

for sq. 1
 $[t_0, t_1] \ni a_i$ want $g(a_i) \in U \cup V$
 p.c.

Proof of thm 59.1:

Let $g : I \rightarrow X$ be a loop in $X = U \cup V$.

Since U and V are open, $I \subset g^{-1}(U) \cup g^{-1}(V)$.

By the Lebesgue number lemma, $\exists \delta > 0$ such that if $A \subset X$ with $\text{diam}(A) < \delta$, then $A \subset g^{-1}(U)$ or $A \subset g^{-1}(V)$.

Thus if $I = \cup [a_i, a_{i+1}]$ where $\text{diam}([a_i, a_{i+1}]) < \delta$, then $[a_i, a_{i+1}] \subset g^{-1}(U)$ or $[a_i, a_{i+1}] \subset g^{-1}(V)$.
 Thus $g([a_i, a_{i+1}]) \subset g(g^{-1}(U)) \subset U$ or $g([a_i, a_{i+1}]) \subset g(g^{-1}(V)) \subset V$.

If there exists a_k such that $g(a_k) \notin U \cap V$, replace the intervals $[a_{k-1}, a_k]$ and $[a_k, a_{k+1}]$ with $[a_{k-1}, a_{k+1}]$.

If $g(a_k) \in U$, then $g(a_k) \notin V$. Hence $g([a_{k-1}, a_k]) \subset U$ & $g([a_k, a_{k+1}]) \subset U$. Thus $g([a_{k-1}, a_{k+1}]) \subset U$.
 Similarly, $g(a_k) \in V$ implies $g([a_{k-1}, a_{k+1}]) \subset V$.

Thus we can write $I = \cup [c_i, c_{i+1}]$ where $c_i \in U \cap V \forall i$ and $g([c_i, c_{i+1}]) \subset U$ or $g([c_i, c_{i+1}]) \subset V$.

Since $U \cap V$ is path connected, \exists a path α_i between x_0 and c_i .

Thus $g = (\alpha_0 * g_1 * \bar{\alpha}_1) (\alpha_1 * g_2 * \bar{\alpha}_2) \cdots (\alpha_{n-1} * g_n * \bar{\alpha}_n)$ where for each i , $(\alpha_{i-1} * g_i * \bar{\alpha}_i)$ is in $\pi_1(U)$ or in $\pi_1(V)$.

I.e., $j : \pi_1(U) * \pi_1(V) \rightarrow \pi_1(X)$ induced by the two inclusion maps is surjective.

WLOG $g_i \in U$ where $i=0$

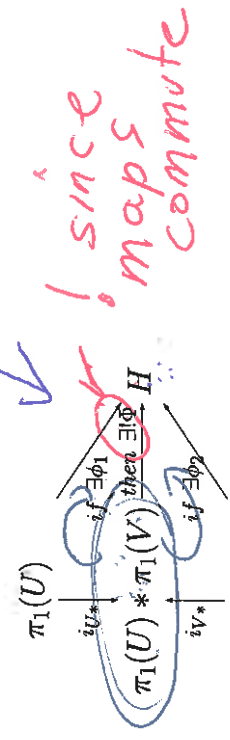
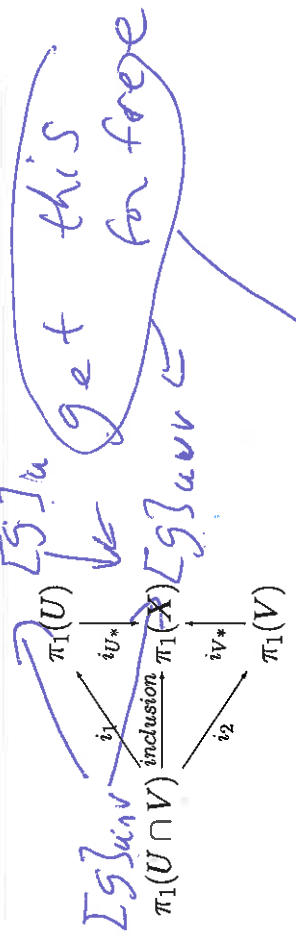
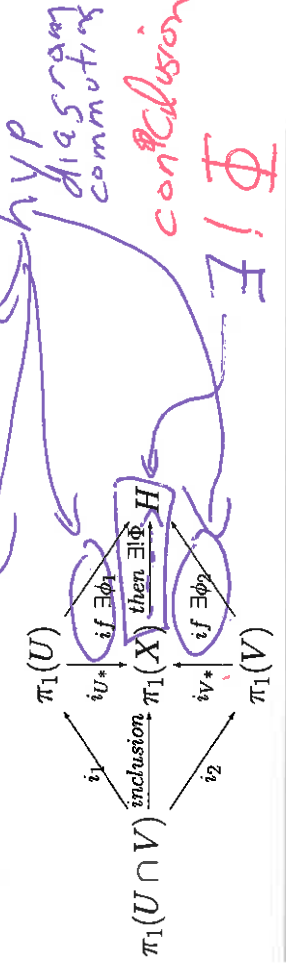
only need $U \cap V$ p.c.

$j : \pi_1(U) * \pi_1(V) \rightarrow \pi_1(X)$ induced by the two inclusion maps is surjective.

Thus $\pi_1(X) = \pi_1(U) * \pi_1(V) / \ker(j)$

SVK

Thm 70.1: $U, V, U \cup V$ open and path-connected.



If $g = g_1 * g_2 * \dots * g_n = \phi_1(g_1) * \dots$
 $g_i \in U$ where $i=0$