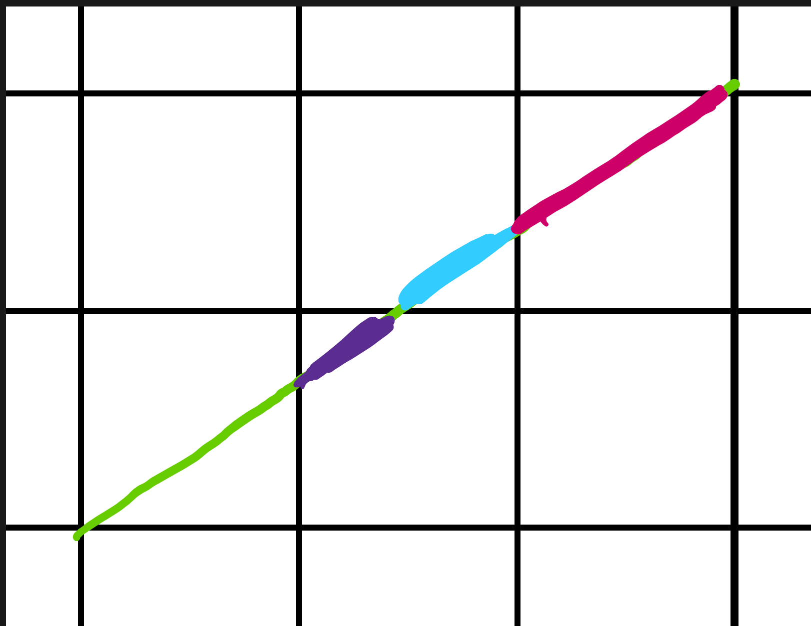
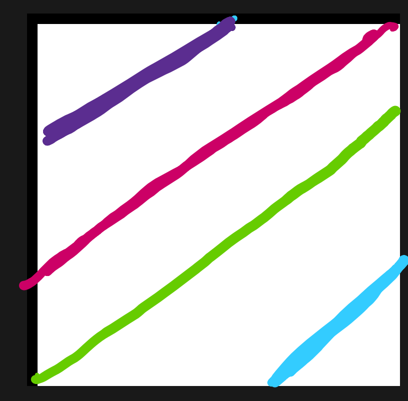
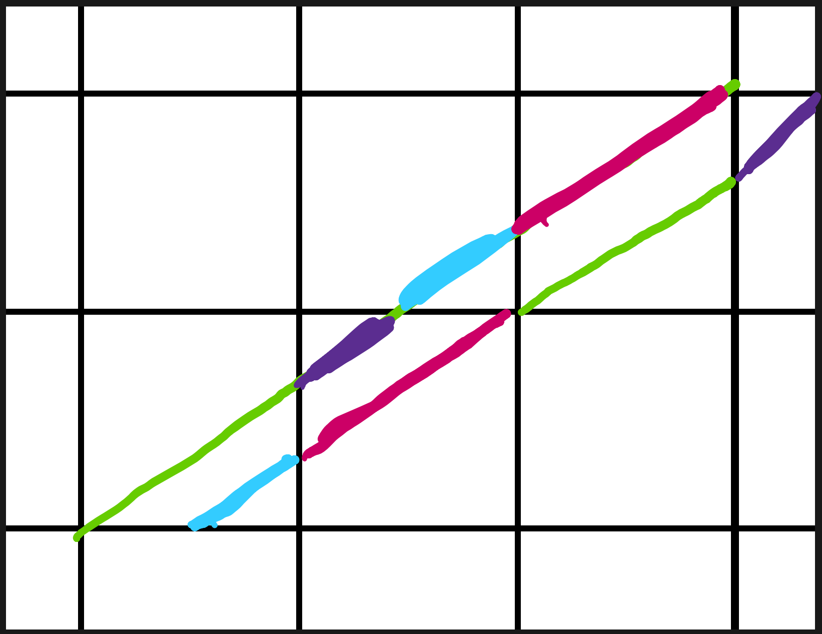
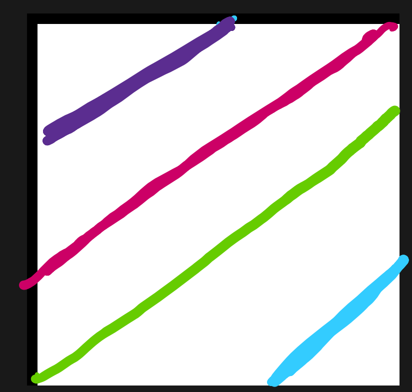
$m \to 3l + 2m$

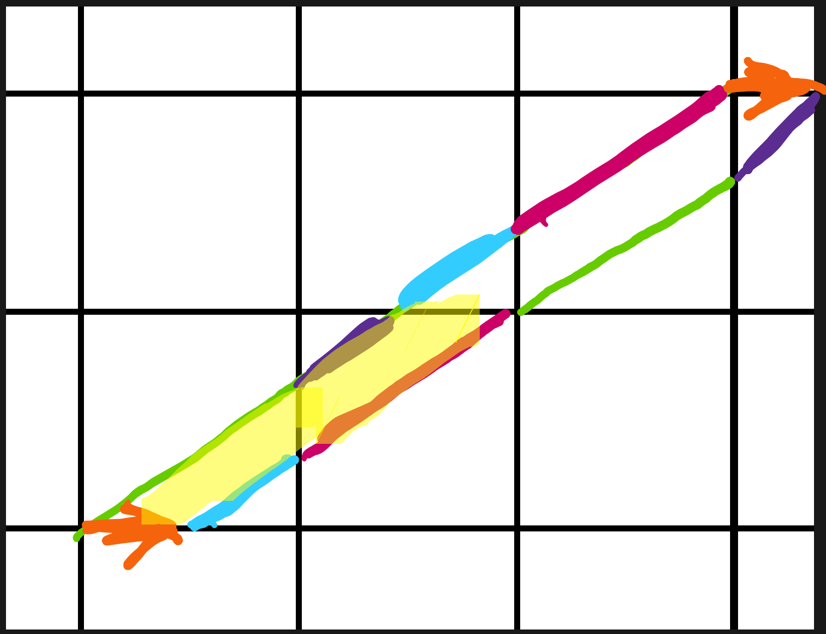




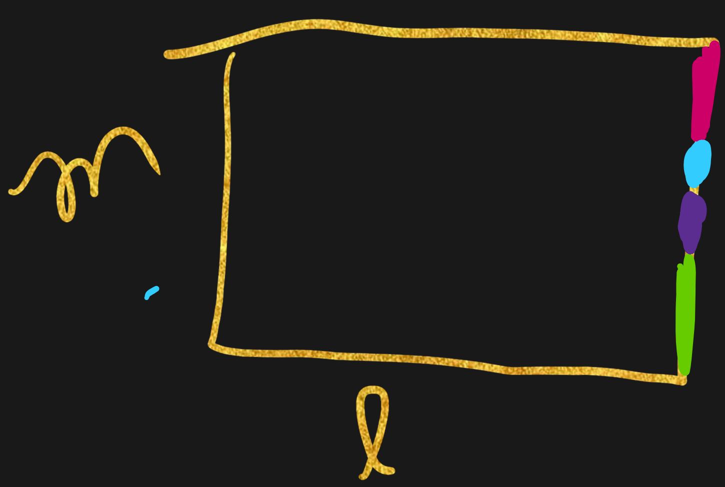
$m \to 3\ell + 2m$

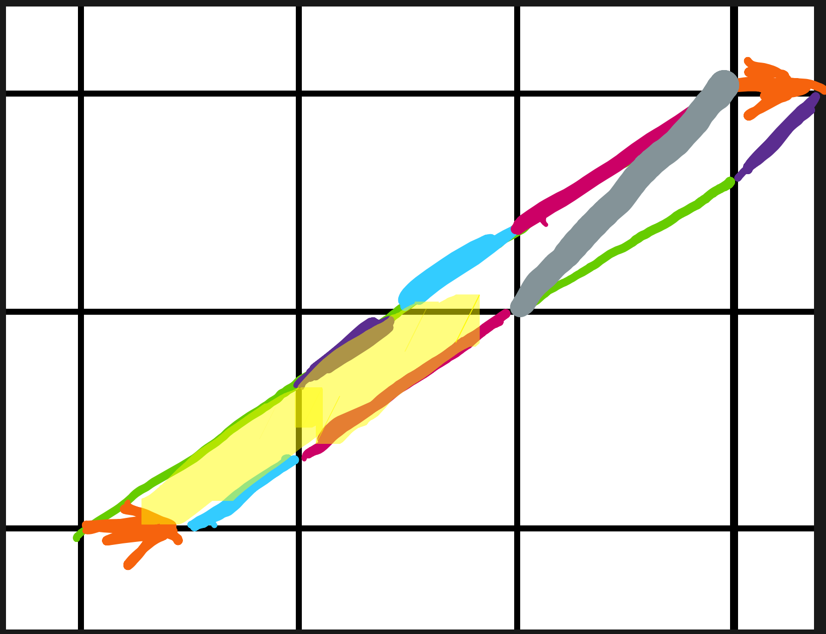






$$m \to 3l + 2m$$

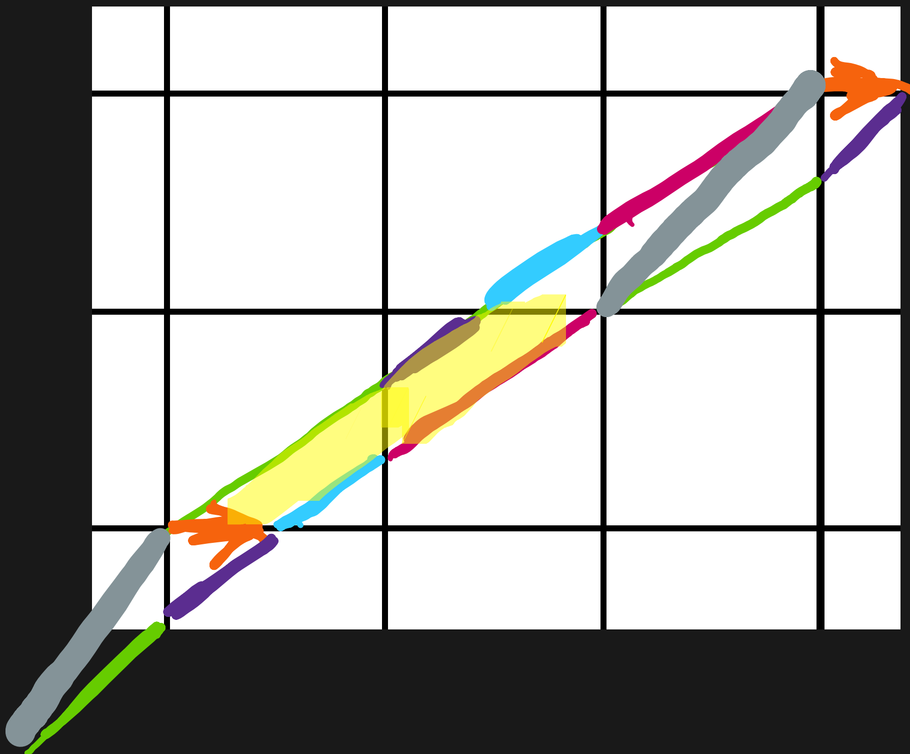




$$m \to 3l + 2m$$

$$l \to l + m$$

$$\Delta(m,l) = \begin{vmatrix} 3 & 2 \\ 1 & 1 \end{vmatrix} = 3 - 2 = 1$$



$$m \to 3l + 2m$$

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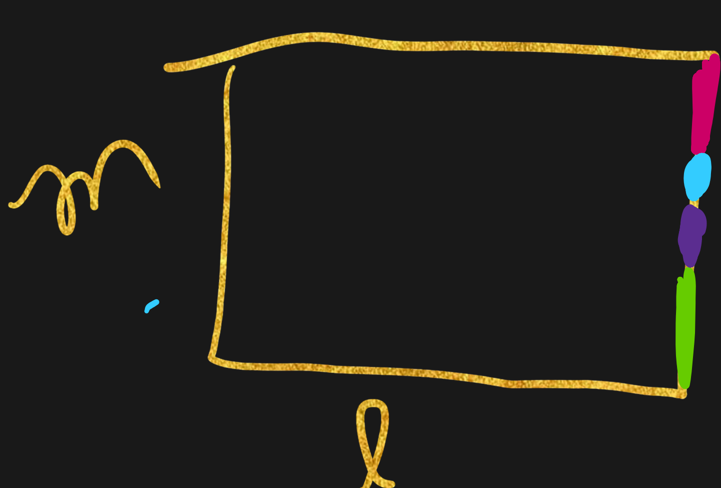
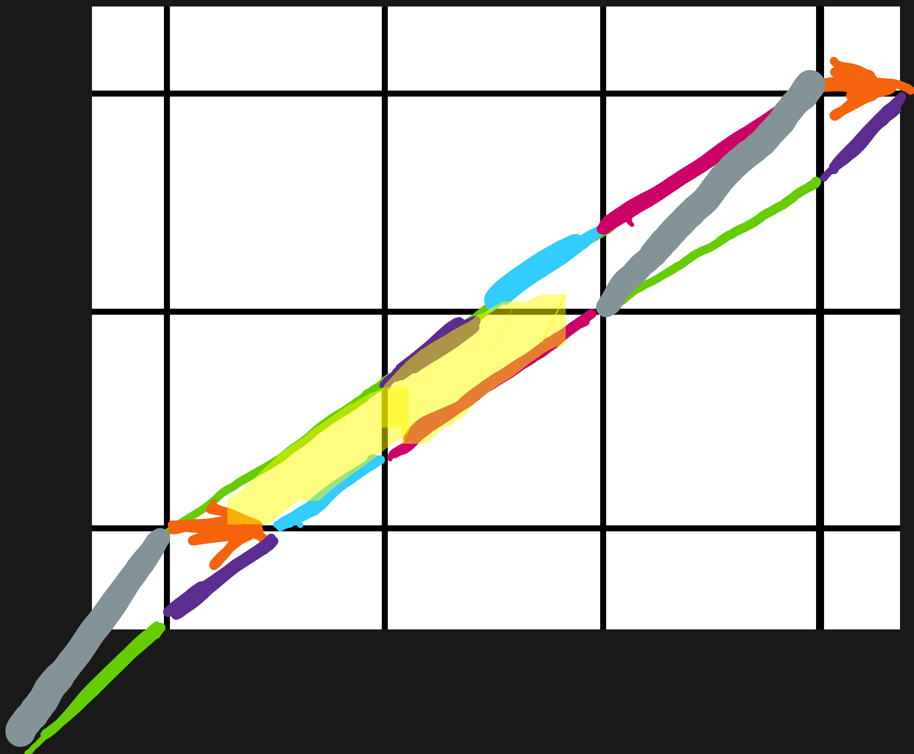
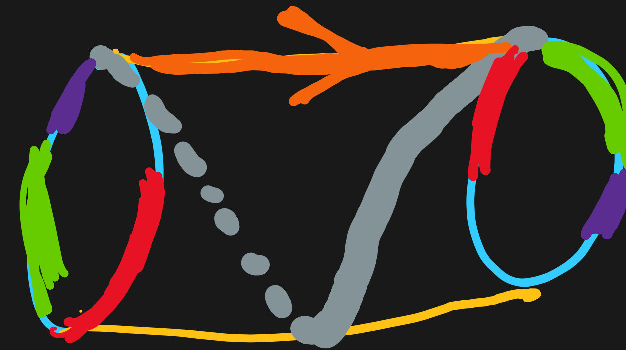






$\partial V \to \partial V$

$m \to 3m + 2l$

$l \to m + l$

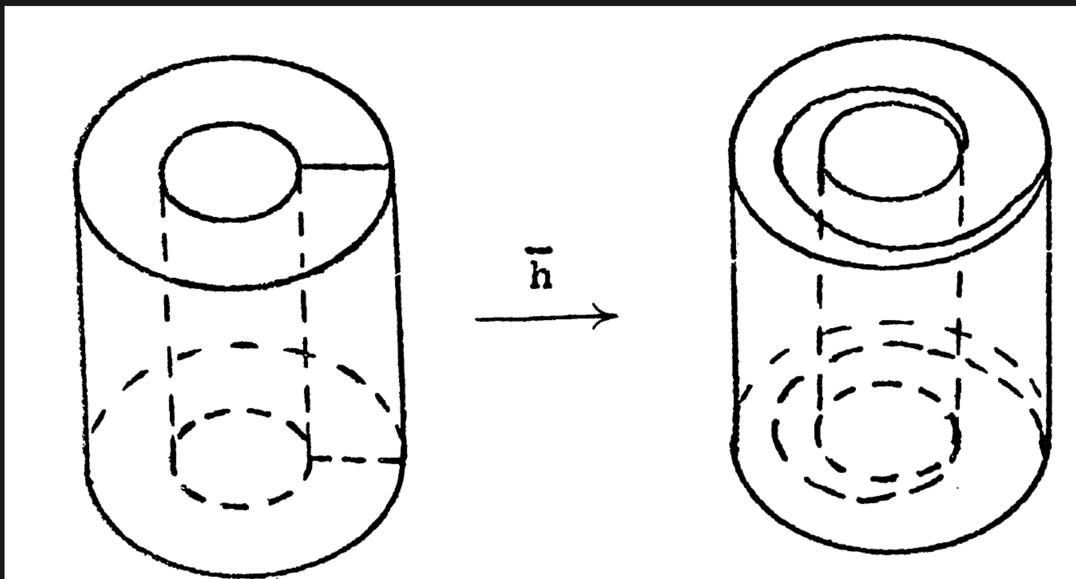
homeomorphism

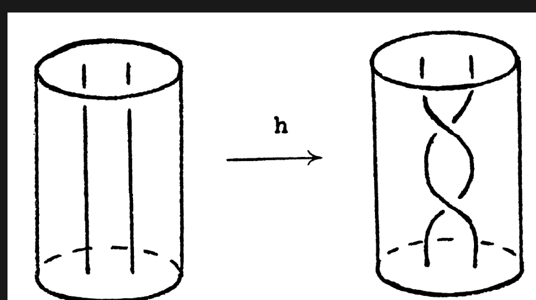
http://dx.doi.org/10.1090/conm/044/813107

Contemporary Mathematics
Volume 44, 1985

THE UNKNOTTING NUMBER OF A CLASSICAL KNOT

W. B. Raymond Lickorish





Lemma 1. *If k has unknotting number equal to one, then M_k is obtained by $n/2$-surgery on some knot in S^3, n being an odd integer.*

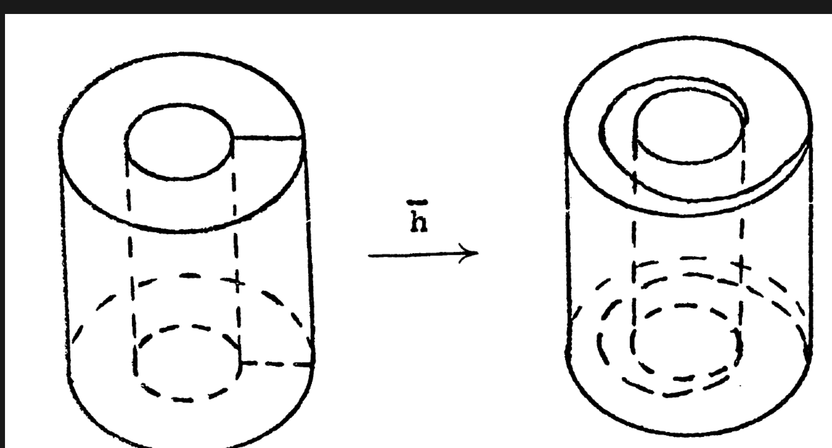
Proof. Let A be the annulus $\{re^{i\theta} : 1 \le r \le 2\} \subset \mathbb{C}$, and let $\tau : A \to A$ be the twisting homeomorphism defined by

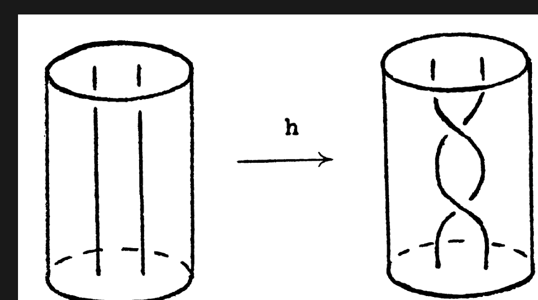
$$\tau(re^{i\theta}) = re^{i(\theta+2\pi r)}.$$

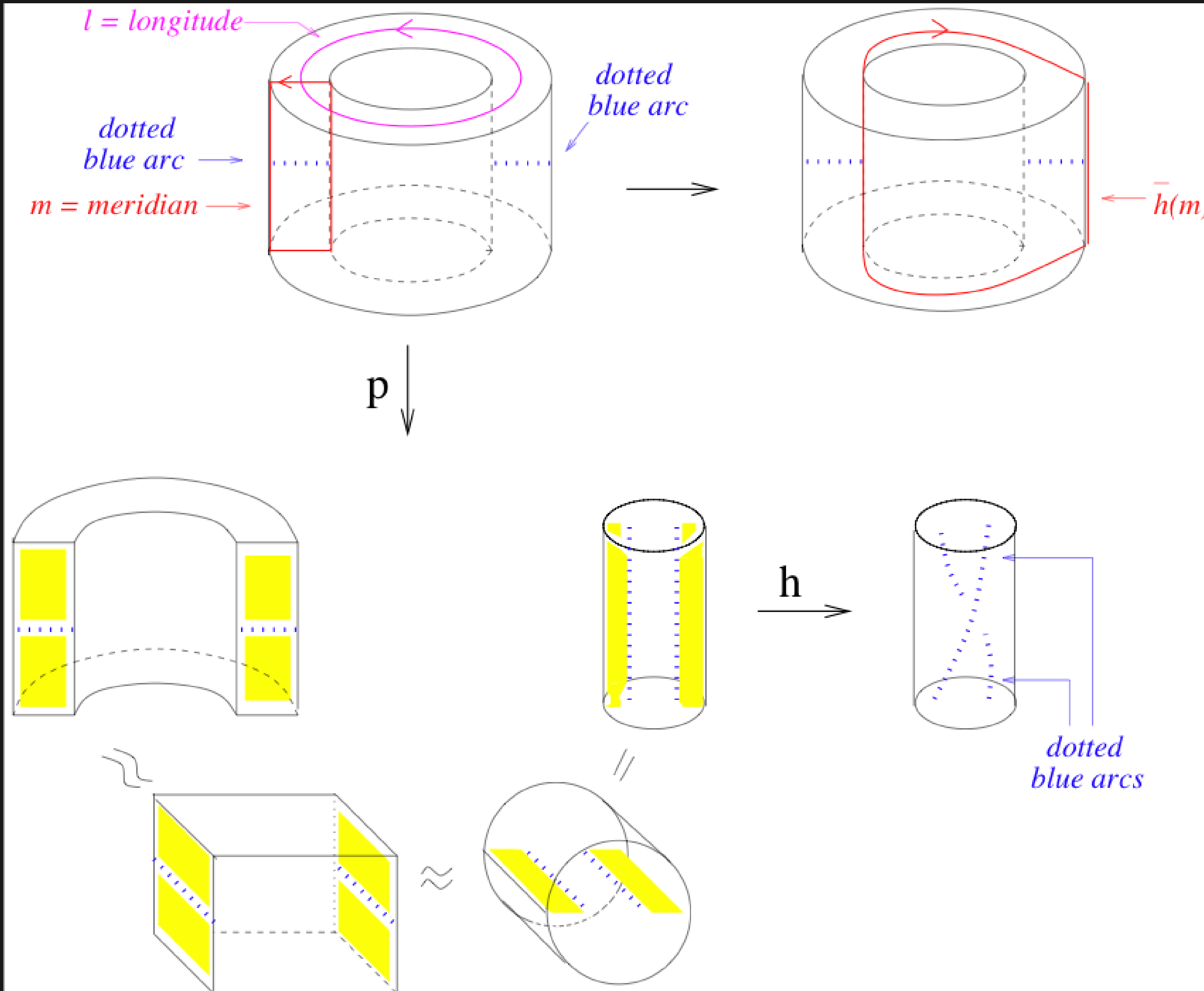
Let ρ be the rotation about the real axis of the solid torus $T = A \times [-1,1] \subset \mathbb{C} \times \mathbb{R}$ given by $\rho(re^{i\theta}, t) = (re^{-i\theta}, -t)$. Define a homeomorphism $\overline{h}$ from the boundary of T to itself by

$$\overline{h}(re^{i\theta}, 1) = (\tau re^{i\theta}, 1)$$
$$\overline{h}(re^{i\theta}, -1) = (\tau^{-1} re^{i\theta}, -1)$$

$\overline{h}$ being the identity on the remainder of ∂T. This $\overline{h}$ commutes with $\rho|\partial T$ and so induces a homeomorphism on the quotient space $h : \partial T/\rho \to \partial T/\rho$.







l = longitude
dotted
blue arc
dotted
blue arc
m = meridian
h̄(m)
p
h
dotted
blue arcs

Classification of rational tangles is simpler:

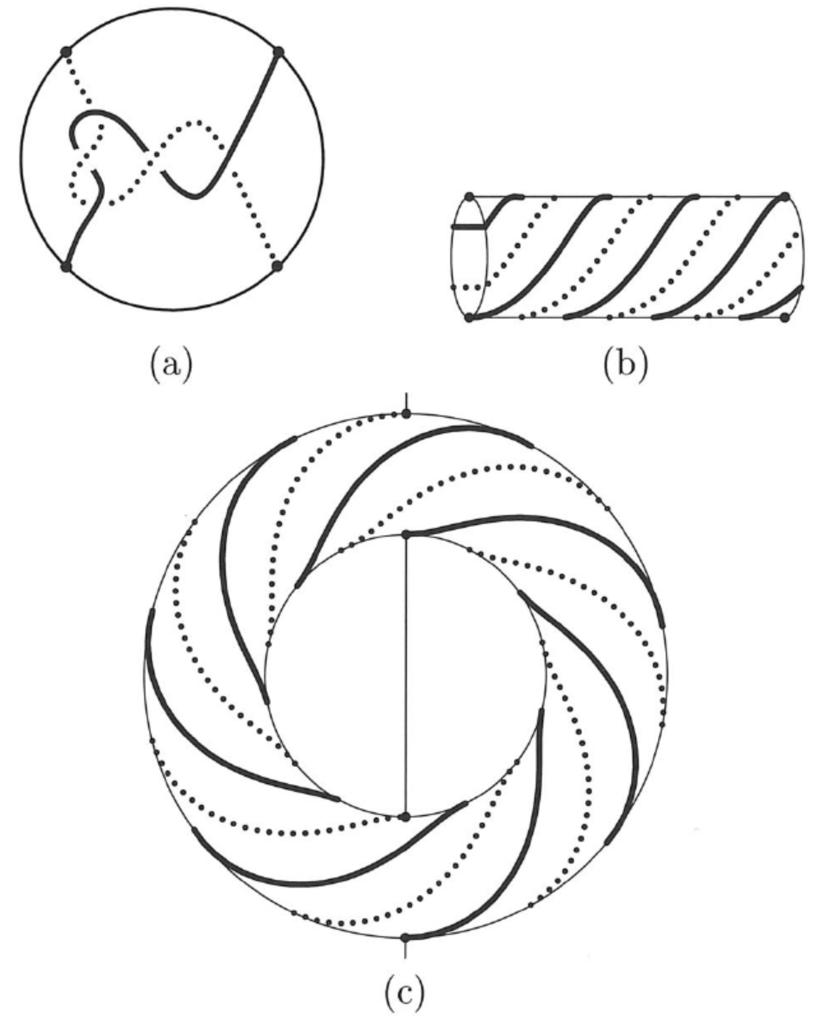
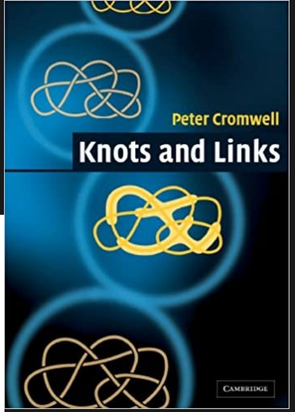
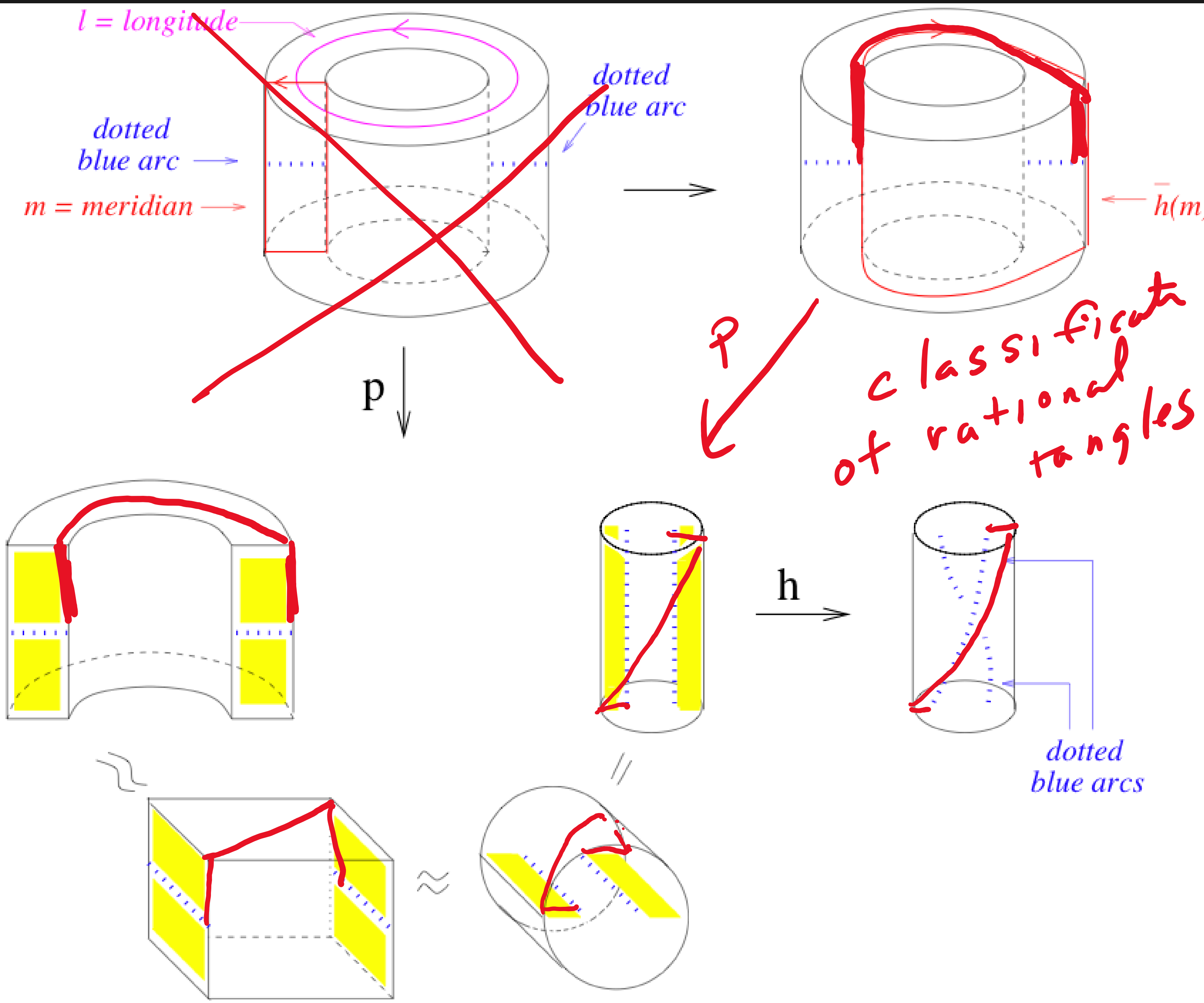


Figure 8.6. A rational tangle can be isotoped to lie on the boundary, then lifted to the covering torus.

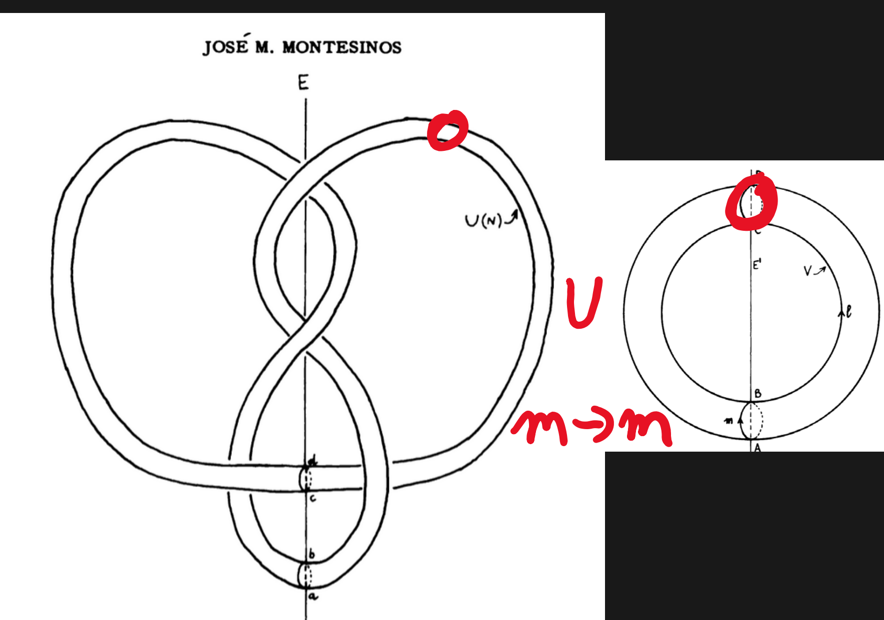


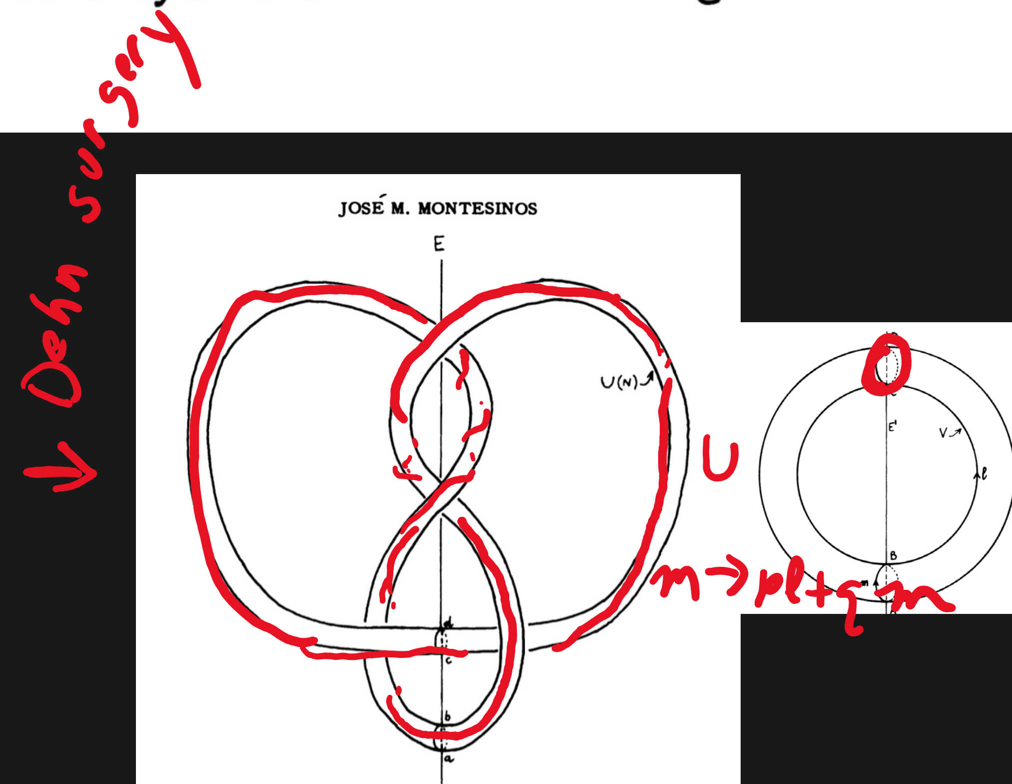


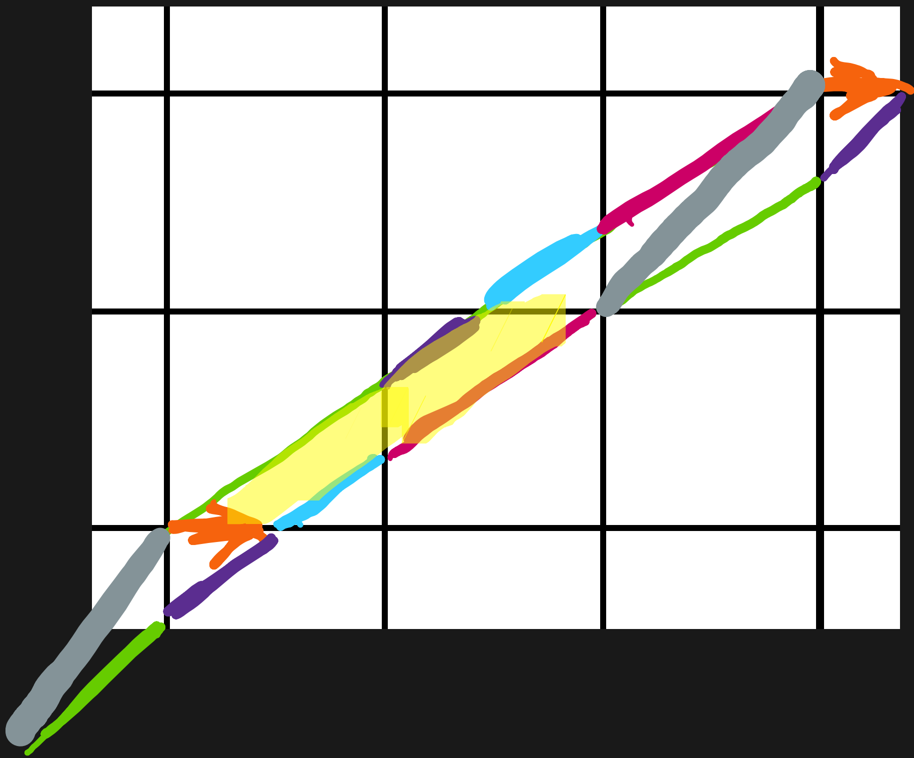
l = longitude
dotted blue arc
dotted blue arc
m = meridian
$\bar{h}(m)$
p
p
classification of rational tangles
h
dotted blue arcs

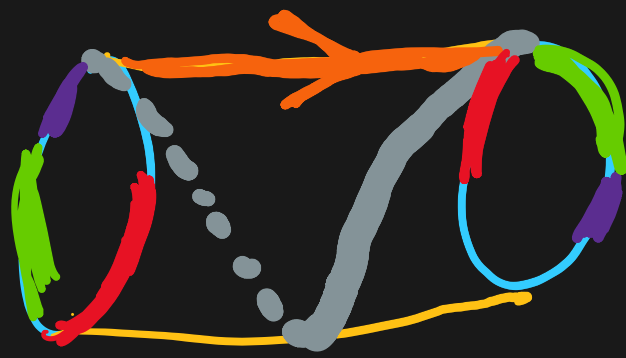
J. M. Montesinos, *Surgery on links and double branched coverings of S^3.* Ann. of Math. Studies **84**, (1975), 227–259.

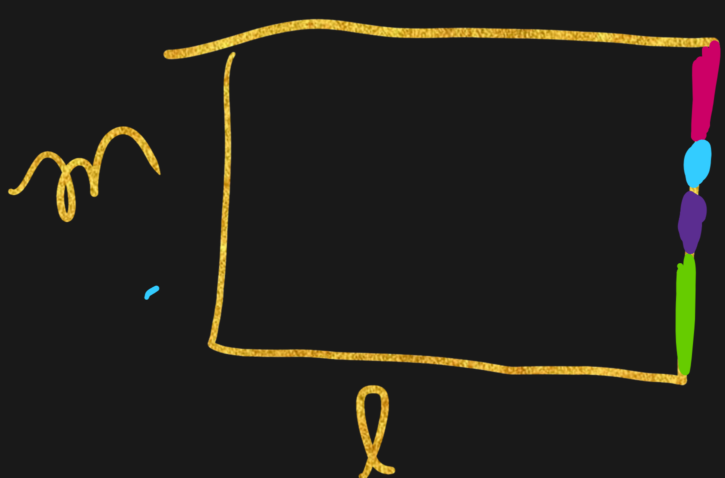
THEOREM 1. *Let M be a closed, orientable 3-manifold that is obtained by doing surgery on a strongly-invertible link L of n components. Then M is a 2-fold cyclic covering of S^3 branched over a link of at most n+1 components. Conversely, every 2-fold cyclic branched covering of S^3 can be obtained in this fashion.*











$\partial V \to \partial V$

$m \to 3m+2l$

$l \to m+l$

homeomorphism

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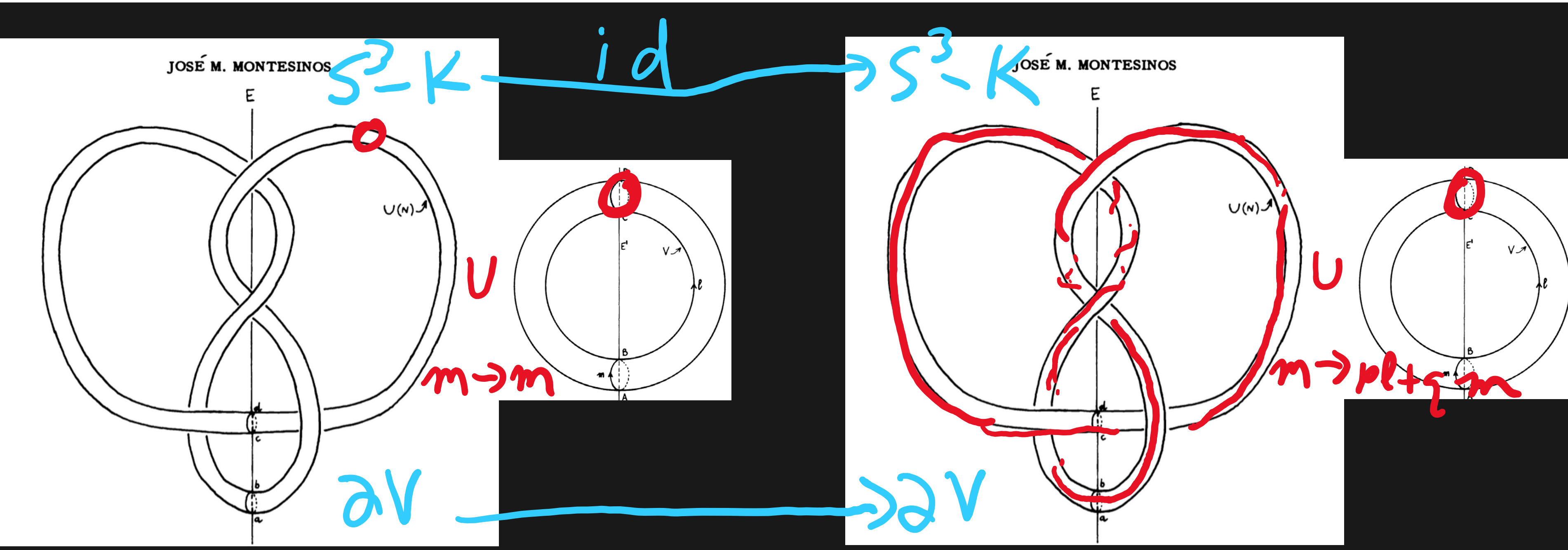

http://dx.doi.org/10.1090/conm/044/813107

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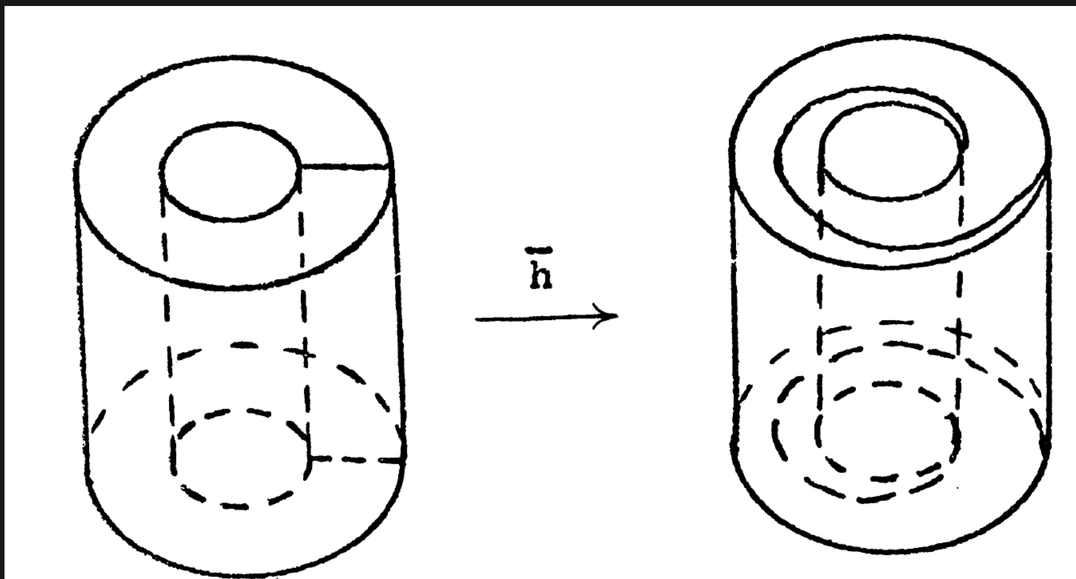
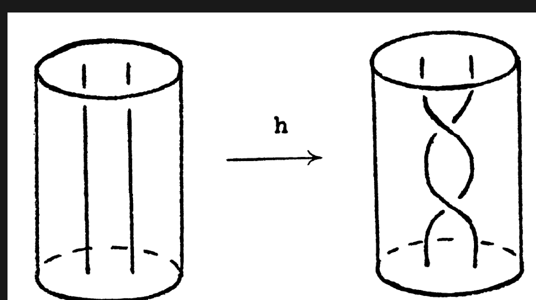
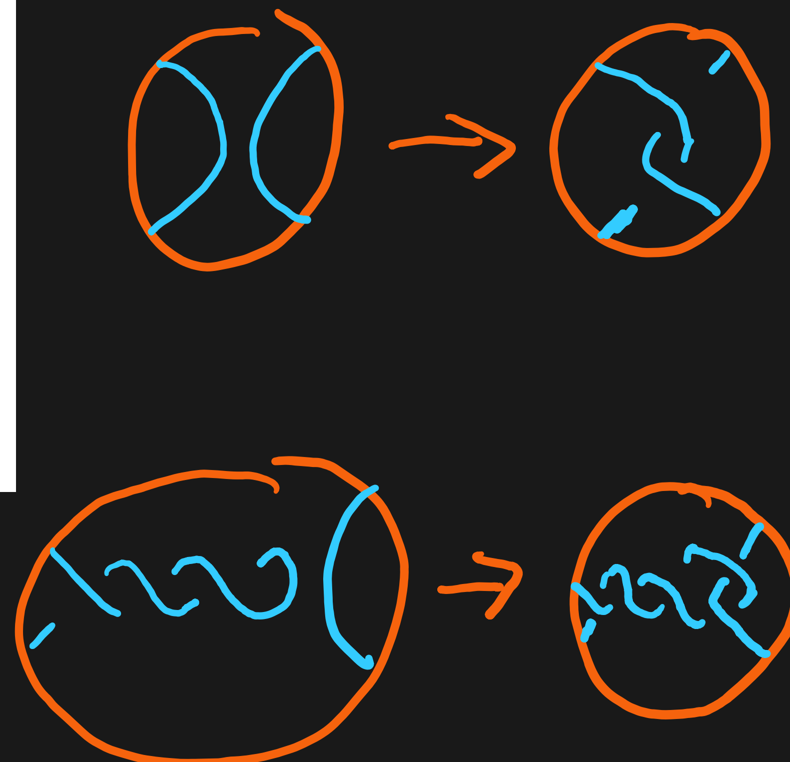
THE UNKNOTTING NUMBER OF A CLASSICAL KNOT

W. B. Raymond Lickorish







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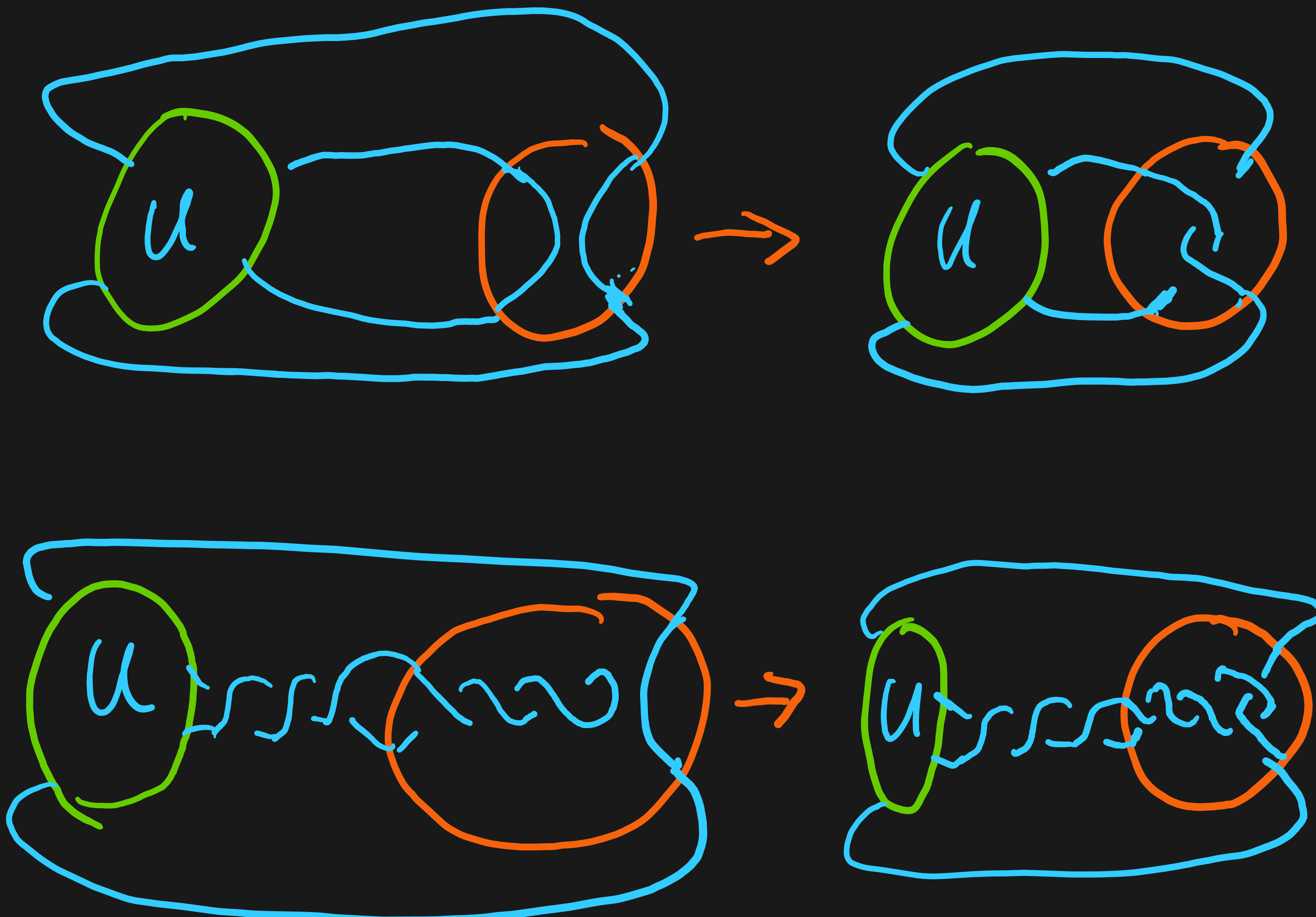
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U
U
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HW 2: Choose 1 problem

Option 2: Relate tangle equations on previous slide to double branch cover.

Hint: