Euler characteristic (simple form):

χ = number of vertices – number of edges + number of faces

Or in short-hand,

$$\chi = |V| - |E| + |F|$$

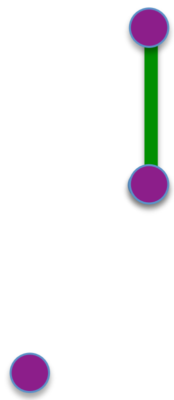
where V = set of vertices

E = set of edges

F = set of faces

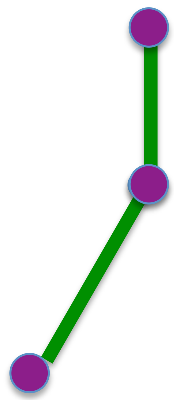
& the notation |X| = the number of elements in the set X.

$$\chi = |V| - |E| + |F|$$



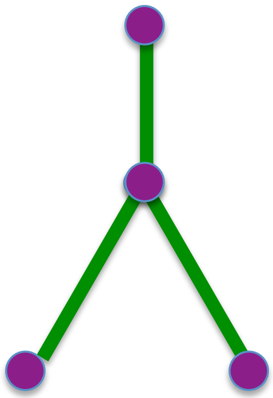
$$\chi = 3 - 1 = 2$$

$\chi = |V| - |E| + |F|$



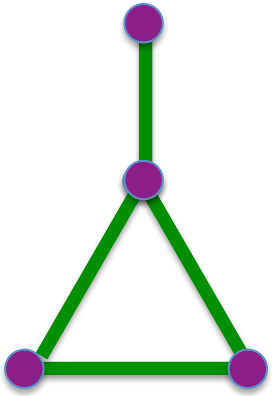
$\chi = 3 - 2 = 1$

$\chi = |V| - |E| + |F|$



$\chi = 4 - 3 = 1$

$$\chi = |V| - |E| + |F|$$



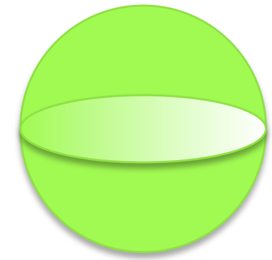
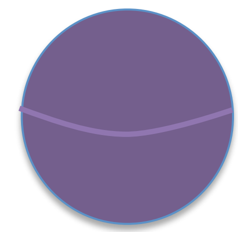
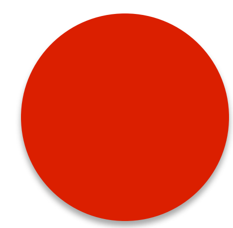
$$\chi = 4 - 4 = 0$$

The Euler characteristic is a topological invariant

That means that if two objects are topologically the same, they have the same Euler characteristic.

Example:

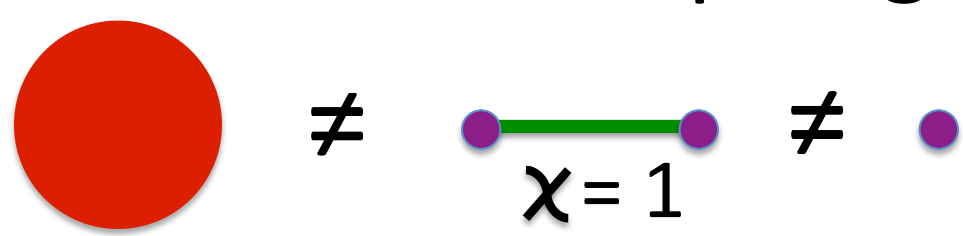
$\chi = 1$ $\chi = 1$

Euler characteristic		
2	sphere $= \{ x \text{ in } R^3 : \ \|\|x\|\| = 1 \}$	
1	ball $= \{ x \text{ in } R^3 : \ \|\|x\|\| \leq 1 \}$	
	disk $= \{ x \text{ in } R^2 : \ \|\|x\|\| \leq 1 \}$	
	closed interval $= \{ x \text{ in } R : \ \|\|x\|\| \leq 1 \}$	

The Euler characteristic is a topological invariant

That means that if two objects are topologically the same, they have the same Euler characteristic.

But objects with the same Euler characteristic need not be topologically equivalent.



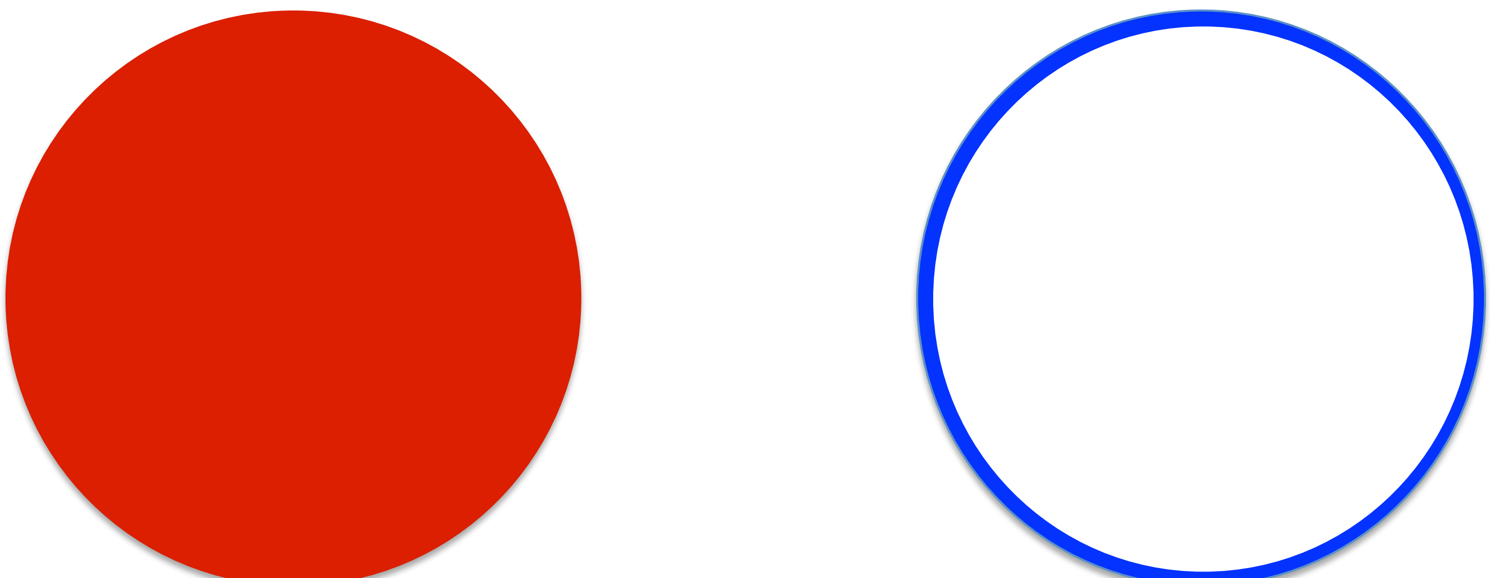
Let R be a subset of X

A *deformation retract* of X onto R is a continuous map $F: X \times [0, 1] \rightarrow X$, $F(x, t) = f_t(x)$ such that

f_0 is the identity map,

$f_1(X) = R$, and

$f_t(r) = r$ for all r in R.



If R is a deformation retract of X, then $\chi(R) = \chi(X)$.

Let R be a subset of X

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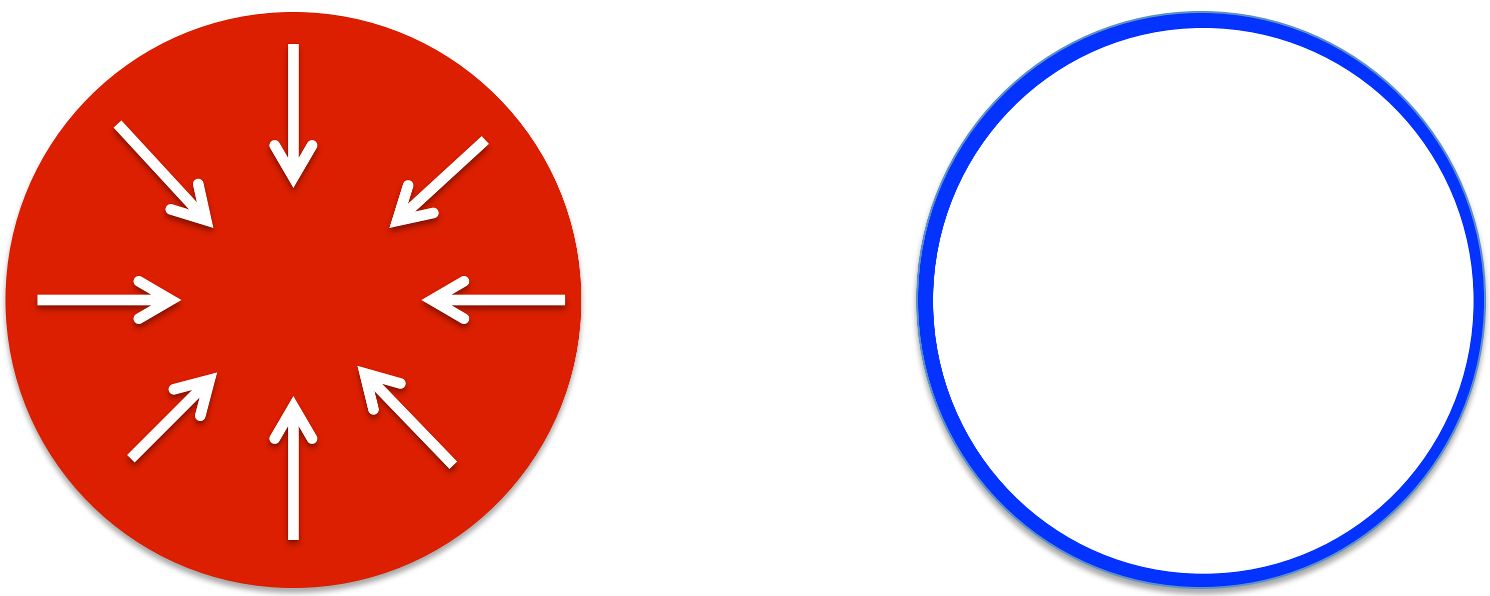
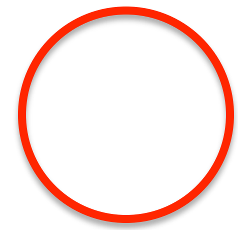
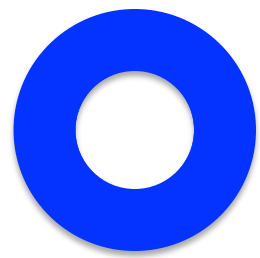
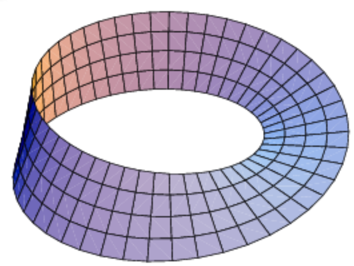
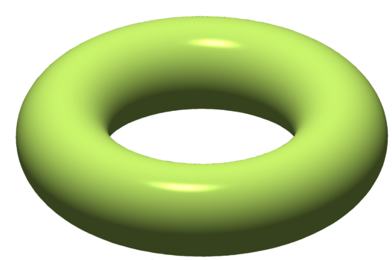
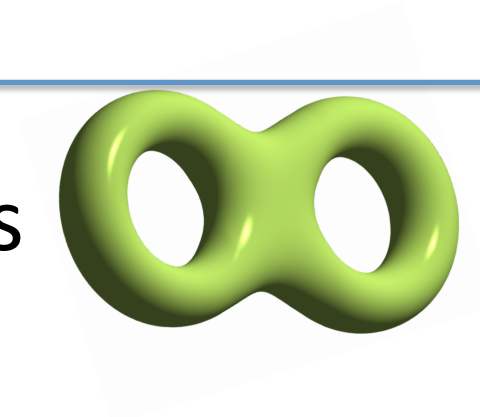
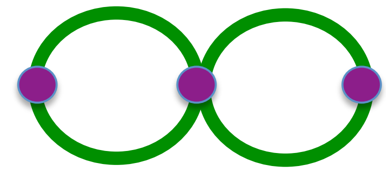
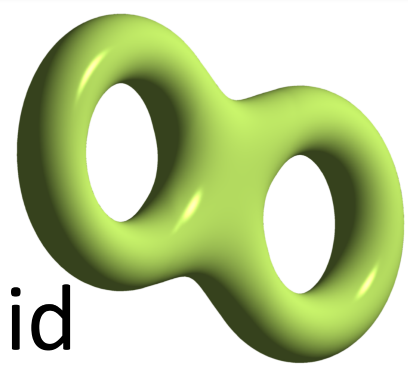


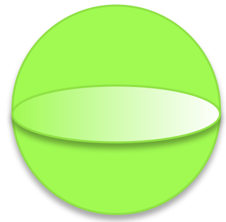
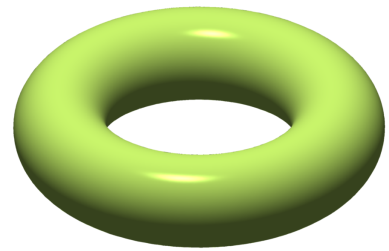
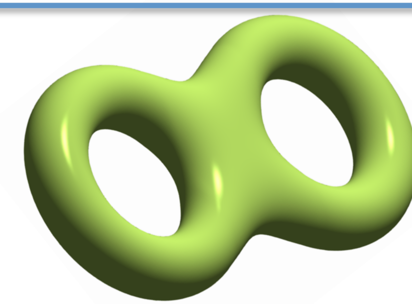
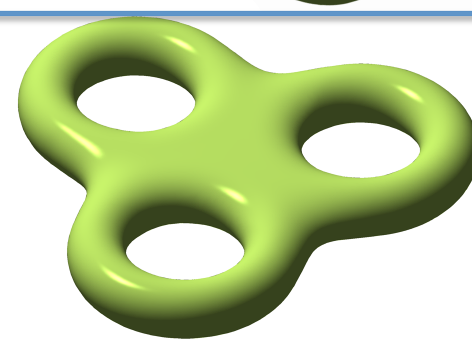
If R is a deformation retract of X, then $\chi(R) = \chi(X)$.

Euler characteristic	
0	S^1 = circle = { x in R^2 : ||x|| = 1 }  Annulus  Mobius band  Solid torus = S^1 x disk Torus = S^1 x S^1 

Mobius band and torus images from https://en.wikipedia.org/wiki/Euler_characteristic

Euler characteristic	
-1	Solid double torus  The graph: 
-2	Double torus = genus 2 torus = boundary of solid double torus 

Genus n tori images from https://en.wikipedia.org/wiki/Euler_characteristic

Euler characteristic	2-dimensional orientable surface without boundary
2	sphere 
0	$S^1 \times S^1$ = torus 
-2	genus 2 torus 
-4	genus 3 torus 

Genus n tori images from https://en.wikipedia.org/wiki/Euler_characteristic