We will use: to prove p implies q, one should assume the hypothesis p and prove the conclusion q.

Use induction to prove that if a simple connected graph G has at least 3 vertices, and if each vertex is of degree 2, then G is a cycle.

Proof by induction on $|V(G)|$

Base case: Suppose $|V(G)| = 3$. Then each vertex of G is adjacent to the other two vertices of G. Thus G is the complete graph on 3 vertices. Thus $G = K_3$. Note K_3 is a cycle.

Let $S(n)$ be the statement that "If G is a simple connected graph with n vertices, and if each vertex is of degree 2, then G is a cycle."

We need to prove that $S(n)$ implies $S(n+1)$.

Thus we assume the hypothesis, $S(n)$, and prove the conclusion, $S(n+1)$.

Induction hypothesis: If G' is a simple connected graph with n vertices, and if each vertex is of degree 2, then G' is a cycle.

Claim: If G is a simple connected graph with $n+1$ vertices, and if each vertex is of degree 2, then G is a cycle.

To prove that the claim is true, we again assume the hypothesis and prove the conclusion:

Suppose G is a simple connected graph with $n + 1$ vertices, and each vertex is of degree 2.

Claim: G is a cycle.

We need to use both the hypothesis that G is a simple connected graph with $n + 1$ vertices, and each vertex is of degree 2, as well as the induction hypothesis. To use the induction hypothesis, we need to create a graph with n vertices that satisfies the hypotheses of the induction hypothesis.

Let v be a vertex of G. Let $N(v) = \{x, y\}$

Let $G^* = G - v$. Note G^* does not satisfy the hypotheses of the induction hypothesis (G^* has 2 vertices with degree 1, but we need all vertices to have degree 2 to use the induction hypothesis). So instead

Let $G' = (V', E')$ where $V' = V(G) - v$ and $E' = E(G) \cup \{< x, y >\} - \{< v, x >, < y, v >\}$

Note G' is a simple connected graph with n vertices, and each vertex is of degree 2. Thus by the induction hypothesis, G' is a cycle.

We now need to show G is a cycle. Sometimes it helps to be specific.

G' is a cycle means we can write G' as the cycle, $x, u_1, u_2, ..., u_{n-2}, y, x$.

Then G is the cycle $v, x, u_1, u_2, ..., u_{n-2}, y, v$

Note induction gives you lots of hypothesis to work with.

Let $S(n)$ be the statement that $p(n)$ implies $q(n)$

where $p(n)$ is the hypothesis that depends on n and $q(n)$ is the conclusion that depends on n (for example if G has n vertices).

To use induction, you prove

(1) Base case: $S(n_0)$ is true.

and that

(2) $S(n)$ implies $S(n+1)$

(1) and (2) implies that $S(n_0)$ is true which implies that $S(n_0+1)$ is true which implies that $S(n_0+2)$ is true which implies that $S(n_0+3)$ is true which implies that ....

Thus $S(m)$ is true for any integer $m \geq n_0$.

To prove $S(n)$ implies $S(n+1)$, you assume hypothesis and prove conclusion:

Induction hypothesis: Suppose $S(n)$ is true.

Claim: $S(n+1)$ is true

Note $S(n+1)$ is the statement that $p(n+1)$ implies $q(n+1)$. Thus we need to prove:

Claim: $p(n+1)$ implies $q(n+1)$

To prove $p(n+1)$ implies $q(n+1)$, you assume hypothesis and prove conclusion:

Suppose $p(n+1)$ holds.

Claim $q(n+1)$ holds.

Thus to prove the claim that $q(n+1)$ is true, you can use both $p(n+1)$ and $S(n)$. Thus you take an arbitrary graph, G, satisfying the hypothesis $p(n+1)$, modify this graph, creating a new graph, G' so that you can use $S(n)$.