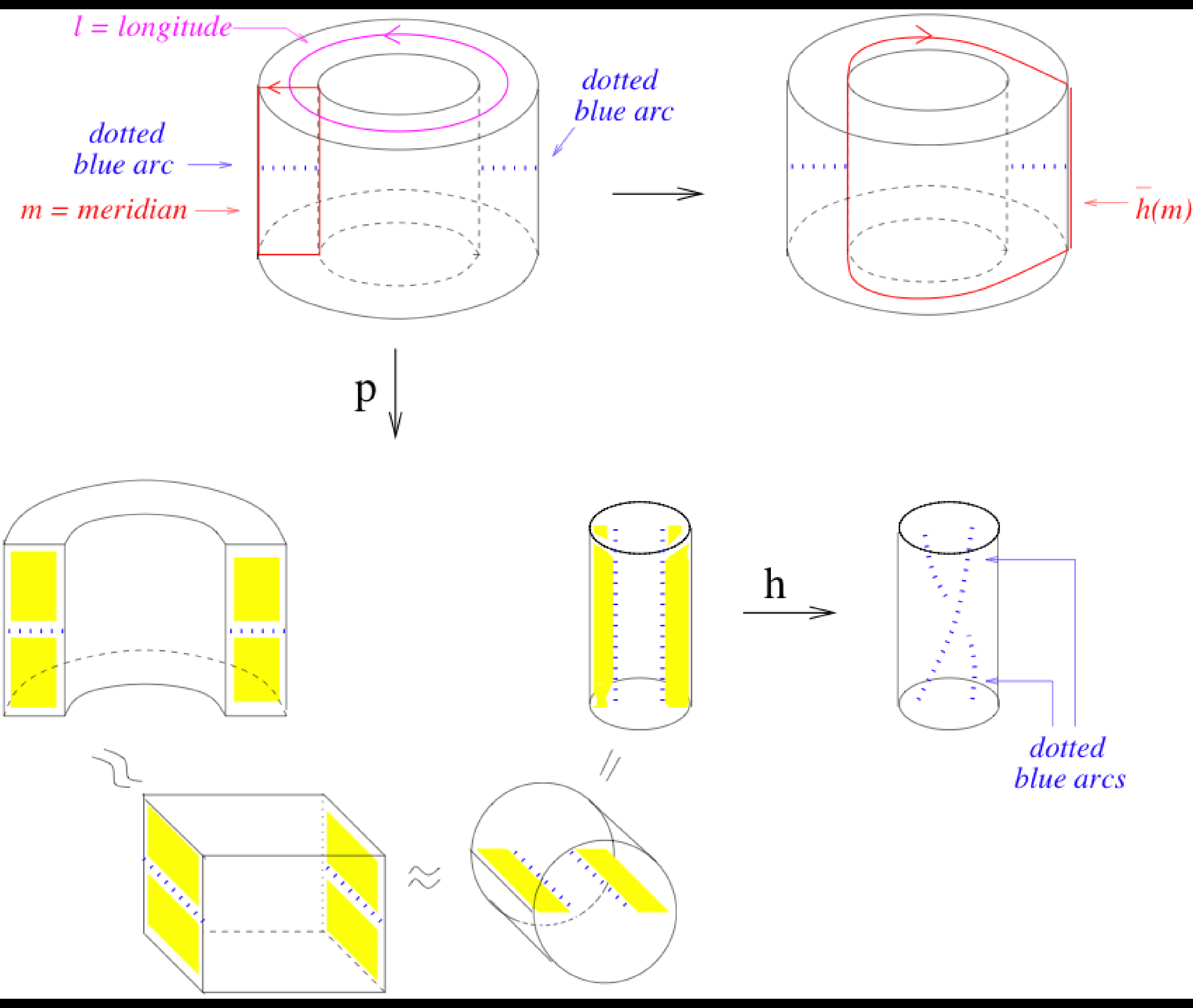
l = longitude
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blue arc
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p
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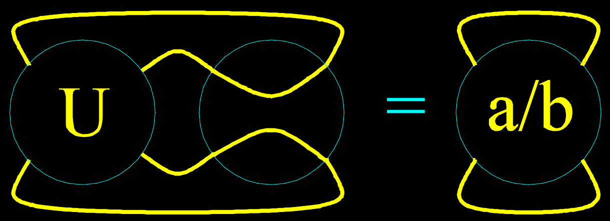
dotted
blue arcs

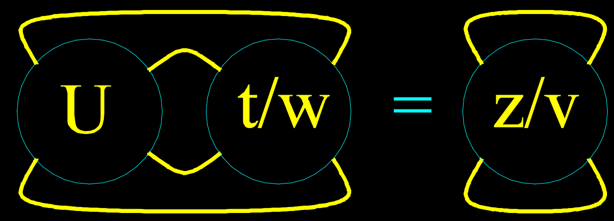
M. Culler, C. Gordon, J. Luecke, P. Shalen (1987). Dehn surgery on knots. The Annals of Mathematics 125 (2): 237-300. https://marc-culler.info/static/home/papers/CyclicSurgery.pdf

CYCLIC SURGERY THEOREM. *Suppose that M is not a Seifert fibered space. If* $\pi_1(M(r))$ *and* $\pi_1(M(s))$ *are cyclic, then* $\Delta(r, s) \leq 1$. *Hence there are at most three slopes r such that* $\pi_1(M(r))$ *is cyclic.*

f(m') = curve of slope *r* ~ *pl* + *qm*; *r* = *q/p* ,

in terms of a bases: *l* = longitude and *m* = meridian of boundary of *N(K)*

If 

and 

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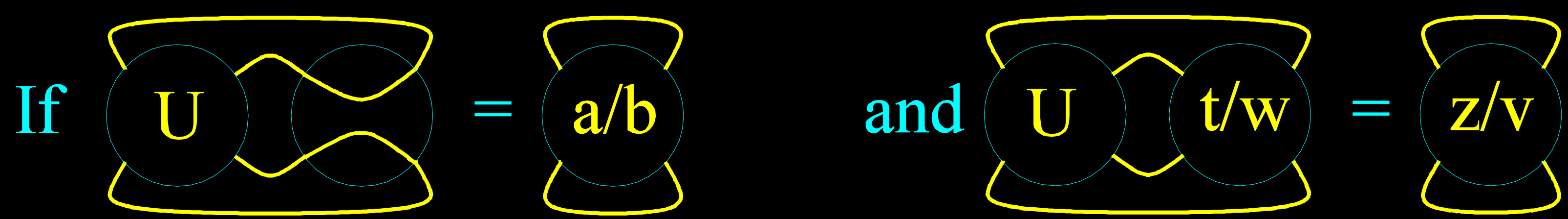
CYCLIC SURGERY THEOREM. *Suppose that M is not a Seifert fibered space. If $\pi_1(M(r))$ and $\pi_1(M(s))$ are cyclic, then $\Delta(r, s) \leq 1$. Hence there are at most three slopes r such that $\pi_1(M(r))$ is cyclic.*



The proof of the Cyclic Surgery Theorem gives a rather stronger result. Let us define a closed 3-manifold L to be *small* if

(*) there exists no incompressible surface in L; and

(**) there exists no representation of $\pi_1(L)$ into $PSL_2(\mathbf{C})$ with non-cyclic image.

Then in both the statement and proof of the Cyclic Surgery Theorem, the hypothesis that $M(r)$ and $M(s)$ have cyclic fundamental groups may be replaced by the condition that they are small. (A connected sum of two non-trivial lens spaces violates (**) because a free product of two cyclic groups is Fuchsian and hence embeds in $PSL_2(\mathbf{R})$.)