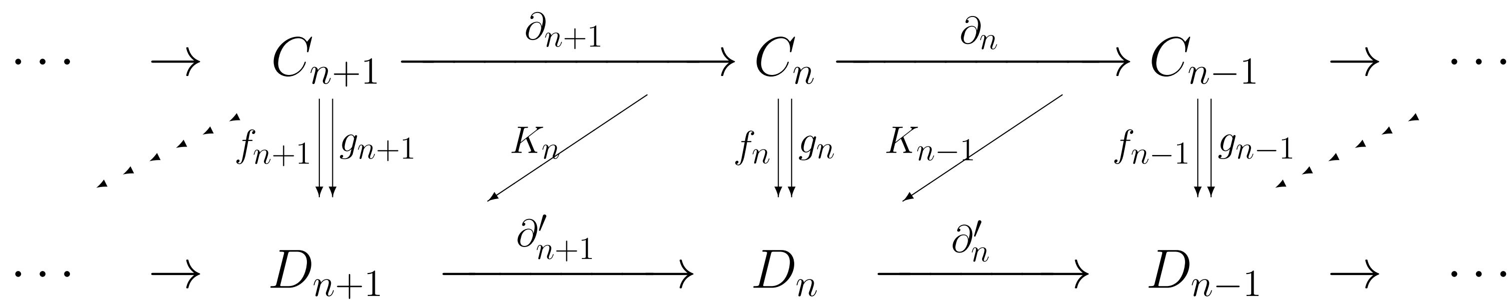
The following diagram does NOT commute:



The chain maps $f_n, g_n : C_n \to D_n$ are chain homotopic if $\exists K_n : C_n \to D_n$ such that

$$\partial'_{n+1}K_n + K_{n-1}\partial_n = f_n - g_n$$

Claim: $f_* = g_* : H_\bullet^C \to H_\bullet^D$

Proof: If $\alpha \in Z_n^C$, then $\partial\alpha = 0$.

$$\partial'_{n+1}K_n(\alpha) + K_{n-1}\partial_n(\alpha) = f_n(\alpha) - g_n(\alpha)$$

$$f_n(\alpha) = \partial'_{n+1}K_n(\alpha) + g_n(\alpha)$$

$$[f_n(\alpha)] = [\partial'_{n+1}K_n(\alpha) + g_n(\alpha)] = [g_n(\alpha)]$$

Thus $f_*(\alpha) = g_*(\alpha)$