. . Sketches of Topology . .

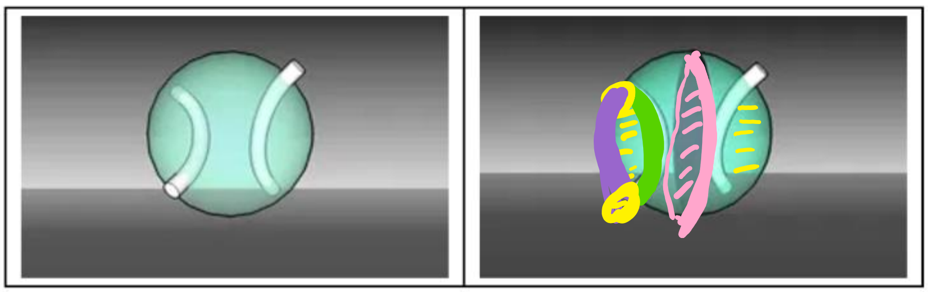
visualizations of low dimensional topology

Double Branched Covers of Rational Tangles and the Montesinos Trick

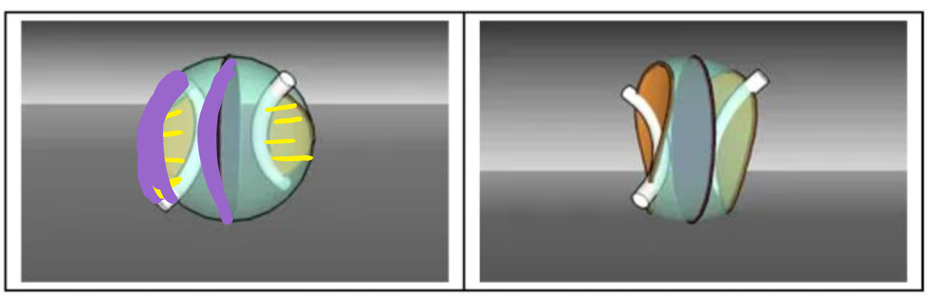
Here we see a rational tangle in a ball. The ball is solid, but we make it translucent to see inside. By a rational tangle we mean two locally unknotted arcs that may be separated by a properly embedded disk. Think of the disk, shown in a dark red, as a meridional disk.

not locally knotted

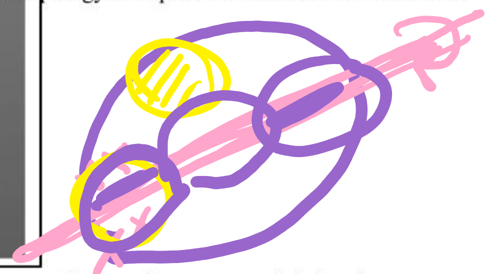
2-string



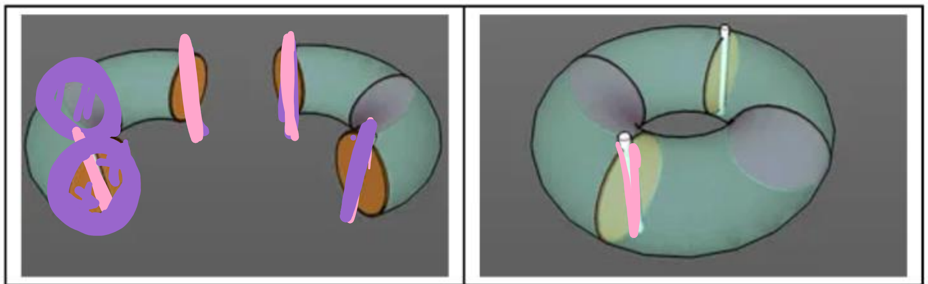
Since the arcs of the tangle are unknotted and separated, they can be pushed onto the boundary sphere while keeping their endpoints fixed. The two "half" disks that are orange (but look more yellow through the translucent ball) guide this isotopy. We can slice open the ball along these two half disks. This makes a "plug" of a sort.



Stretching, bending, and twisting a bit distorts this plug into half of a solid torus.



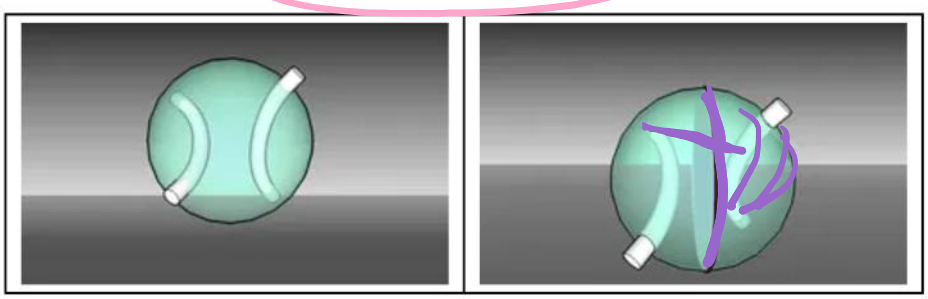
We can then attach two copies of this to make a solid torus. (This is not a mirrored copy — think of bending the second copy of the plug in the other direction.) Each of the original arcs of the tangle become an arc in the solid torus, but everything else is doubled. In particular, the red disk in the tangle becomes two meridional disks of the solid torus.



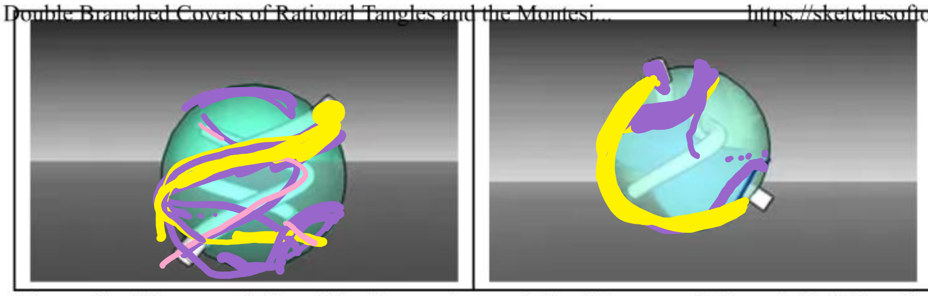
We say that this solid torus is the double branched cover of the tangle. Or more precisely, the solid torus is the double cover of the ball branched over the two arcs of the tangle.

One of the fun things about topology (and just about everything else in life) is seeing what happens when you make small changes.

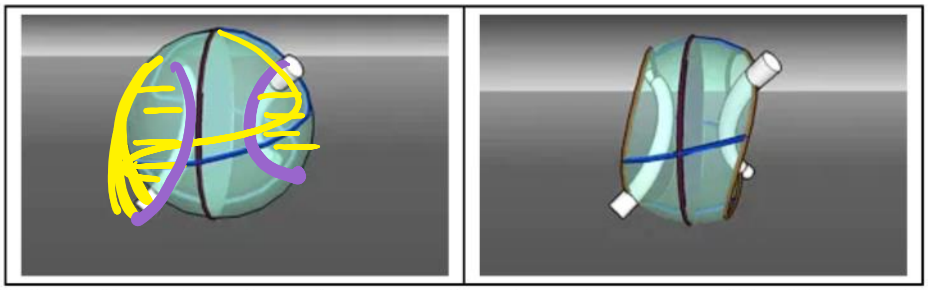
What if we wanted to replace our original tangle, shown again down here,

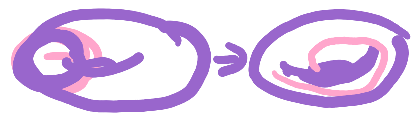


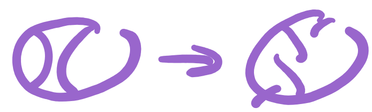
with this other tangle? This new tangle is related to the original one by a single crossing change. We've kept the boundary sphere and the endpoints of the tangle the same, but we've changed how the arcs sit in the ball. This new tangle has the blue meridional disk shown. (The ball has been rotated a bit to better show off the disk.)



Since the blue meridian (the boundary of the blue meridional disk) is all that's needed to define the blue meridional disk and this new tangle, we can see how the blue meridian sits on the sphere with respect to the red meridian. Notice how the blue meridian meets the red meridian four times. So where does this blue meridian go in the double branched cover of the original tangle? Let's slice the original tangle open again and see.

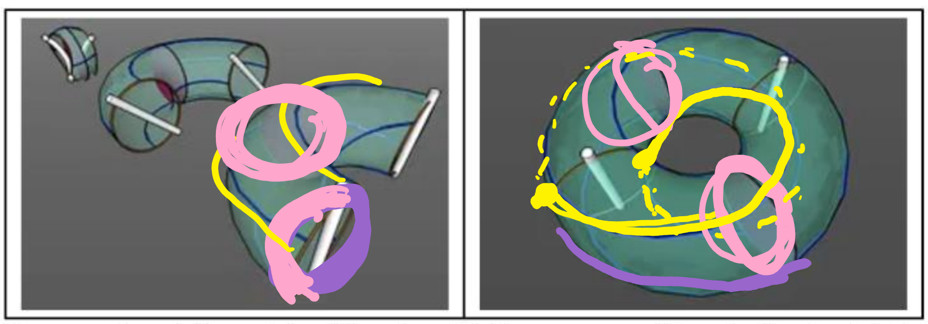




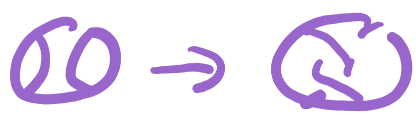


Follow the earlier steps above carrying along the pieces of the blue meridian. (To keep the picture from being too cluttered, we only show the orange boundaries of the "slicing" disks.) When put together the blue meridian becomes two curves on the boundary of the original solid torus. Each of these blue curves crosses each red meridian twice. Just as the blue curve on the boundary sphere of the original tangle didn't bound a disk disjoint from the two original arcs, these blue meridians don't bound disks in the solid torus.

$2(m) \to 2m$



$2l + m$



If we replaced the original tangle with the new one, then in the double branched cover we would replace the solid torus with the red meridian with a solid torus where the blue meridians bounded disks. Exchanging solid tori like this is a process called Dehn surgery. This method of relating the swapping of rational tangles to Dehn surgery is sometimes called the Montesinos Trick.

$2(\frac{1}{2}m) \to 1m$

~ by Ken Baker on January 25, 2008.

Posted in double branched covers, tangles

Tags: crossing change, double branched cover, half integer surgery, montesinos trick, rational tangle, solid torus

Sunday, September 13, 2020 10:55 PM

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A calculus for rational tangles: applications to DNA recombination

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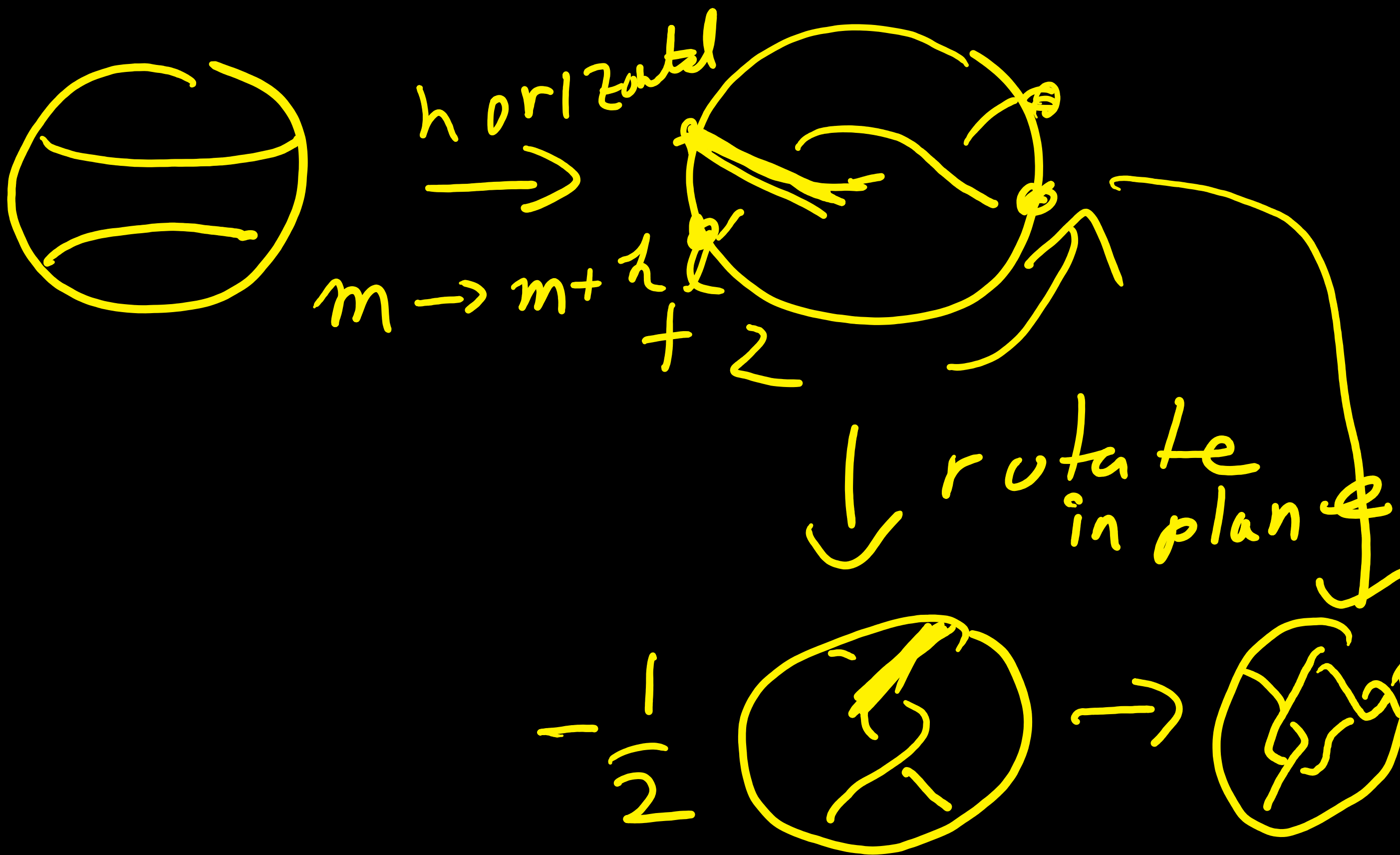
$$\boxed{\begin{pmatrix} u & v' \\ v & u' \end{pmatrix} = \begin{pmatrix} 1 & a_{2k} \\ 0 & 1 \end{pmatrix}\begin{pmatrix} 1 & 0 \\ a_{2k-1} & 1 \end{pmatrix}\cdots\begin{pmatrix} 1 & 0 \\ a_1 & 1 \end{pmatrix}}$$

$$u/v = \beta/\alpha$$ ← continued frach

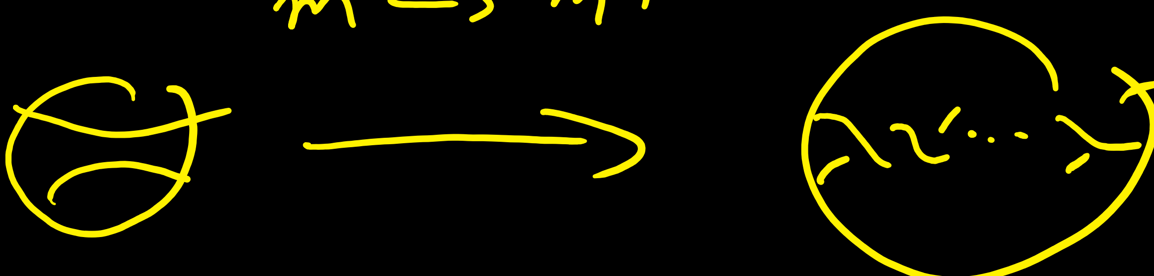
Warnings:

I will be/have been imprecise/wrong
today/last week

1.) signs 2.) longitude



$m \to m + h\ell$



$m \to \ell$

$$m \longrightarrow m + k\ell$$

$$\ell \longrightarrow \ell$$

$$am + b\ell \longrightarrow$$

$$\begin{array}{cc} & \begin{array}{cc} m & \ell \end{array} \\ \begin{array}{c} m \\ \ell \end{array} & \begin{bmatrix} 1 & k \\ 0 & 1 \end{bmatrix} \end{array} \begin{bmatrix} a \\ b \end{bmatrix} \rightarrow$$