Lemma 27.5 (*The Lebesgue number lemma*)
If $\mathcal{U}$ is an open covering of the compact metric space X, then $\delta > 0$ such that if $A \subset X$ with $\text{diam}(A) < \delta$, then $\exists\, U \in \mathcal{U}$ such that $A \subset U$.

Thm 59.1: Suppose $X = U \cup V$ where
U, V are open and $U \cap V$ is path connected. Let $i_U : U \to X$ and $i_V : V \to X$ be inclusion maps. Then $\pi_1(X)$ is generated by the images of i_{U*} and i_{V*}.

I.e., If $g \in \pi_1(X)$, the $g = g_1 * g_2 * .... * g_n$ where for each i, g_i is in either $i_{U*}(\pi_1(U))$ or $i_{V*}(\pi_1(V))$.

I.e., $j : \pi_1(U) * \pi_1(V) \to \pi_1(X)$ induced by the two inclusion maps is surjective.

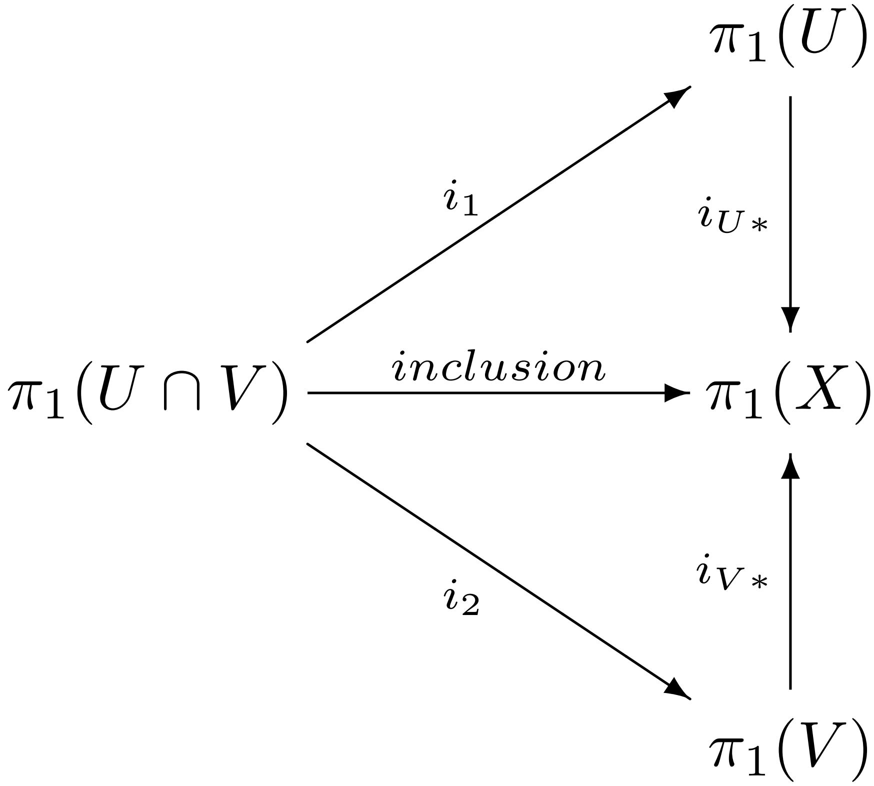
I.e, $\pi_1(X) = \pi_1(U) * \pi_1(V)/ker(j)$

$= < a_1, ..., a_i, b_1, ..., b_j \mid s_1, ..., s_l, t_1, ..., t_m > /ker(j)$

Theorem 70.2. $\ker(j)$ = least normal subgroup generated by $\{i_U(c_1)^{-1} i_V(c_1), ..., i_U(c_n)^{-1} i_V(c_n)\}$.

I.e., $\pi_1(X) = < a_1, ..., a_i, b_1, ..., b_j \mid s_1, ..., s_l, t_1, ..., t_m,$
$i_U(c_1)^{-1} i_V(c_1), ..., i_U(c_n)^{-1} i_V(c_n) >$

The following maps are all induced by inclusion



Thm 70.1: $U, V, U \cap V$ open and path-connected.