https://services.math.duke.edu/education/prep03/teams/Texas%20Southern%20University/TSU_PREP.pps

From Coloring Maps to Avoiding Conflicts

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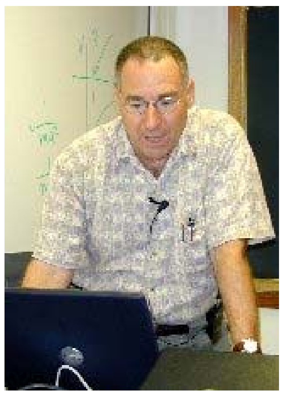
Thanks!



Frank

David

Lang

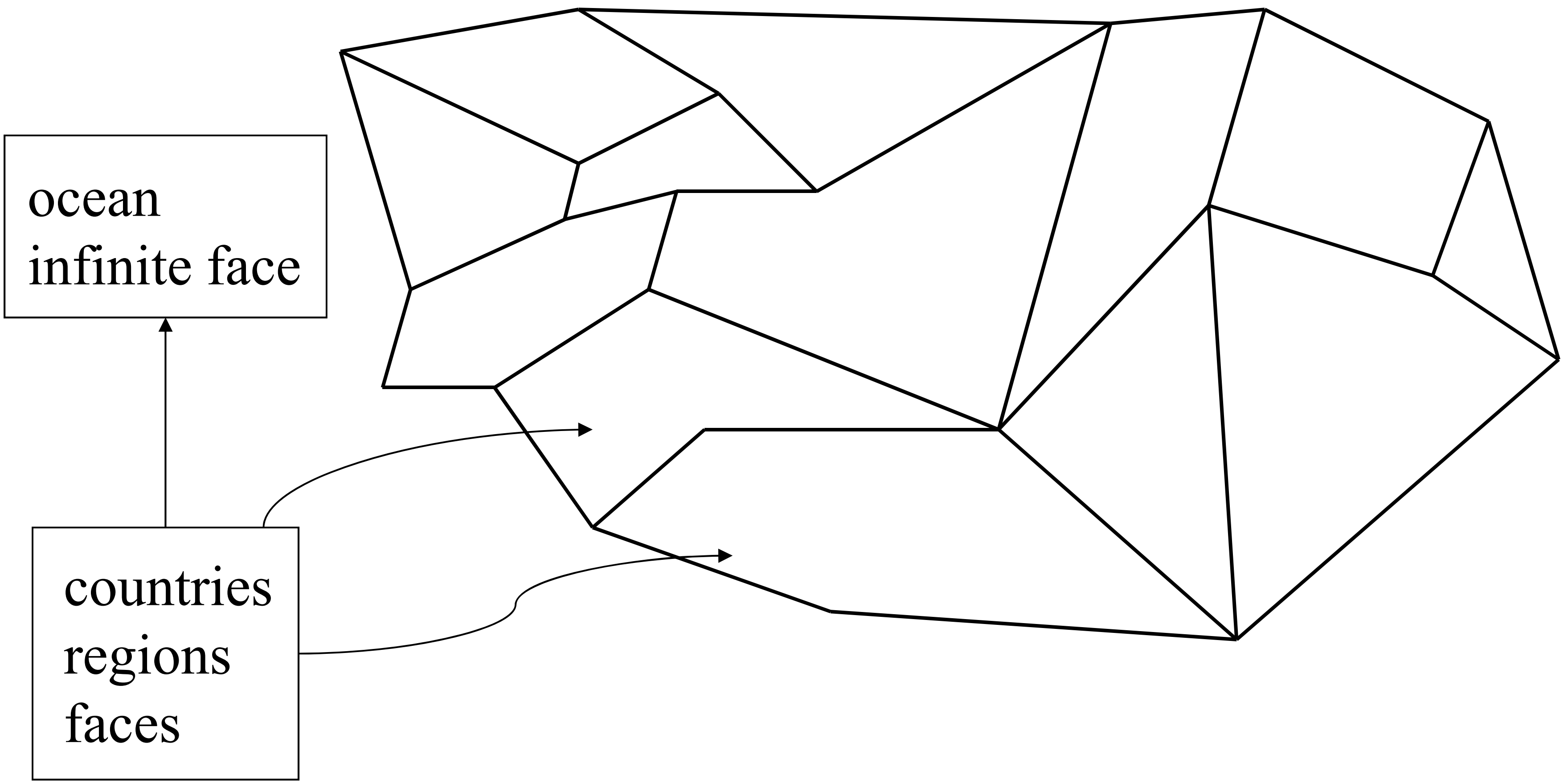
Andrew

- Directors:
  - *Lang Moore*
  - *David Smith*
  - *Frank Wattenberg*
- Funding Agencies

MAA

Map Coloring

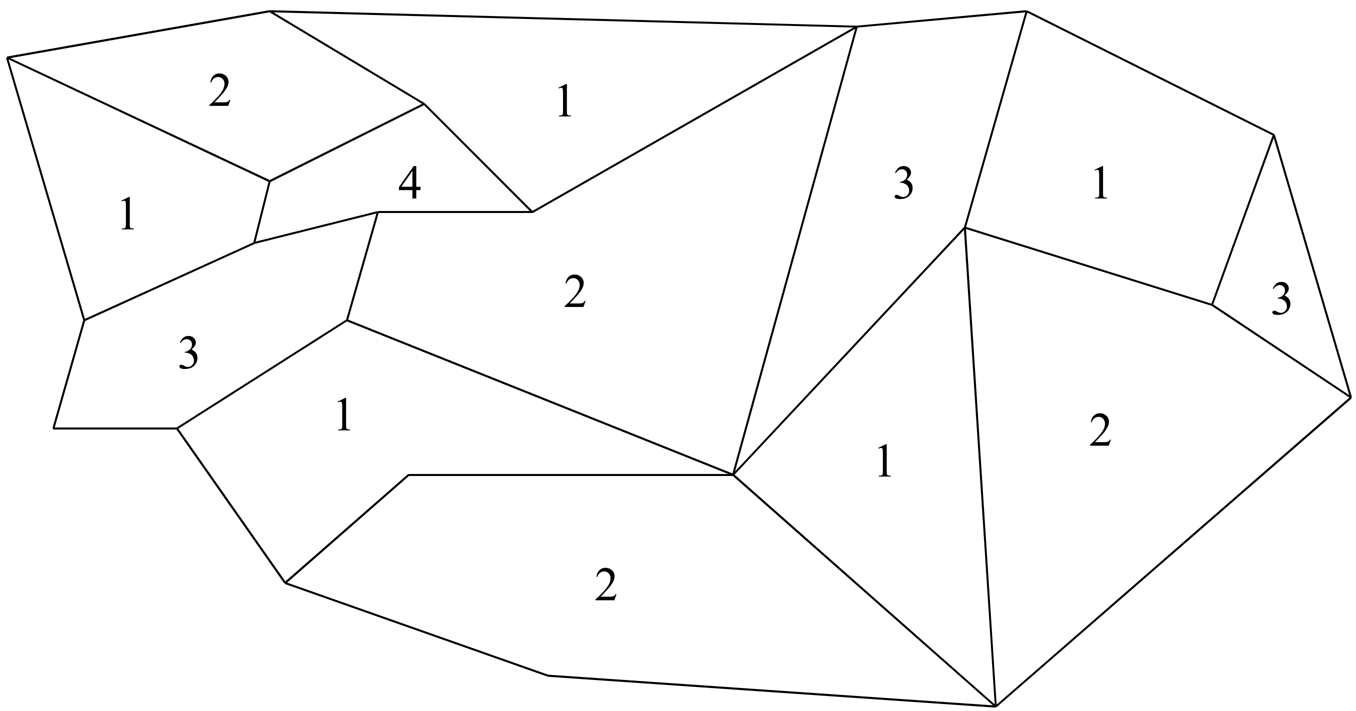
Countries with a common boundary must have different colors.



Four Color Problem

1852 letter by Augustus de Morgan to Sir William Hamilton:

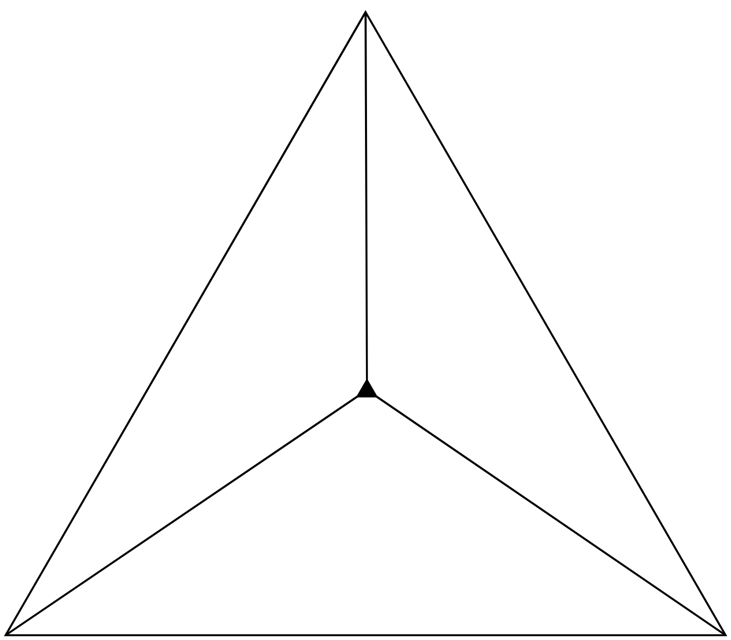
Four colors are required. Do 4 colors suffice?

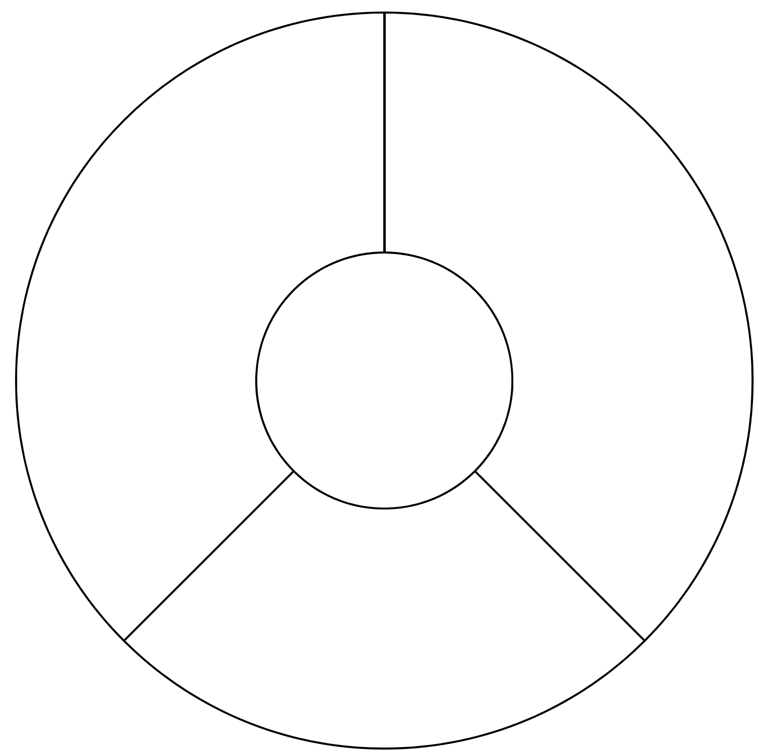


1976: Appel and Haken proved it using an intricate case analysis on a computer.

Exercise:

Draw a map that requires four colors.

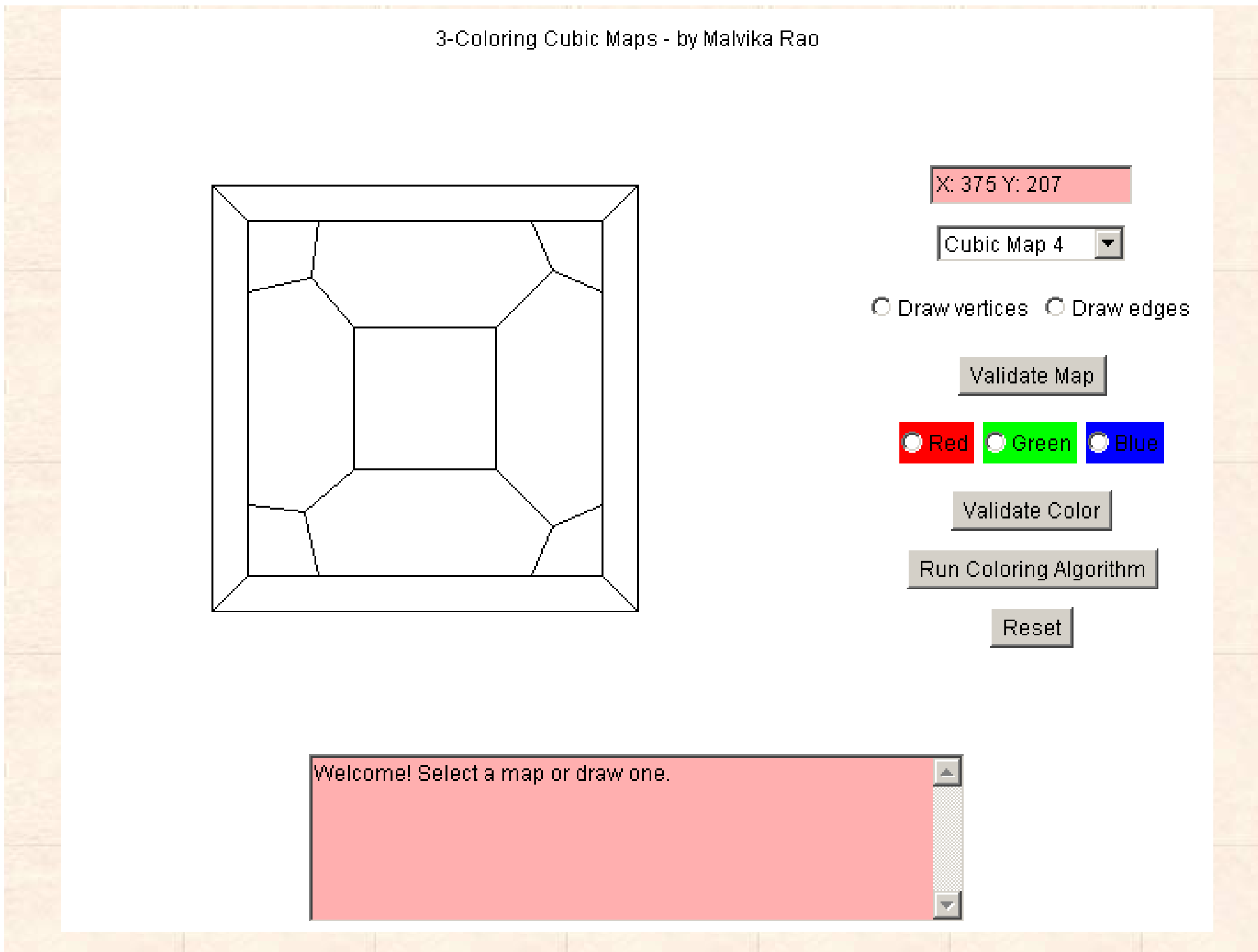




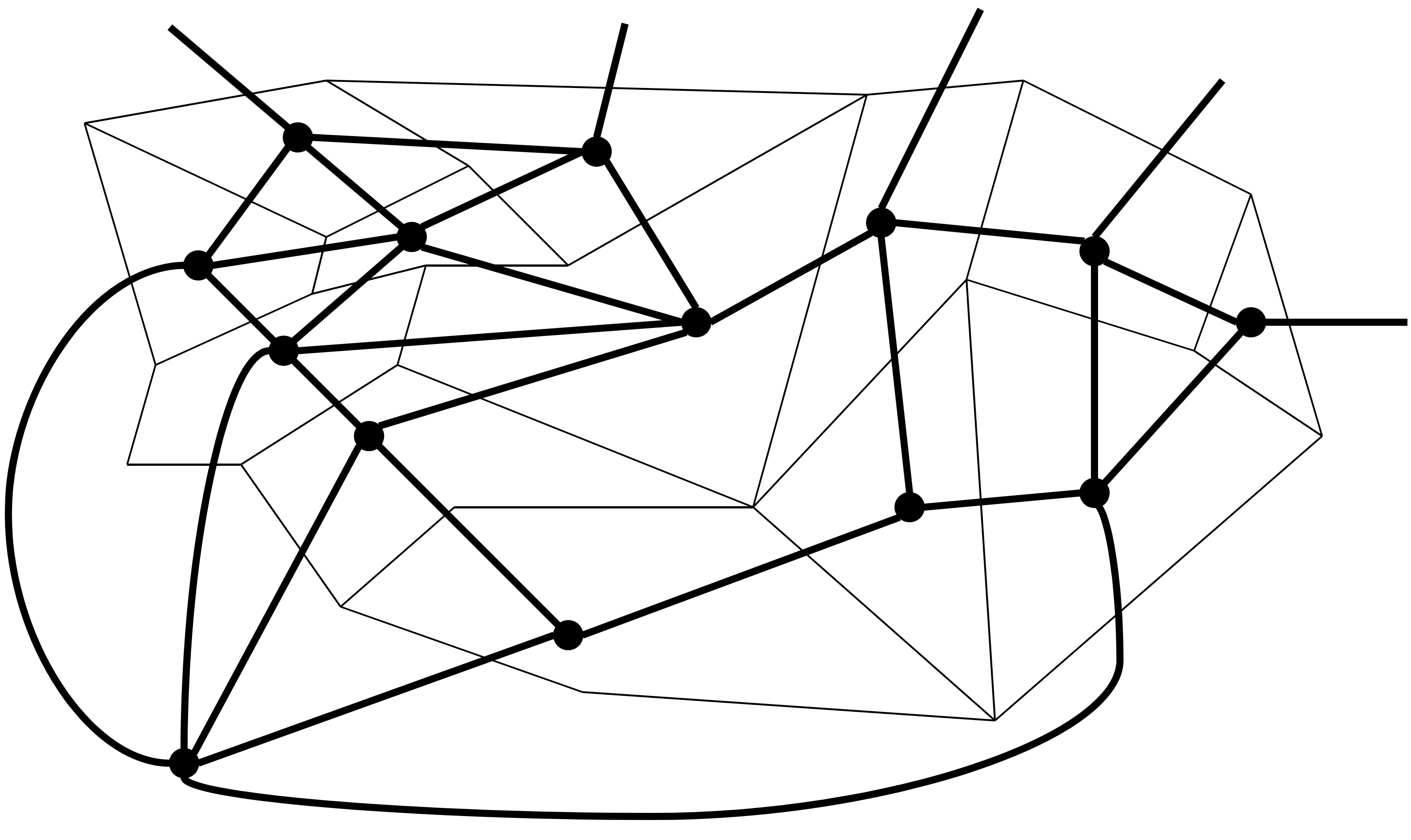
3-Coloring Maps

Computer Science project by Malvika Rao (student), McGill U.

http://www.cs.mcgill.ca/~rao/cs507/MapColoring.html

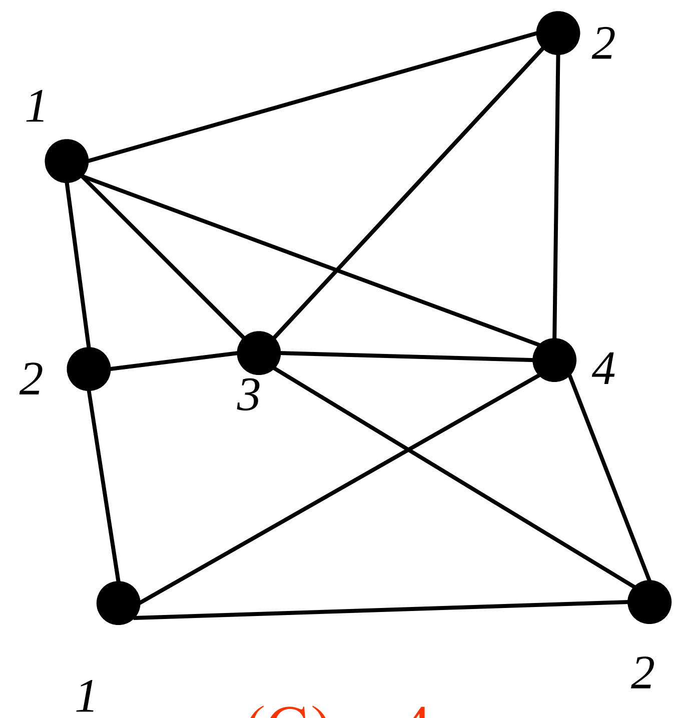


The Dual is a Planar Graph.



Vertex Coloring

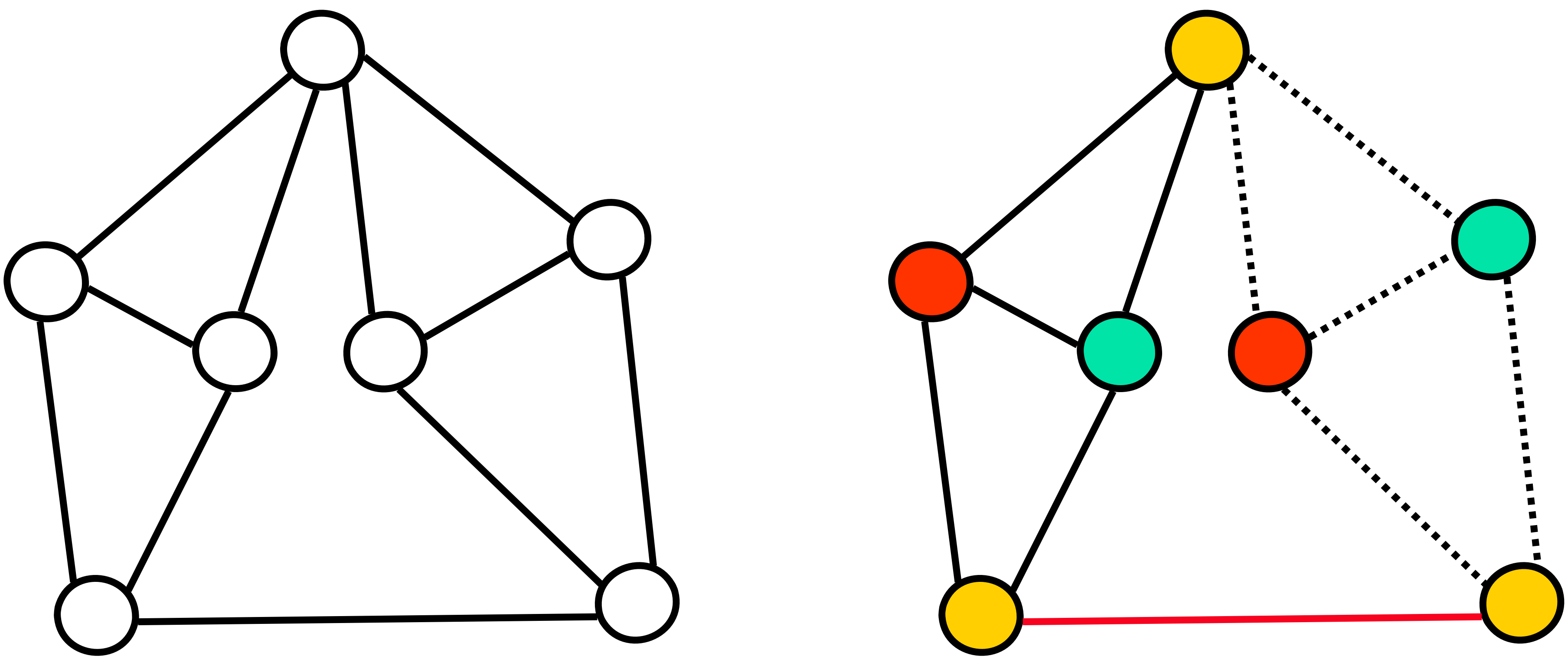
- A *k-coloring* is a labeling $f: V(G) \to \{1,2,\ldots,k\}$.
- A k-coloring is *proper* if $xy \in E(G)$ implies $f(x) \neq f(y)$.
- G is *k-colorable* if it has a proper k-coloring.
- The *chromatic number* $\chi(G)$ is the smallest k such that G is k-colorable.



$\chi(G) = 4$

Exercise:

Prove χ (Moser Graph) = 4.



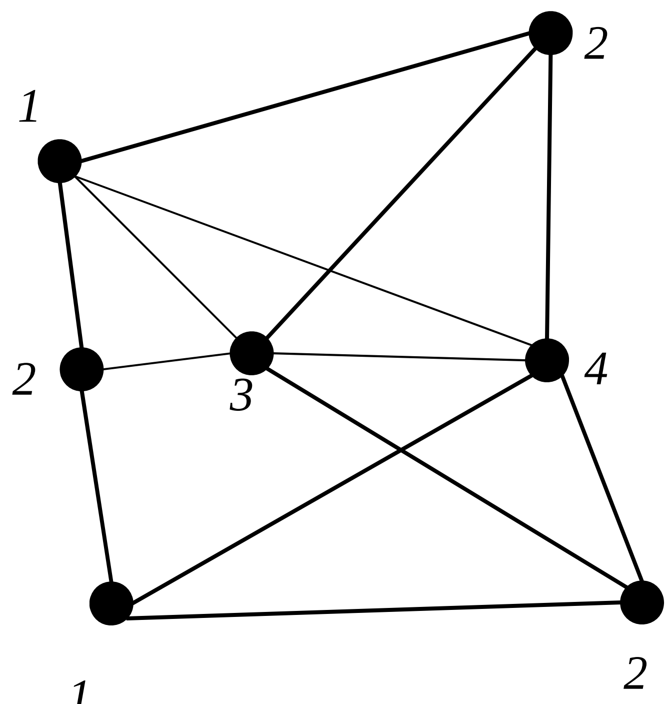
Party Problem

- People P_1, P_2, …, P_n meet for a party, but certain pairs are incompatible.
- *Goal:* Assign people to rooms so that no two people in the same room are incompatible.
- How many rooms are needed?

Solution to the Party Problem

Construct a conflict graph G.

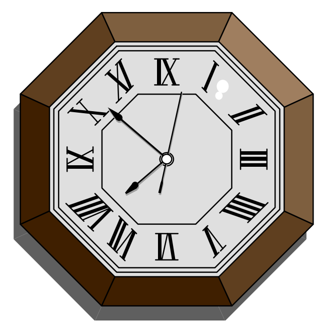
- $V(G) = \{P_1, P_2, \ldots, P_n\}$.
- $P_i, P_j \in E(G)$ iff P_i and P_j are incompatible.
- The chromatic number $\chi(G)$ is the least number of rooms.



$\chi(G) = 4$

Scheduling Problem

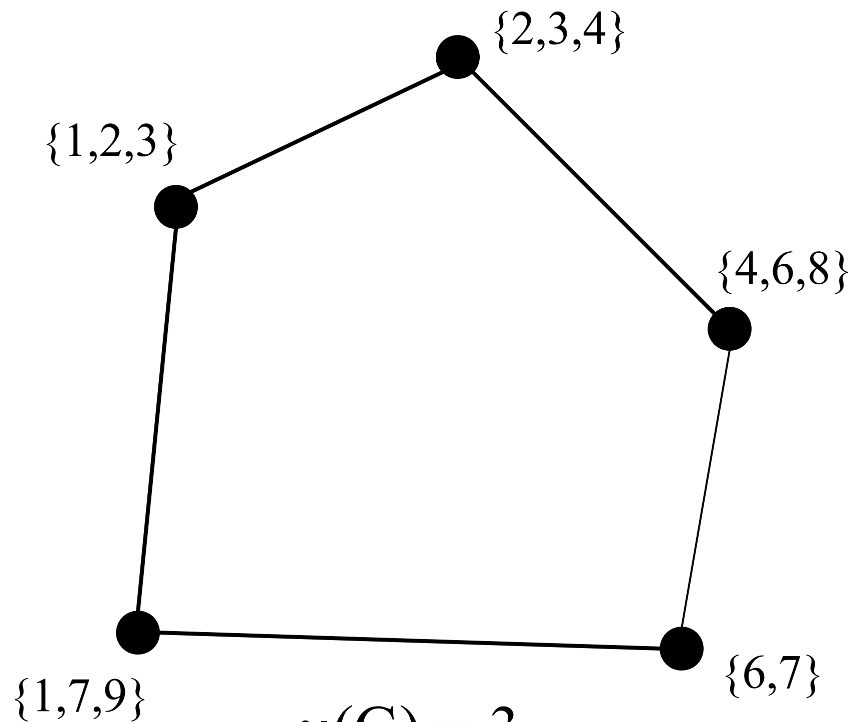
- Five different groups of students {1,2,3}, {6,7}, {1,7,9}, {4,6,8}, {2,3,4} must take exams in the following engineering courses S_1, S_2, S_3, S_4, S_5, respectively.
- *Goal:* Schedule the exams using a minimum number of time periods.



Solution to the Scheduling Problem

Construct a conflict graph G.

- $V(G) = \{S_1, S_2, S_3, S_4, S_5\}$.
- $S_i, S_j \in E(G)$ iff $S_i \cap S_j \neq \varnothing$.
- The chromatic number $\chi(G)$ is the minimum number of time periods.



$\chi(G) = 3$