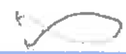
EX

1)  Wedge Sum $S^1 \vee S^1 = S^1 \sqcup S^1 / x_1 \sim x_2$ (disjoint union)

$x_1 \in S^1$, $x_2 \in S^1$

(Take a point on the first circle and identify it with a point on the other circle)

$U =$ , $V =$ , $U \cap V =$ 

$\pi_1(U) = \pi_1(\ldots) = \pi_1(\ldots) = \langle M \mid \ \rangle$

via deformation retract

$\pi_1(V) = \pi_1(\ldots) = \pi_1(\ldots) = \langle L \mid \ \rangle$

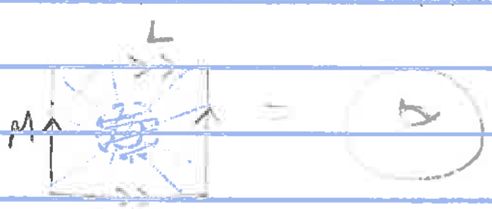
$\pi_1(U \cap V) = \pi_1(\ldots) = \pi_1(\cdot) = \langle \ \rangle = e$

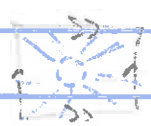
$\pi_1(U \cup V) = \langle M, L \mid \ \rangle \cong \mathbb{Z} * \mathbb{Z}$

$\quad = \pi_1(\infty)$

Note: If $\pi_1(U \cap V)$ is trivial i.e. $\langle \ \rangle = e$ then

$\pi_1(U \cup V) = \pi_1(U) * \pi_1(V)$

2) 

$\pi_1($  $) = \pi_1(\ldots) = \pi_1(\ldots) = \pi_1(\ldots) = \langle M, L \rangle$

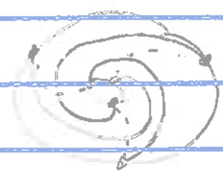
$\pi_1($  $) = \pi_1(\cdot) = \langle \ \rangle = e$

$\pi_1($  $) = \pi_1(\bigcirc) = \langle g \rangle$

$\pi_1($  $)$

$$
\begin{aligned}
&= \langle M, L \mid i_*(g) = j_*(g) \rangle \\
&= \langle M, L \mid MLM^{-1}L^{-1} = e \rangle \leftarrow \text{use either } \partial \text{ or } e \\
&= \langle M, L \mid ML = LM \rangle \\
&= \mathbb{Z}^2 = \mathbb{Z} \oplus \mathbb{Z} \quad \text{Abelian}
\end{aligned}
$$

$\Rightarrow$ Any closed curve on the torus is homotopic to $pL + qM$



$2L + 3M$

SIDE NOTE: Any embedded closed curve is ambient isotopic to $pL + qM$ $(p, q) = 1$ or to a trivial curve.

3) $\bigvee_{i=1}^{3} S^1 =$