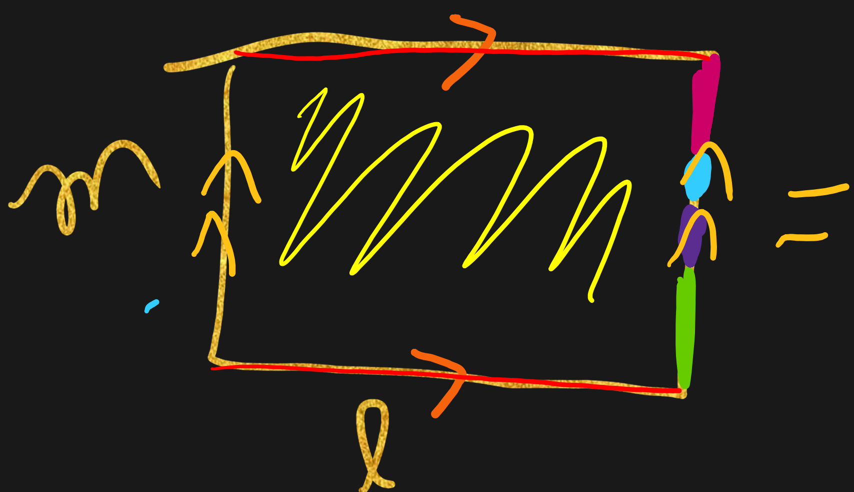
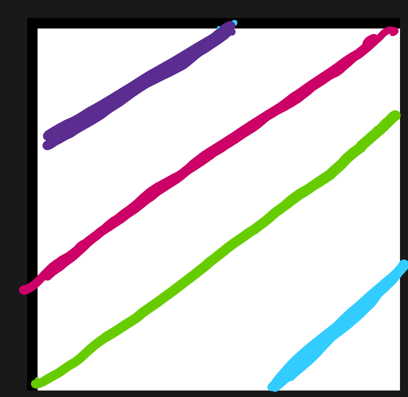
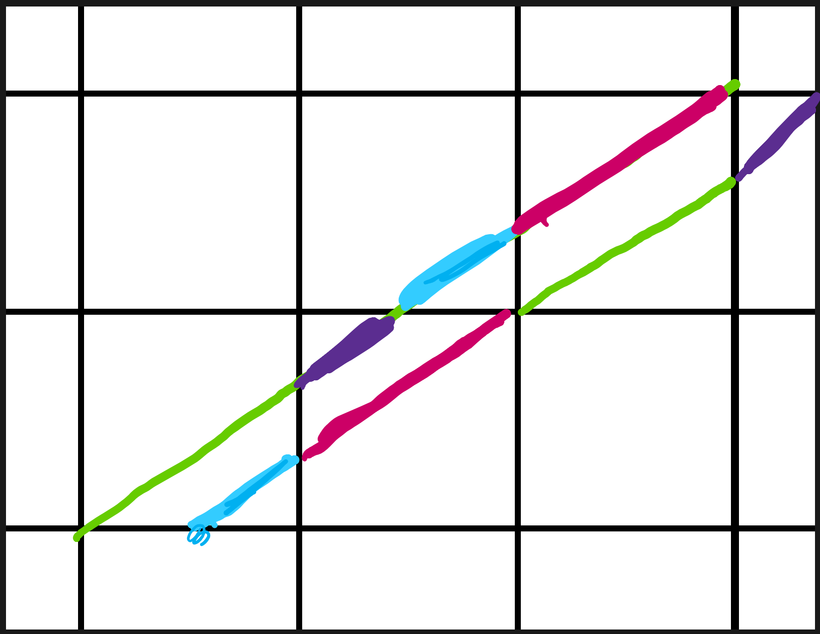
$$m \to 3l + 2m$$

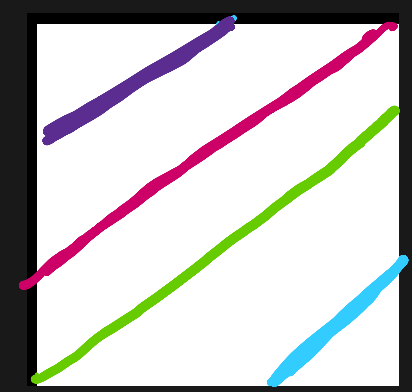
$$T^2 = 2V^3$$

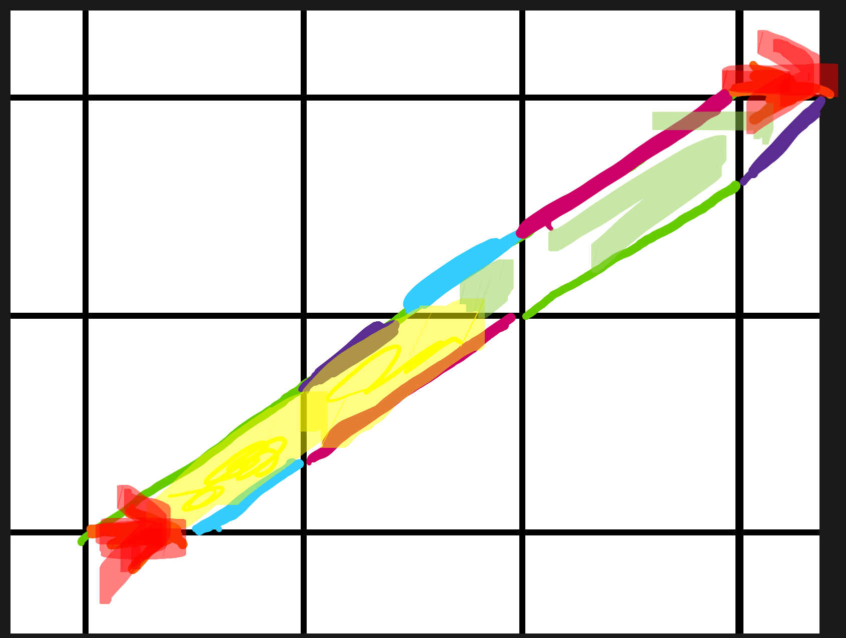




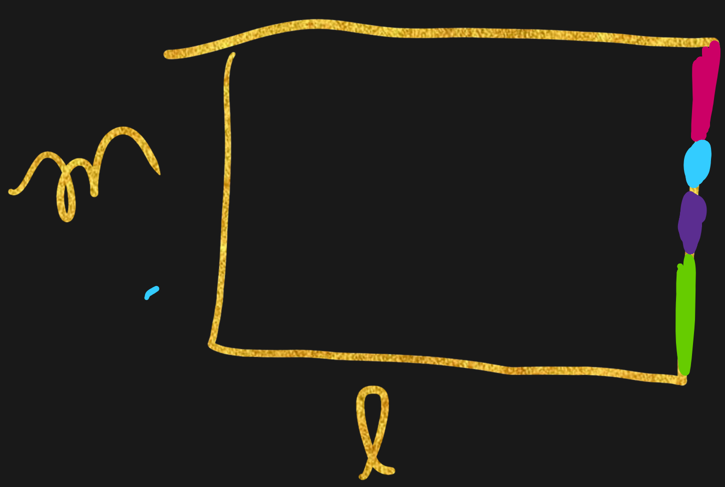
$m \to \underline{3l + 2m}$

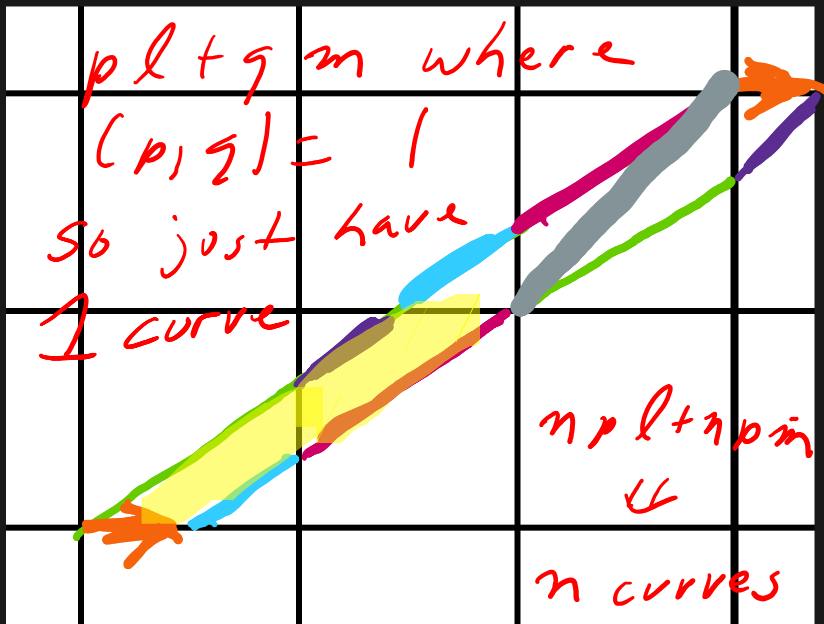






$m \to 3l + 2m$

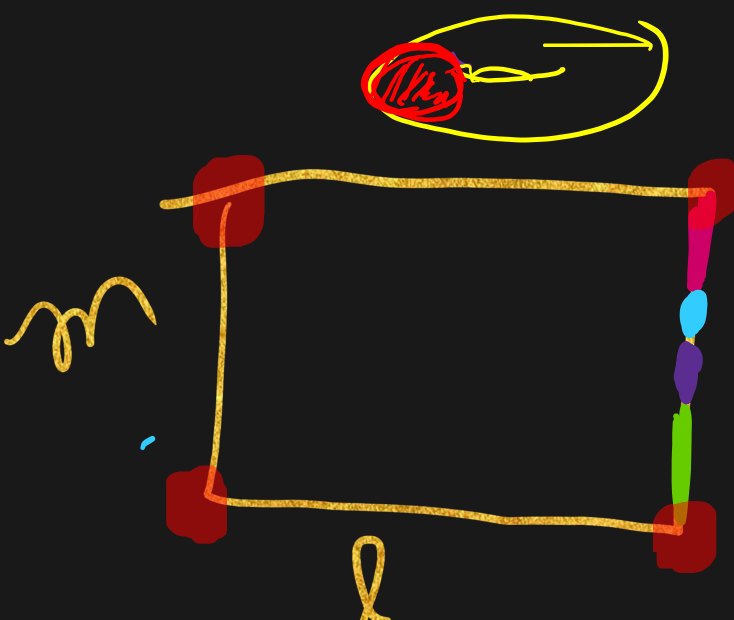




$$\boxed{m \to 3l + 2m}$$

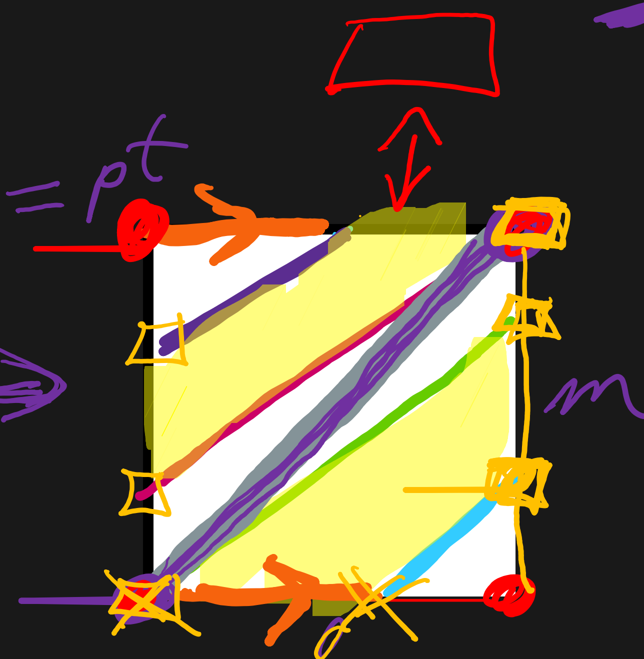
$$\boxed{l \to l + m}$$

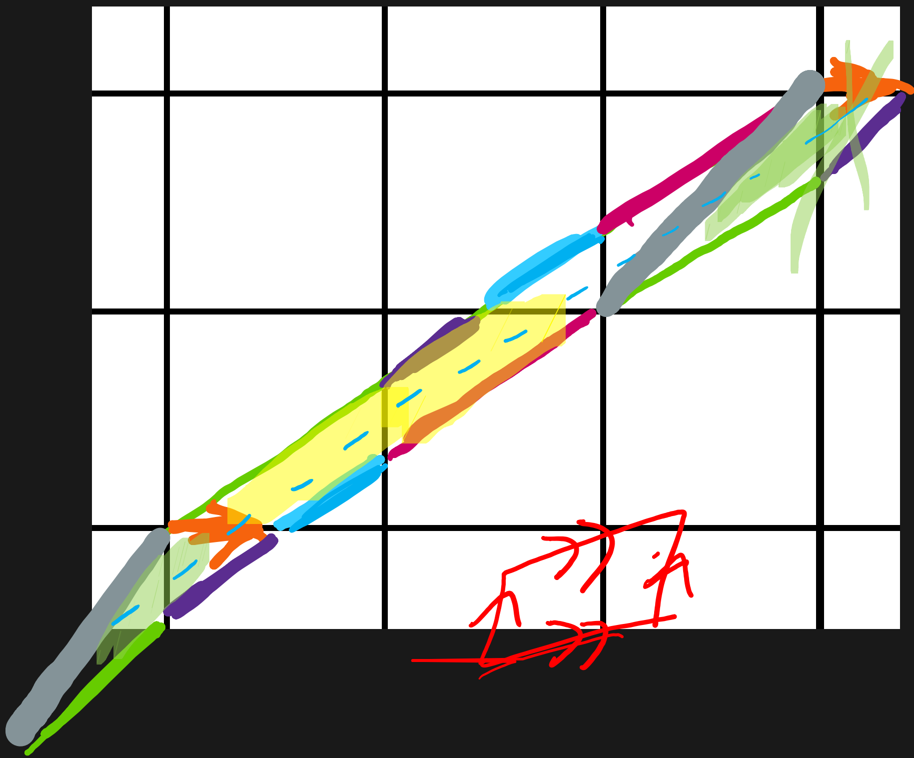
$$\Delta(m,l) = \begin{vmatrix} 3 & 2 \\ 1 & 1 \end{vmatrix} = 3-2 = 1$$



homeo

$h(m \cap l) = pt$

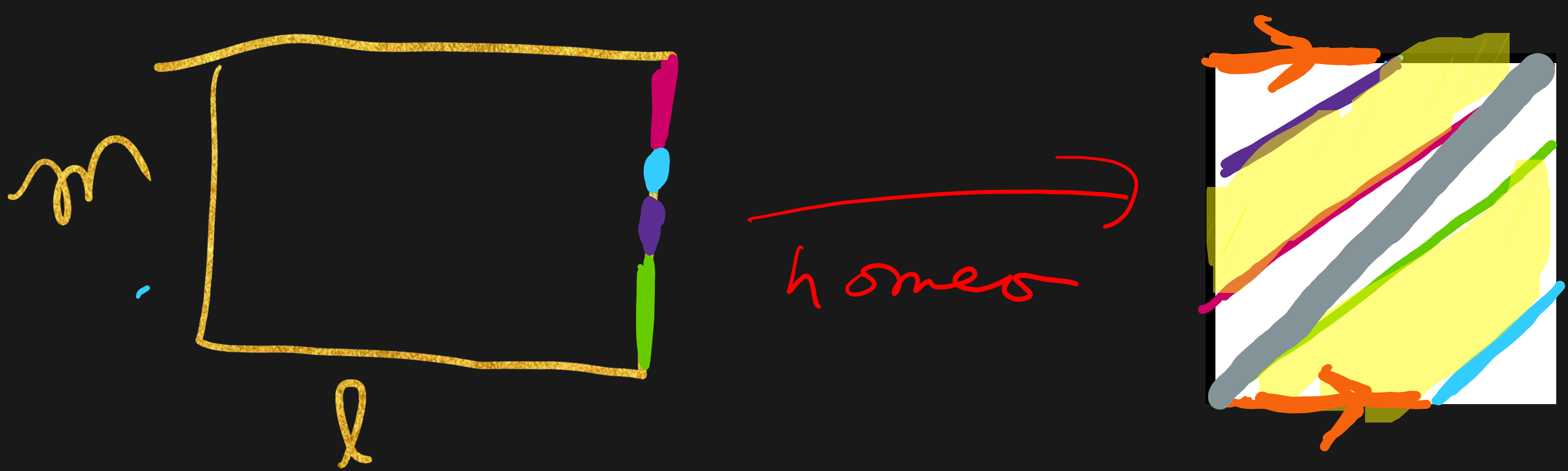


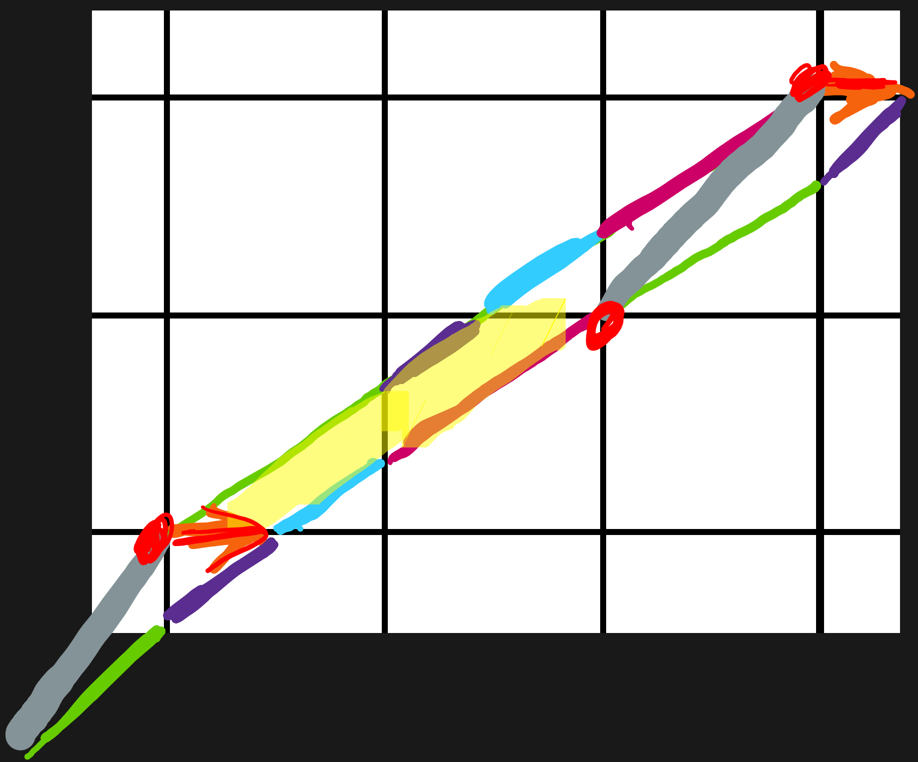


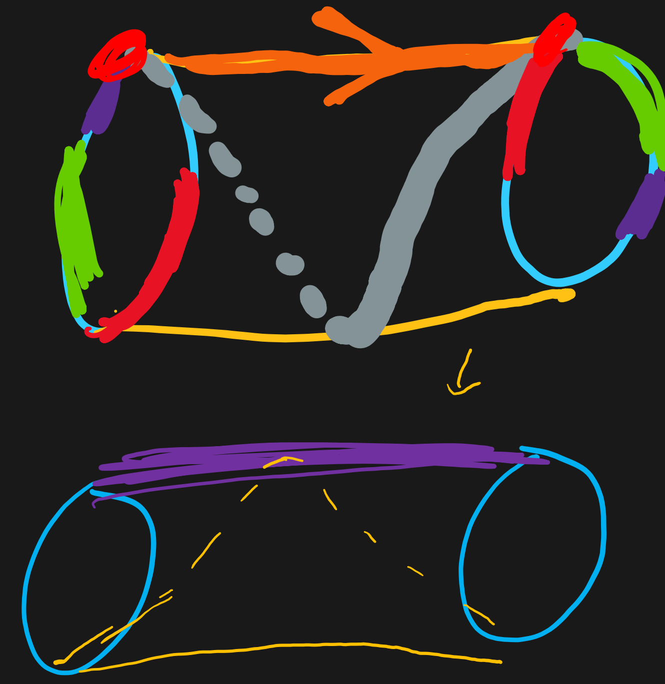
$$\boxed{m \to 3l + 2m}$$

$$l \to l + m$$

$$\Delta(m,l) = \begin{vmatrix} 3 & 2 \\ 1 & 1 \end{vmatrix} = 3-2 = 1$$







$\partial V \to \partial V$

$\xrightarrow{\text{homeomorphism}}$

$m \to 3m + 2l$

$l \to m + l$

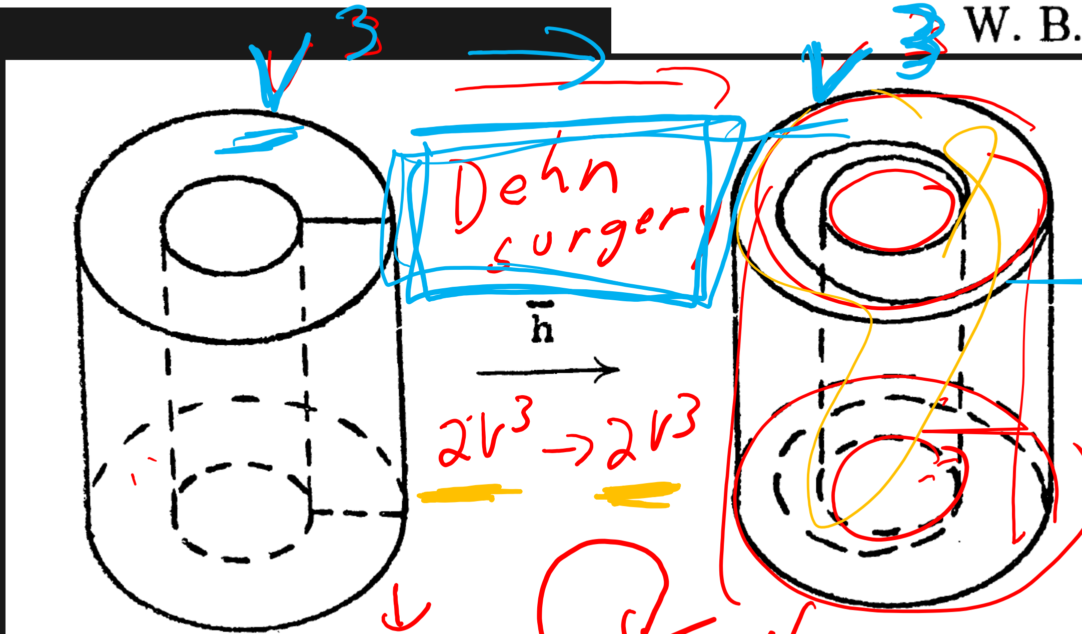
homeomorphism

http://dx.doi.org/10.1090/conm/044/813107

Contemporary Mathematics
Volume 44, 1985

THE UNKNOTTING NUMBER OF A CLASSICAL KNOT

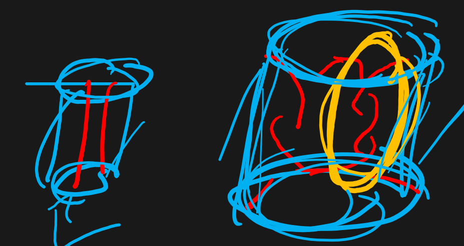
W. B. Raymond Lickorish

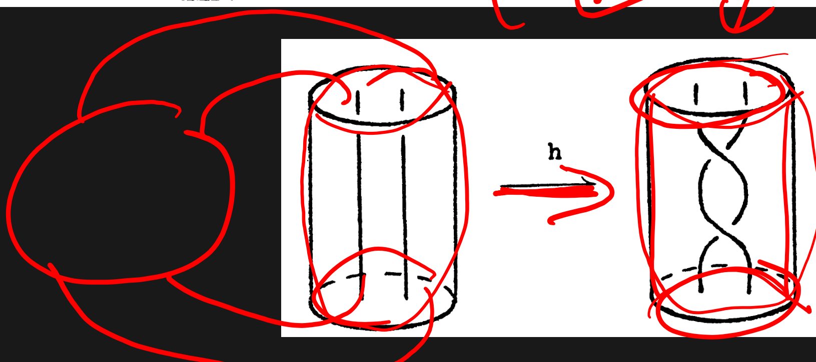


Generaly

$\partial(V^3) \to \partial(N^3) = T^2$

Downstair





$\frac{1}{2}$ - 3

Lemma 1. *If k has unknotting number equal to one, then M_k is obtained by $n/2$-surgery on some knot in S^3, n being an odd integer.*

Proof. Let A be the annulus $\{re^{i\theta} : 1 \le r \le 2\} \subset \mathbb{C}$, and let $\tau : A \to A$ be the twisting homeomorphism defined by

$$\tau(re^{i\theta}) = re^{i(\theta+2\pi r)}.$$

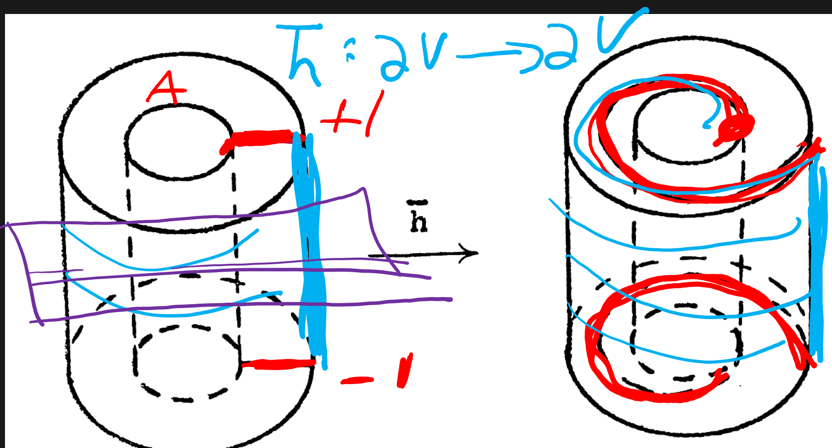
Let ρ be the rotation about the real axis of the solid torus $T = A \times [-1,1] \subset \mathbb{C} \times \mathbb{R}$ given by $\rho(re^{i\theta}, t) = (re^{-i\theta}, -t)$. Define a homeomorphism $\bar{h}$ from the boundary of T to itself by

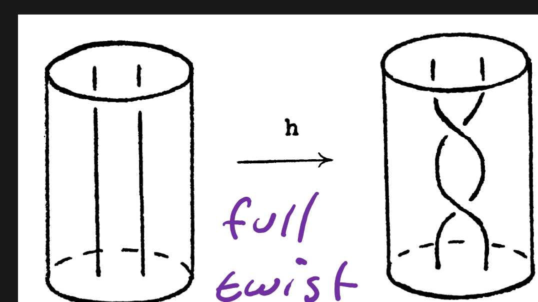
$$S^1 \times D^2 = V = V^3$$

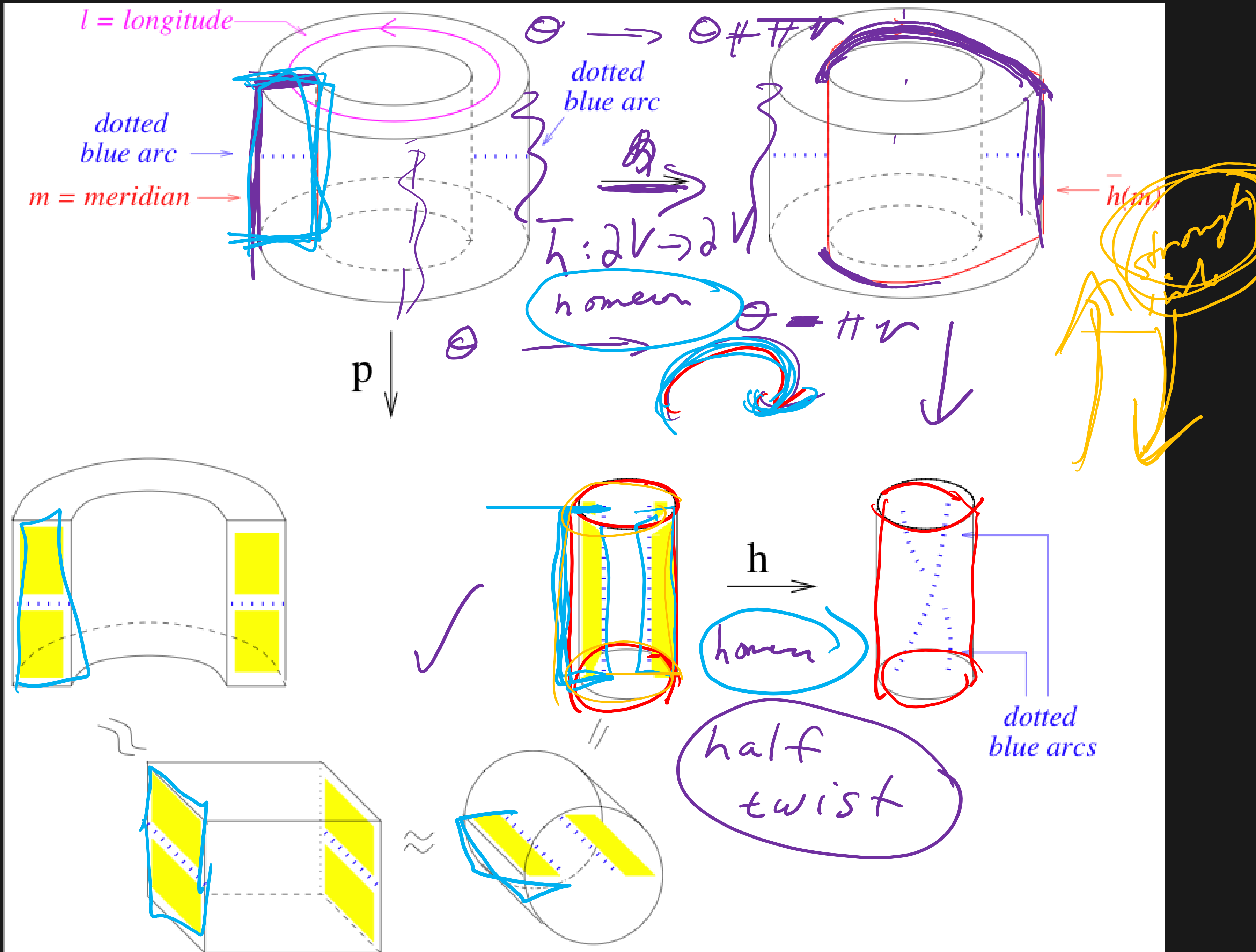
$$\bar{h}(re^{i\theta}, 1) = (\tau re^{i\theta}, 1)$$
$$\bar{h}(re^{i\theta}, -1) = (\tau^{-1} re^{i\theta}, -1)$$

$$\partial V^3 = T^2 \neq T^3$$

$\bar{h}$ being the identity on the remainder of ∂T. This $\bar{h}$ commutes with $\rho|\partial T$ and so induces a homeomorphism on the quotient space $h : \partial T/\rho \to \partial T/\rho$.

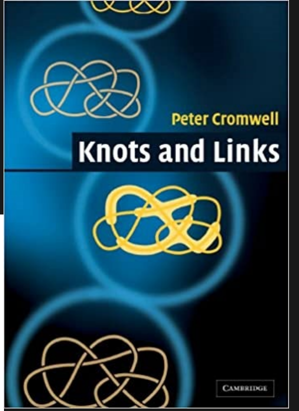






l = longitude
dotted blue arc
dotted blue arc
m = meridian
h(m)
p
h
dotted blue arcs
h : ∂V → ∂V
homeo
homeo
half twist

Classification of rational tangles is simpler:



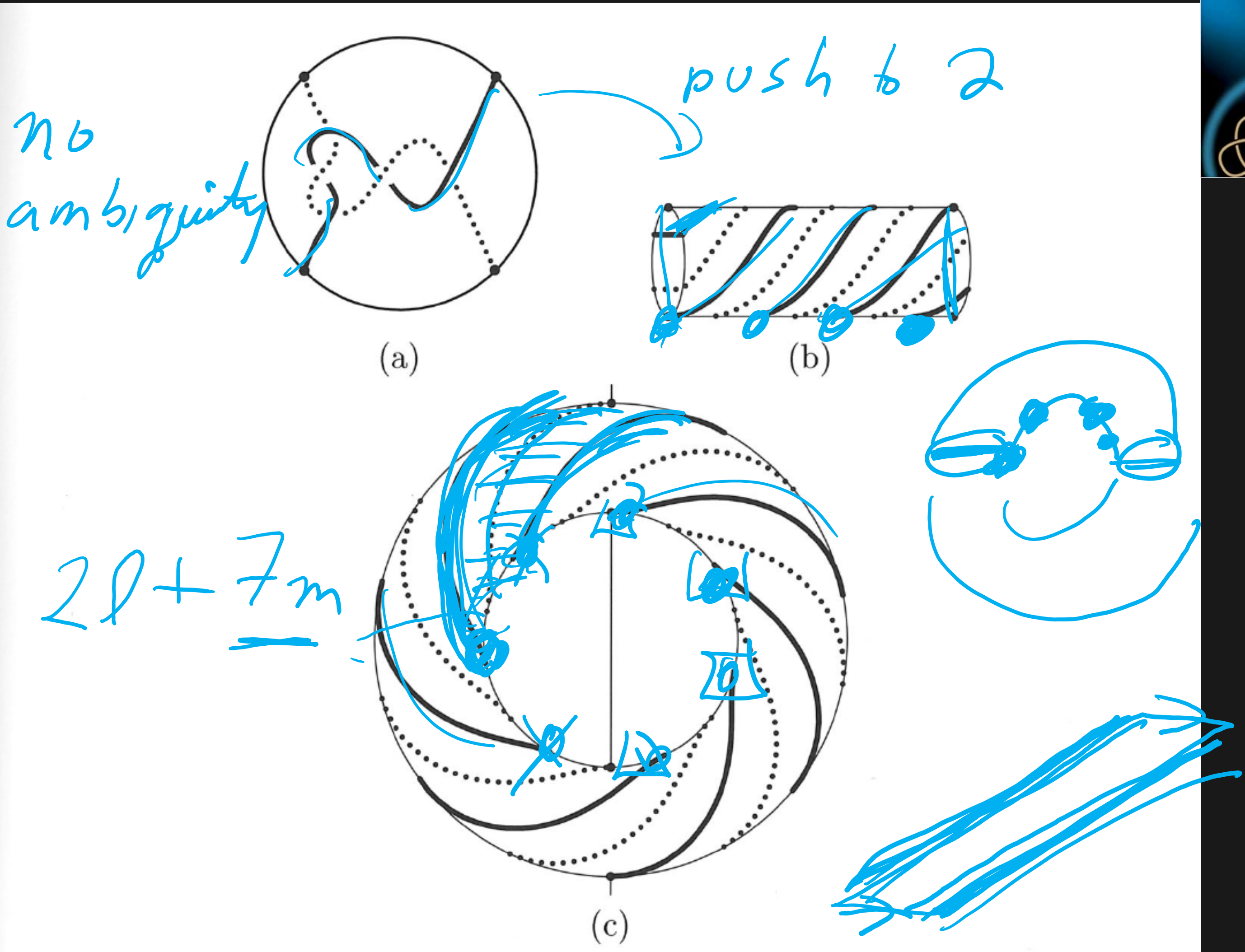
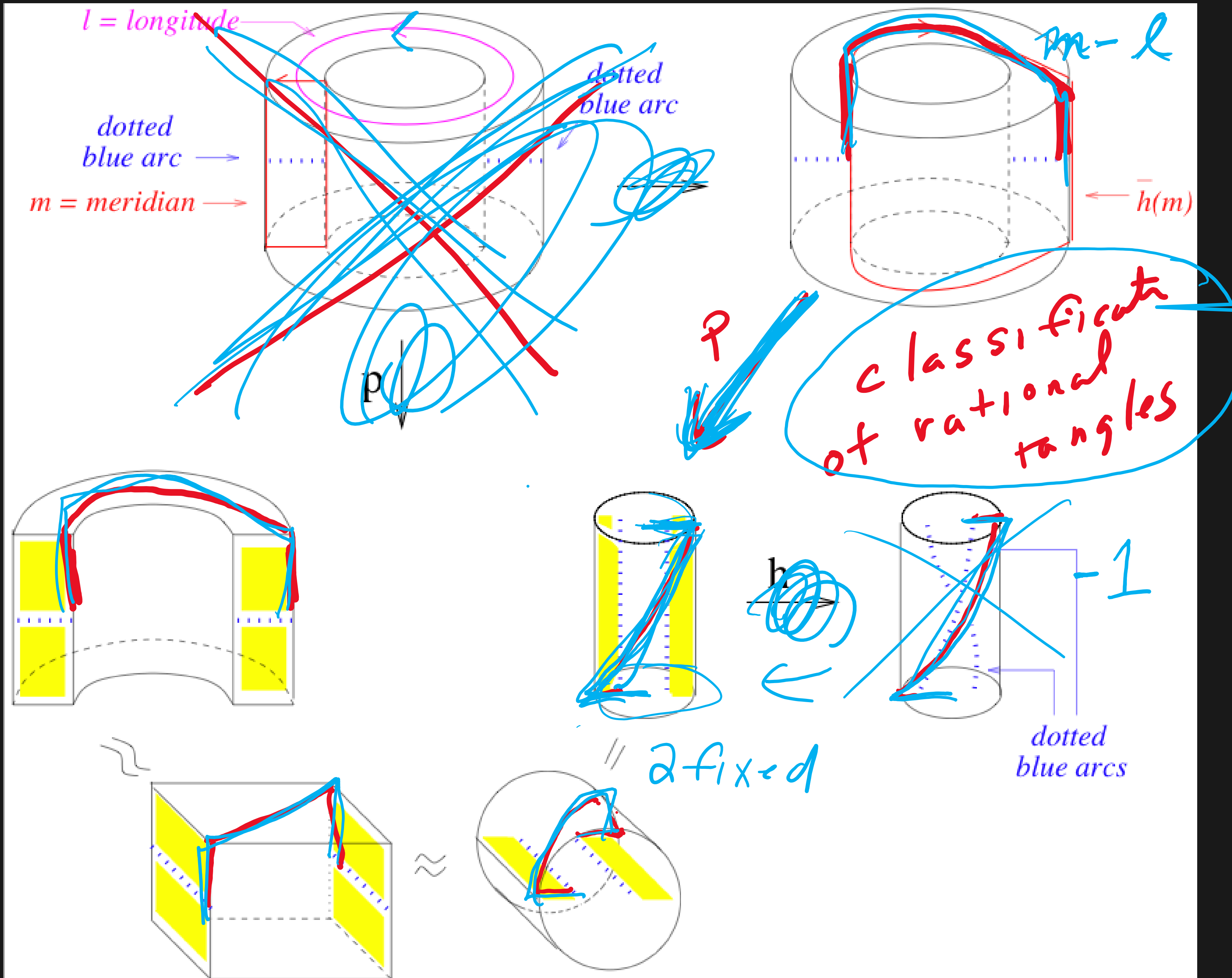


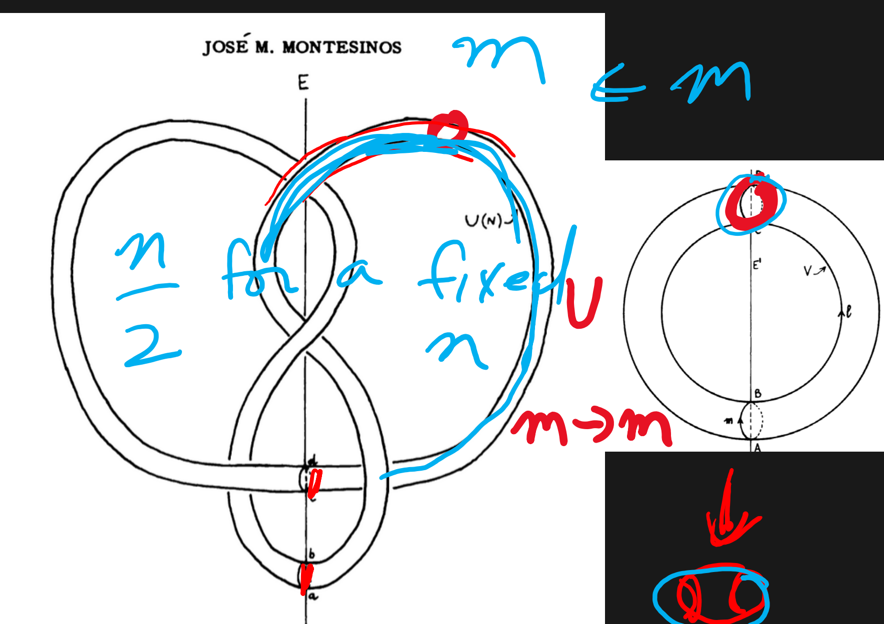
Figure 8.6. A rational tangle can be isotoped to lie on the boundary, then lifted to the covering torus.



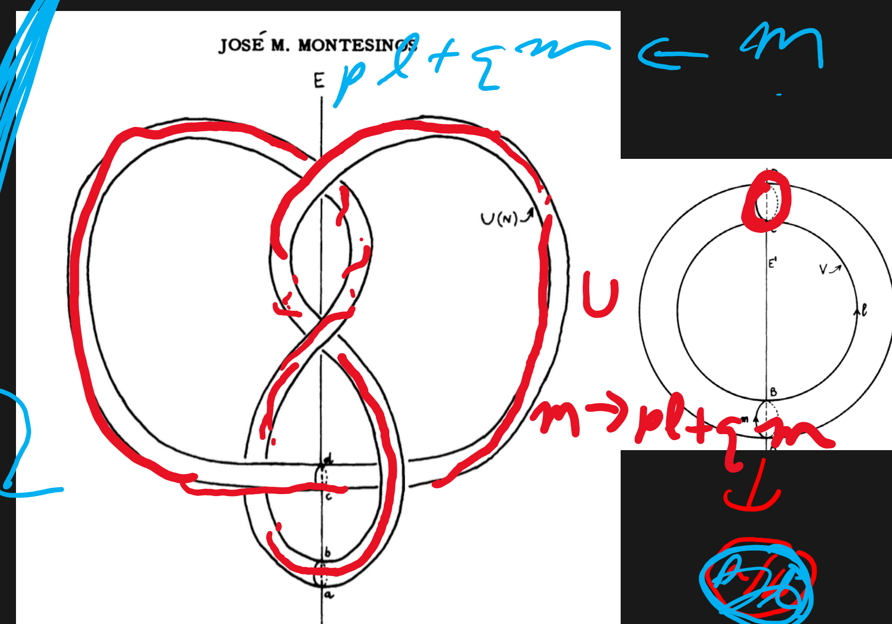
l = longitude
dotted blue arc
m = meridian
dotted blue arc
p
m - l
h(m)
P
classification of rational tangles
h
-1
dotted blue arcs
2 fixed

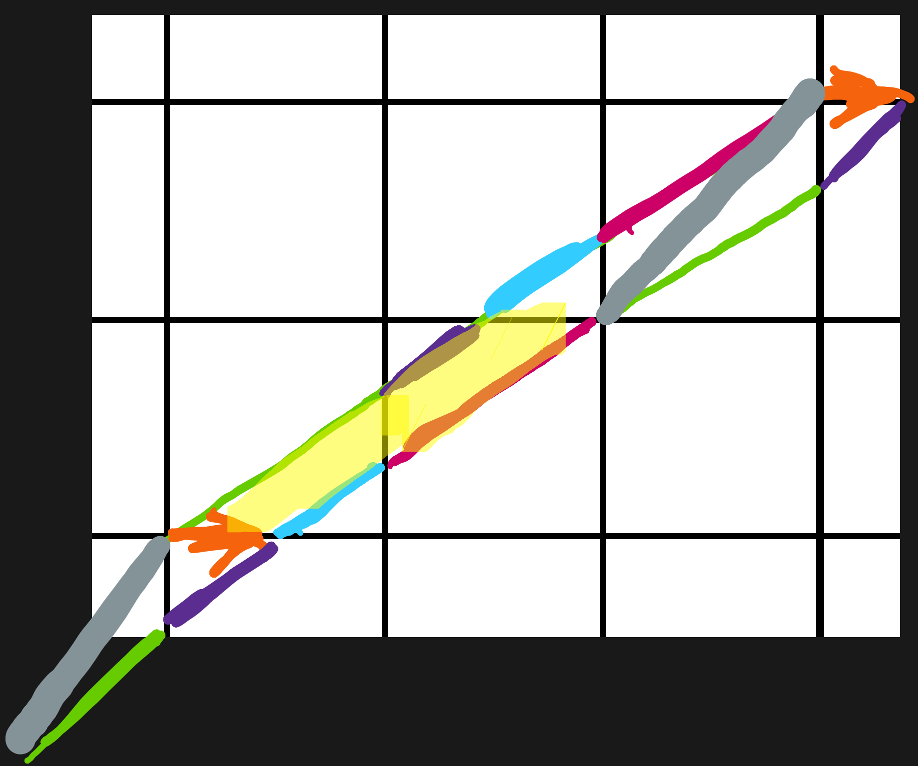
J. M. Montesinos, *Surgery on links and double branched coverings of S^3.* Ann. of Math. Studies 84, (1975), 227–259.

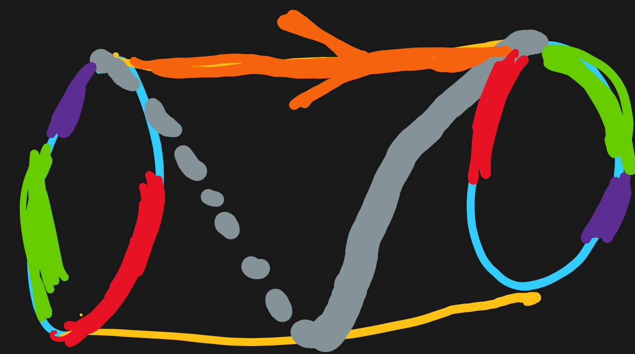
THEOREM 1. *Let* M *be a closed, orientable 3-manifold that is obtained by doing surgery on a strongly-invertible link* L *of* n *components. Then* M *is a 2-fold cyclic covering of* S^3 *branched over a link of at most* n+1 *components. Conversely, every 2-fold cyclic branched covering of* S^3 *can be obtained in this fashion.*



Dehn surgery







$\partial V \to \partial V$

$\longrightarrow$

$m \to 3m + 2l$

$l \to m + l$

homeomorphism

J. M. Montesinos, *Surgery on links and double branched coverings of S^3.* Ann. of Math. Studies 84, (1975), 227–259.

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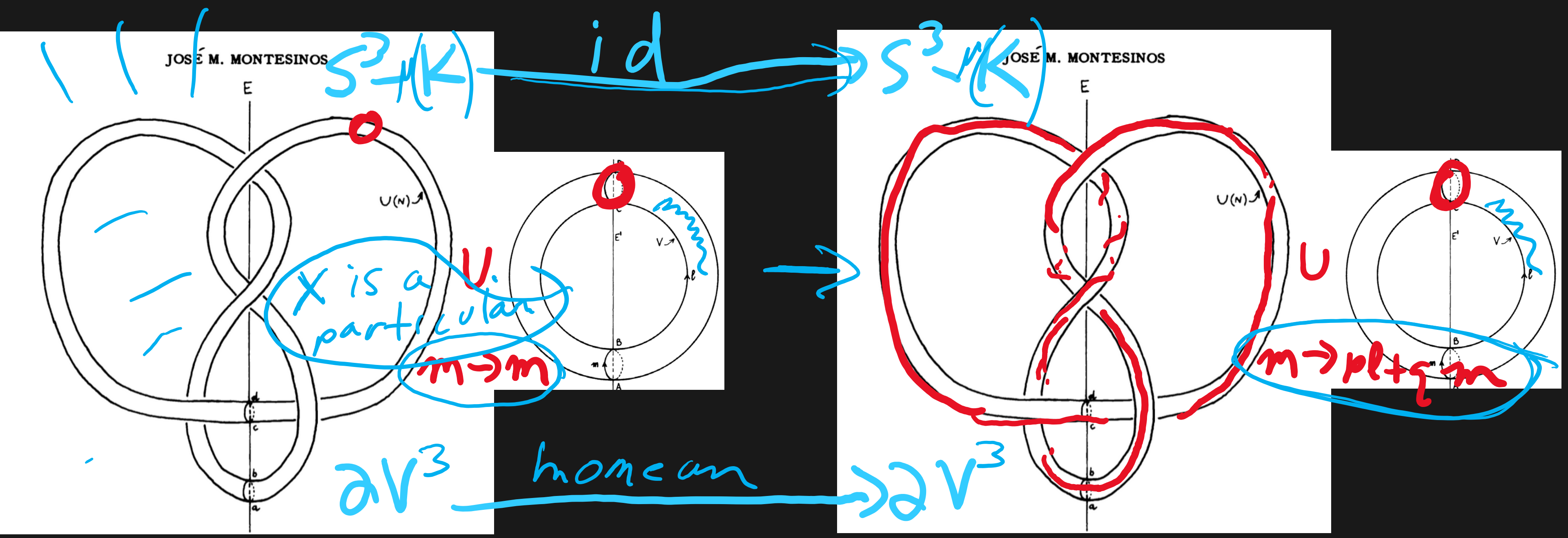
$\partial N(K) = T^2 = \partial V^3$

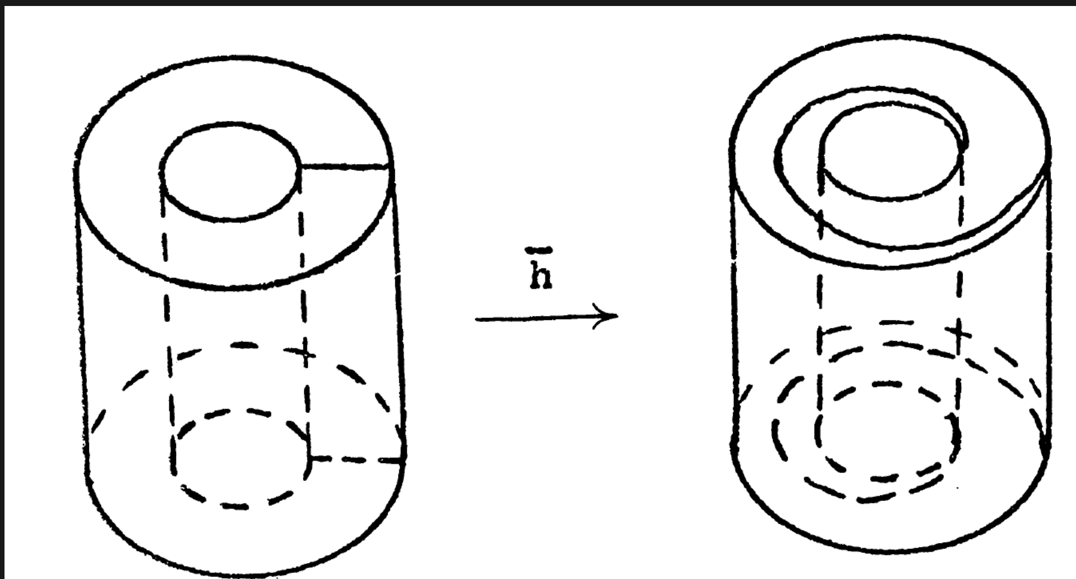


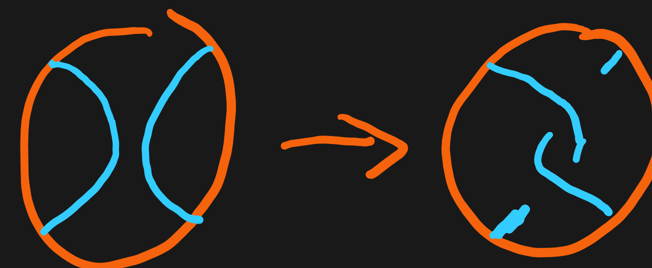
http://dx.doi.org/10.1090/conm/044/813107

Contemporary Mathematics
Volume 44, 1985

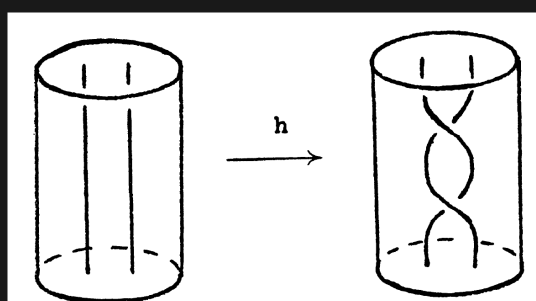
THE UNKNOTTING NUMBER OF A CLASSICAL KNOT

W. B. Raymond Lickorish





$$m \to m + 2\ell \to m + 2(L + Xm)$$
$$= m(1 + Xm) + 2L$$



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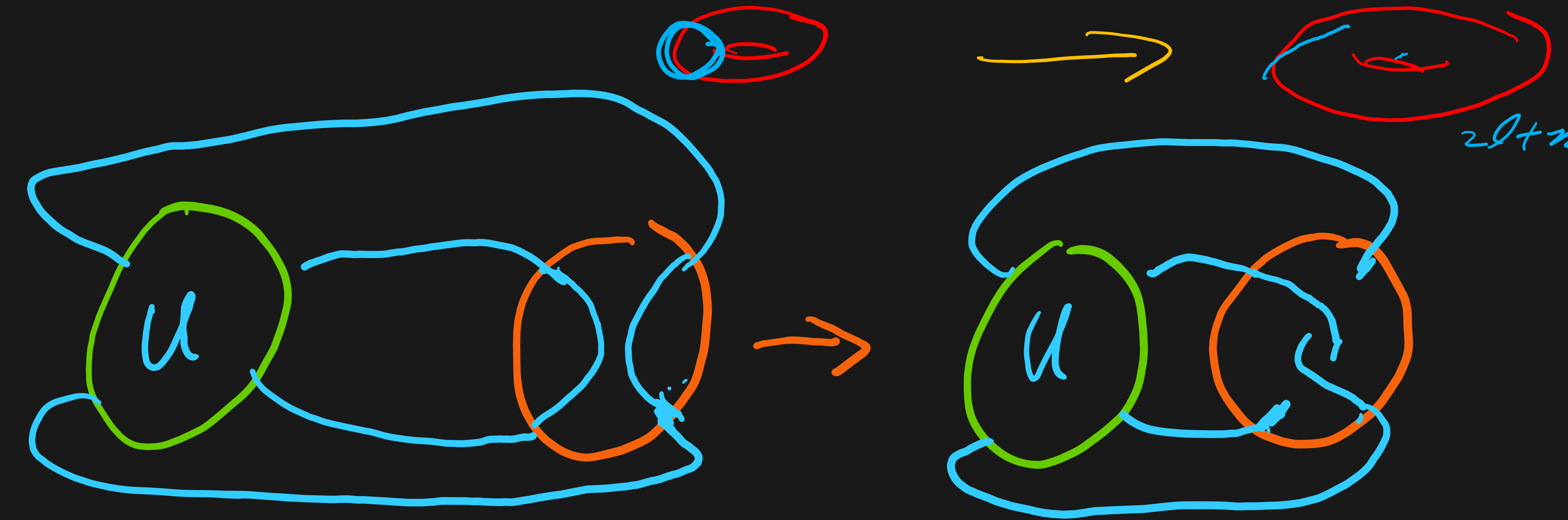
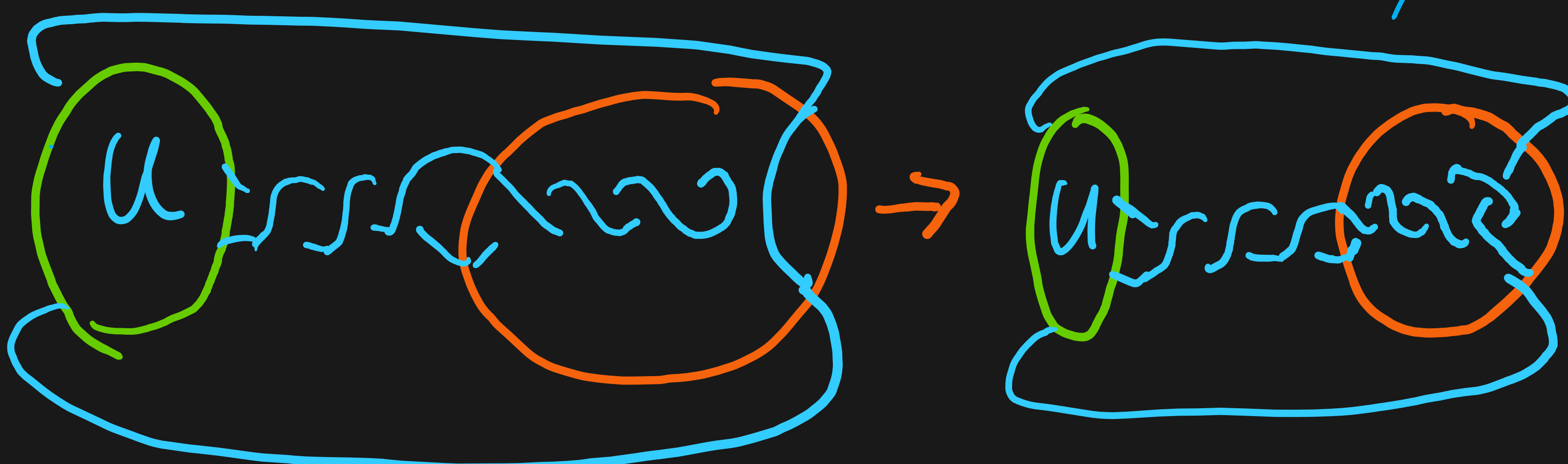


different homeomorph on boundary

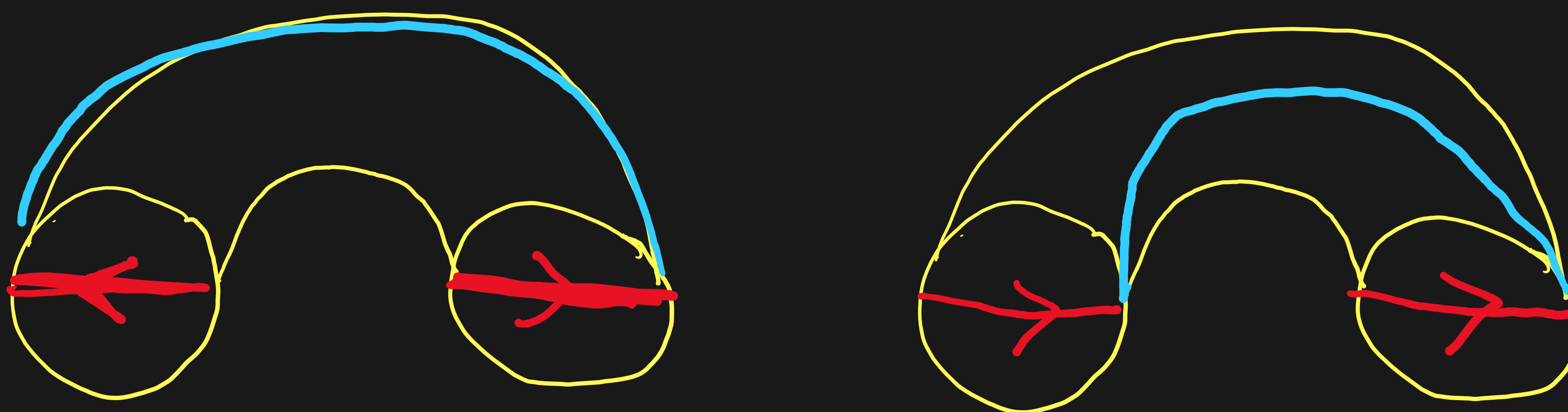


HW 2: Choose 1 problem

post to discussion

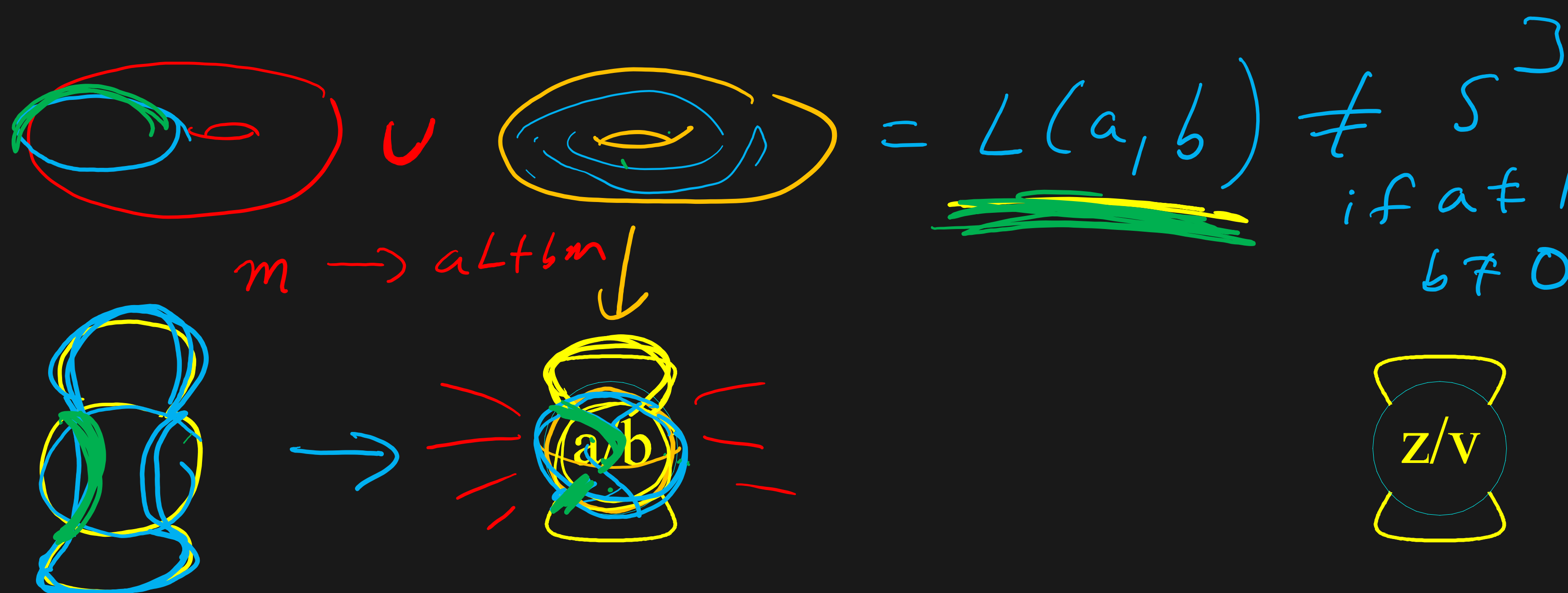
Option 2: Relate tangle equations on previous slide to double branch cover.

Hint:



M. Culler, C. Gordon, J. Luecke, P. Shalen (1987). Dehn surgery on knots. The Annals of Mathematics (Annals of Mathematics) 125 (2): 237-300. https://marc-culler.info/static/home/papers/CyclicSurgery.pdf

CYCLIC SURGERY THEOREM. *Suppose that M is not a Seifert fibered space. If $\pi_1(M(r))$ and $\pi_1(M(s))$ are cyclic, then $\Delta(r, s) \leq 1$. Hence there are at most three slopes r such that $\pi_1(M(r))$ is cyclic.*



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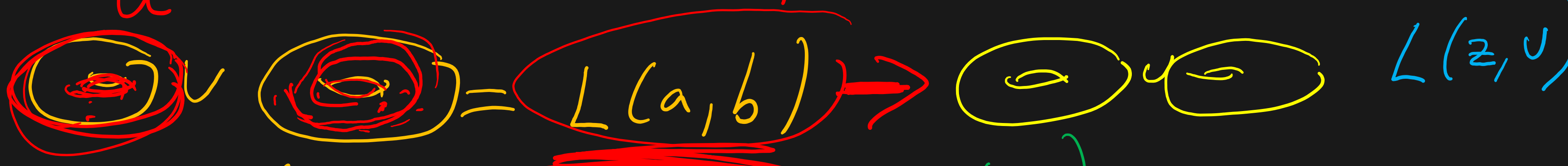
claim: $\tilde{U}$ = complement of torus knot



$m \to al + bm$

$\begin{pmatrix} & \end{pmatrix}\begin{pmatrix} a \\ b \end{pmatrix}$

u + rotn

LHS

RHS