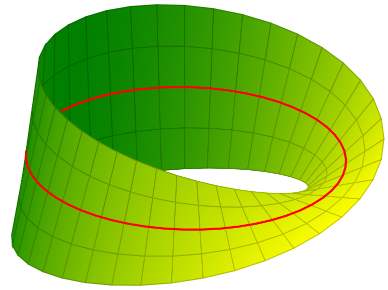
Mobius band = Cross cap



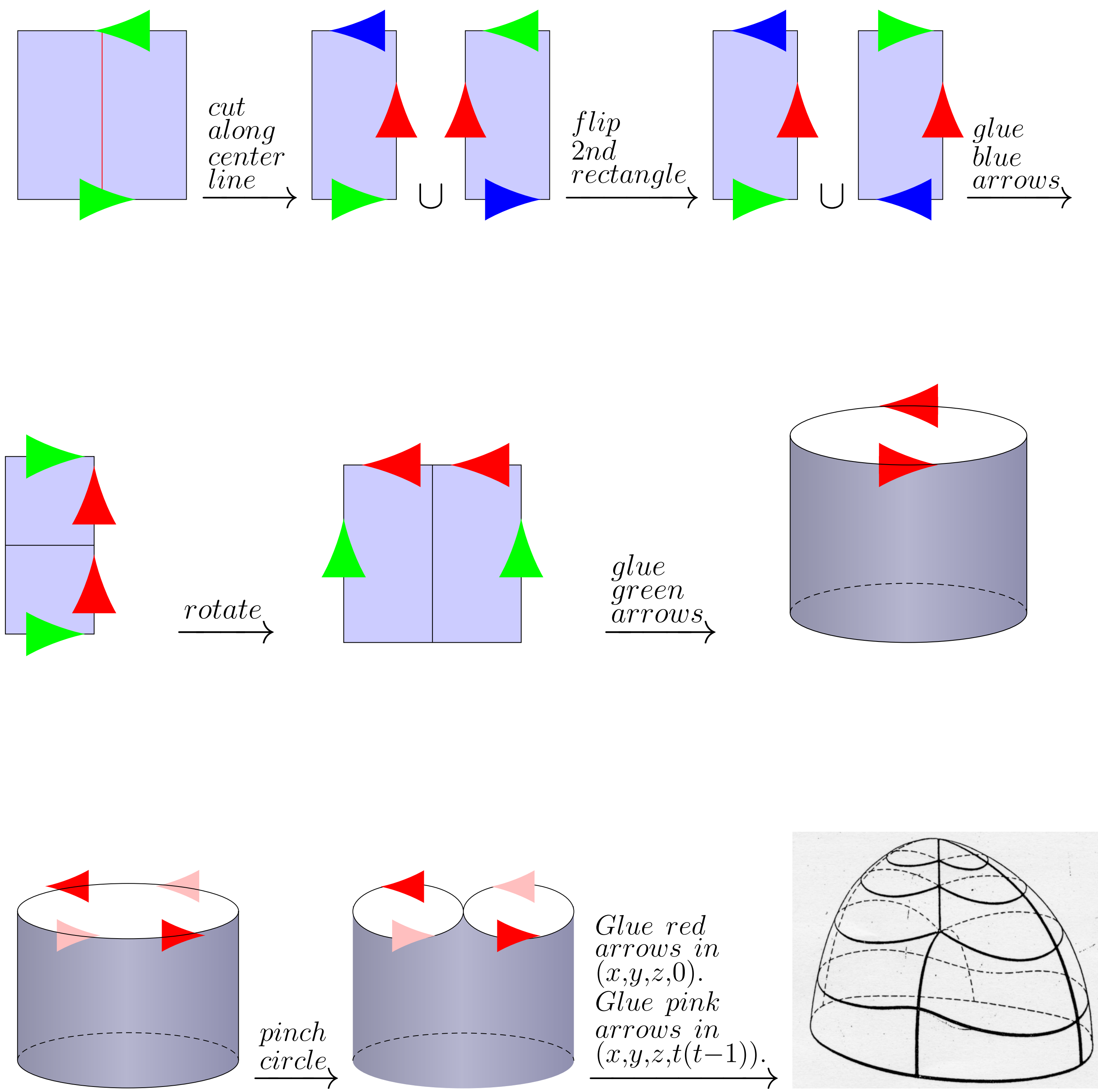


Figure 1: Cross cap in $\mathbb{R}^4$. Last figure from http://www.freud-lacan.com/freud/Champs_specialises/Langues_etrangeres/Anglais/Le_cross_cap_de_Lacan_ou_asphere

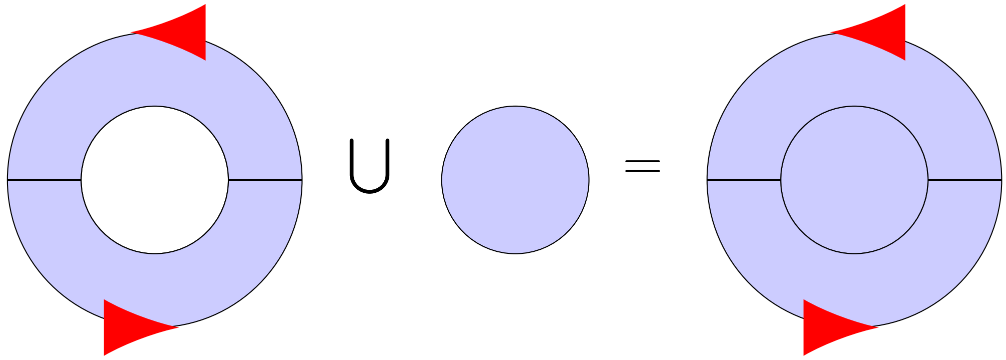


Figure 2: Mobius band $\cup$ disk = projective plane = $\mathbb{RP}^2$

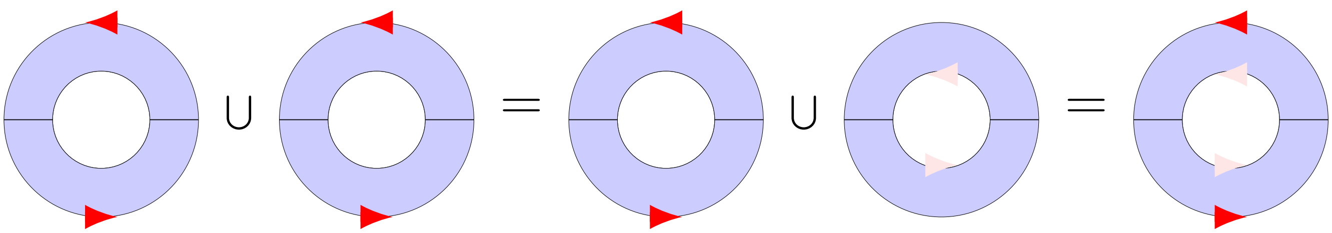


Figure 3: $\mathbb{RP}^2 \# \mathbb{RP}^2$ = Mobius band $\bigcup$ Mobius band = Klein bottle

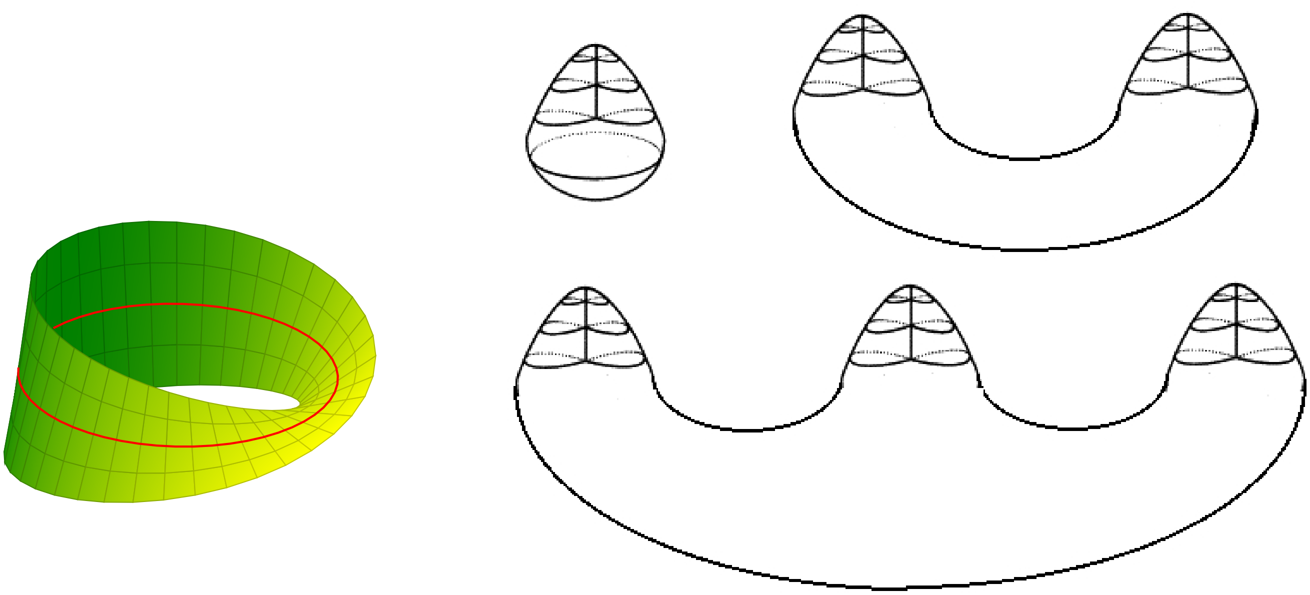


Figure 4: Right figures (connected sum of projective planes) from: people.math.osu.edu/fiedorowicz.1/math655/classification.html

$$S^{n-1} = \{(x_0, ..., x_{n-1}, 0) \mid ||x|| = 1\}$$
$$\subset \{(x_0, ..., x_n) \mid ||x|| = 1\} = S^n$$

Let U^n = closed upper hemisphere

$$= \{(x_0, ..., x_n) \mid ||x|| = 1, x_n \geq 0\}.$$

Let D^n = closed disk $= \{(x_0, ..., x_{n-1}, 0) \mid ||x|| \leq 1\}$.

Then $\pi : U^n \to D^n, \quad \pi(x_0, ..., x_n) = (x_0, ..., x_{n-1}, 0)$,
projection onto the first $n-1$ components is a homeomorphism.

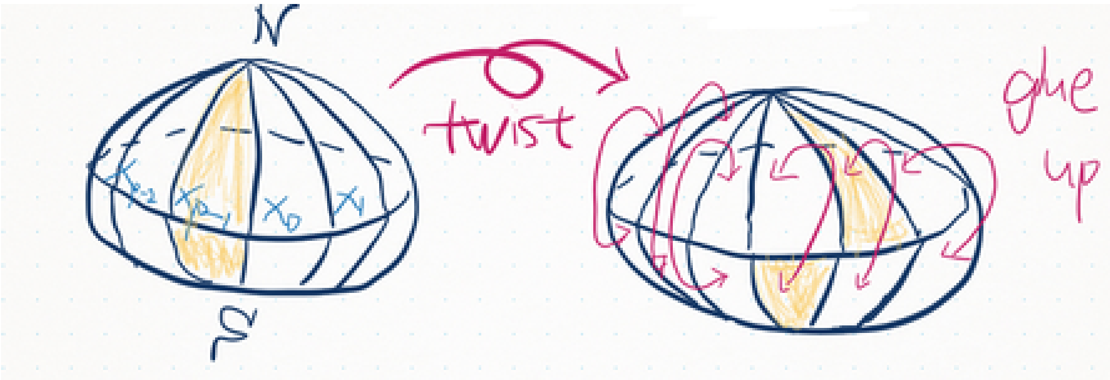
$$\mathbb{R}\mathrm{P}^n = S^n/(x \sim -x)$$
$$= D^n/\sim \text{ where } x \sim -x \text{ for all } x \in \partial D^n = S^{n-1}$$
$$= intD^n \bigsqcup_{\partial D^n} (\partial D^n/(x \sim -x))$$
$$= intD^n \bigsqcup_{\partial D^n = S^{n-1}} (S^{n-1}/(x \sim -x))$$
$$= intD^n \bigsqcup_{\partial D^n = S^{n-1}} \mathbb{R}\mathrm{P}^{n-1}$$

Thus $\mathbb{R}\mathrm{P}^n = \mathbb{R}\mathrm{P}^{n-1} \bigsqcup_{\phi : \partial e^n \to \mathbb{R}\mathrm{P}^{n-1}} e^n$

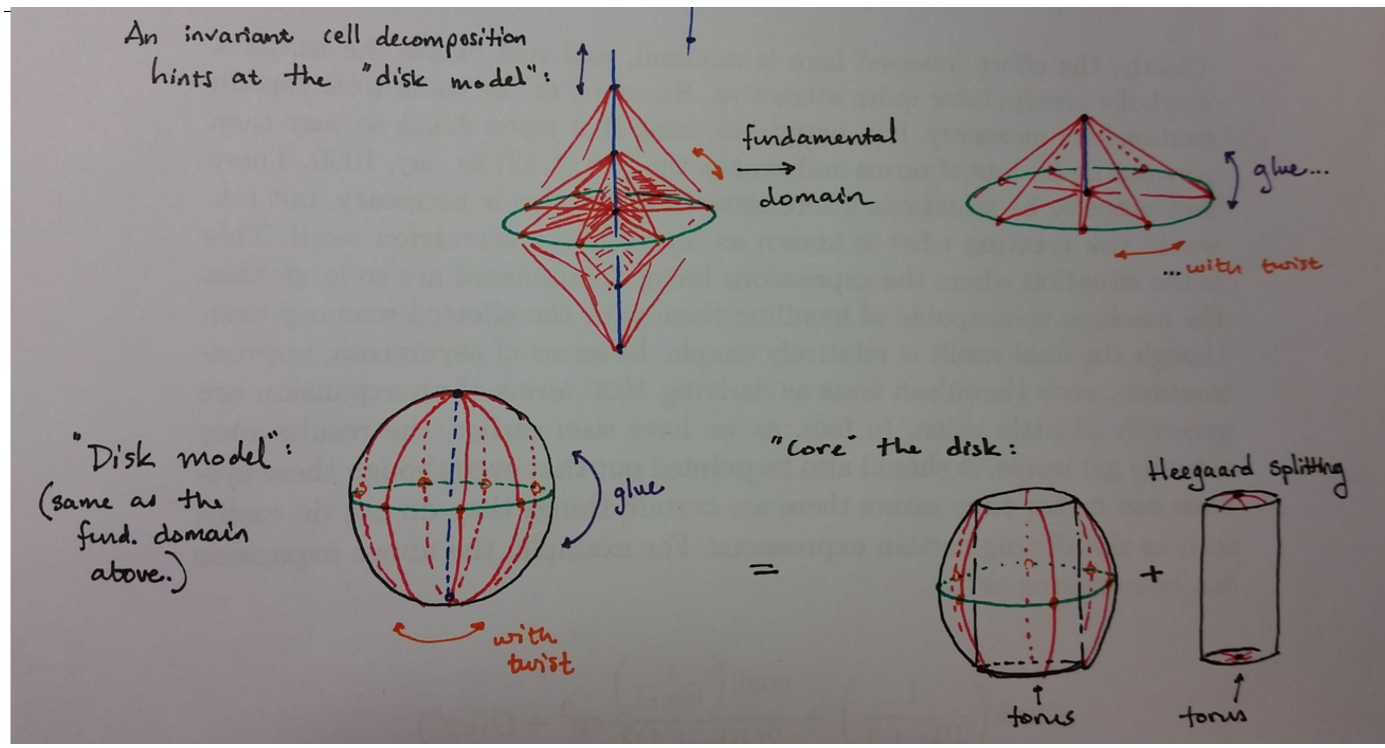
Recall degree of identity map $i : S^n \to S^n$ is 1.

Recall degree of antipodal map ($= n+1$ reflections) is $(-1)^{n+1}$.

3D Lens spaces



https://plus.maths.org/content/dont-judge-black-hole-its-area-2
Don't judge a black hole by its area By Yen Chin Ong



https://math.stackexchange.com/questions/1186778/visualization-of-lens-spaces

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See also Jeffrey R. Weeks - Shape of Space: How to Visualize Surfaces and Three-Dimensional Manifolds: 2nd (second) Edition Hardcover December 12, 2002

and http://www.geometrygames.org/

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KODAI MATH. J.
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EIGENMODES OF LENS AND PRISM SPACES

ROLAND LEHOUCQ, JEAN-PHILIPPE UZAN AND JEFFREY WEEKS

Abstract

Cosmologists are taking a renewed interest in multiconnected spherical 3-manifolds (spherical spaceforms) as possible models for the physical universe. To understand the formation of large scale structures in such a universe, cosmologists express physical quantities, such as density fluctuations in the primordial plasma, as linear combinations of the eigenmodes of the Laplacian, which can then be integrated forward in time. This need for explicit eigenmodes contrasts sharply with previous mathematical investigations, which have focused on questions of isospectrality rather than eigenmodes. The present article provides explicit orthonormal bases for the eigenmodes of lens and prism spaces.