Recall that the local homology of a space X are the homology groups, $H_n(X, X-\{p\})$. If points are closed in X, then by ________________________, $H_n(X, X-\{p\}) \cong H_n(U, U-\{p\})$ where U is an open neighborhood of p.

Thus if $f : X \to Y$ is a homeomorphism,
$H_n(U, U - \{p\}) \cong H_n(X, X - \{p\}) \cong H_n(Y, Y - \{f(p)\}) \cong H_n(f(U), f(U) - \{f(p)\})$.

Recall $f : X \to Y$ is a local homeomorphism if $\forall p \in X, \exists$ open neighborhood U of p such ath $f(U)$ is open and $f : U \to f(U)$ is a homeomorphism.

Thus if X and Y are locally homeomorphic,
$H_n(X, X - \{p\}) \cong H_n(U, U - \{p\}) \cong H_n(f(U), f(U) - \{f(p)\}) \cong H_n(Y, Y - \{f(p)\})$.

1.) Prove Thm 2.26: If nonempty open sets $U \subset \mathbb{R}^m$ and $V \subset \mathbb{R}^n$ are homeomorphic, then $m = n$.

2.) Lemma: Let X be a CW complex of dimension n. Show that $H_k(X, X - \{p\}) = 0$ for $k > n$ and that there exists an $p \in K$ such that $H_n(X, , X - \{p\}) = \mathbb{Z}$. Thus local homology groups can be used to determine the dimension of a CW complex.

3.) Determine the local homology groups of $B^n = \{x \in \mathbb{R}^n \mid ||x|| \leq 1\}$. Show that if $f : B^n \to B^n$ is a homeomorphism, then $f(\partial B^n) = \partial B^n$

4.) Show that if M is an m-dimensional manifold (without boundary), then
$H_i(M, M - \{p\}) \cong \widetilde{H_i}(S^{m-1})$.

5.) If M is an m-dimensional manifold homeomorphic to an n-dimensional manifold N, then $m = n$.

6.) If M and N are manifolds with boundary and $f : M \to N$ is a homeomorphism, does $f(\partial M) = \partial N$?

7.) Note that the boundary of a Mobius band is not homeomorphic to the boundary of an annulus. Show that the Mobius band is not homeomorphic to an annulus.

8.) Let $f : S^1 \times S^1 \to S^1 \times S^1$, $f(x, y) = (x, -y)$. Let M, L be the standard meridian an longitude of $S^1 \times S^1$. Note that $S^1 \times S^1/(M = L = M \cap L) = S^2$. Find the degree of the map $f_1 : S^2 \to S^2$ induced by f. Do the same for the map $g : S^1 \times S^1 \to S^1 \times S^1$, $g(x, y) = (y, -x)$. Describe $f_* : H_2(S^1 \times S^1) \to H_2(S^1 \times S^1)$ and $g_* : H_2(S^1 \times S^1) \to H_2(S^1 \times S^1)$.