

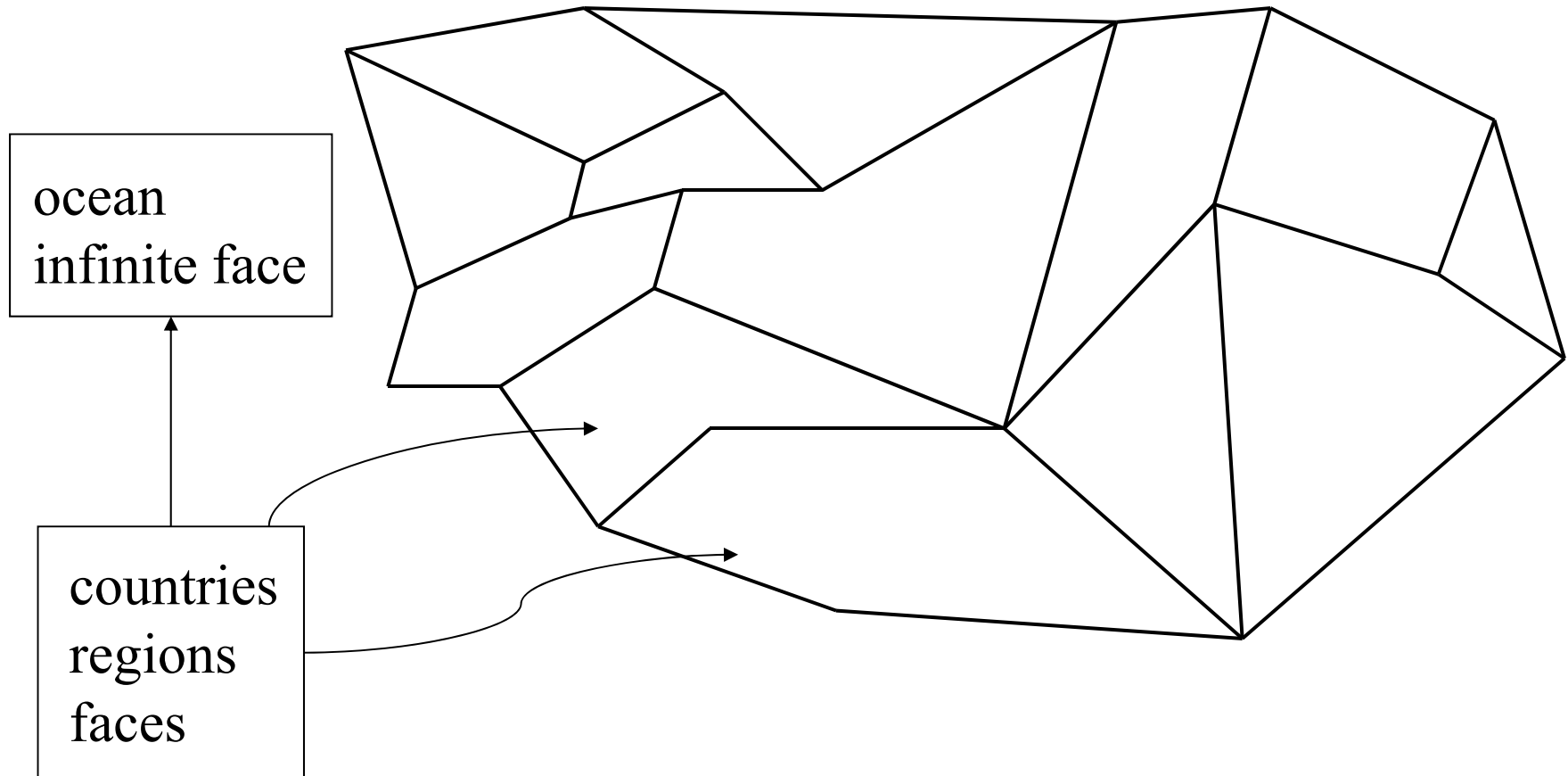
From Coloring Maps to Avoiding Conflicts

*Nathaniel Dean, Robert M. Nehs,
and Tong Wu*

Department of Mathematical Sciences
Texas Southern University
3100 Cleburne Avenue
Houston, TX 77004

Map Coloring

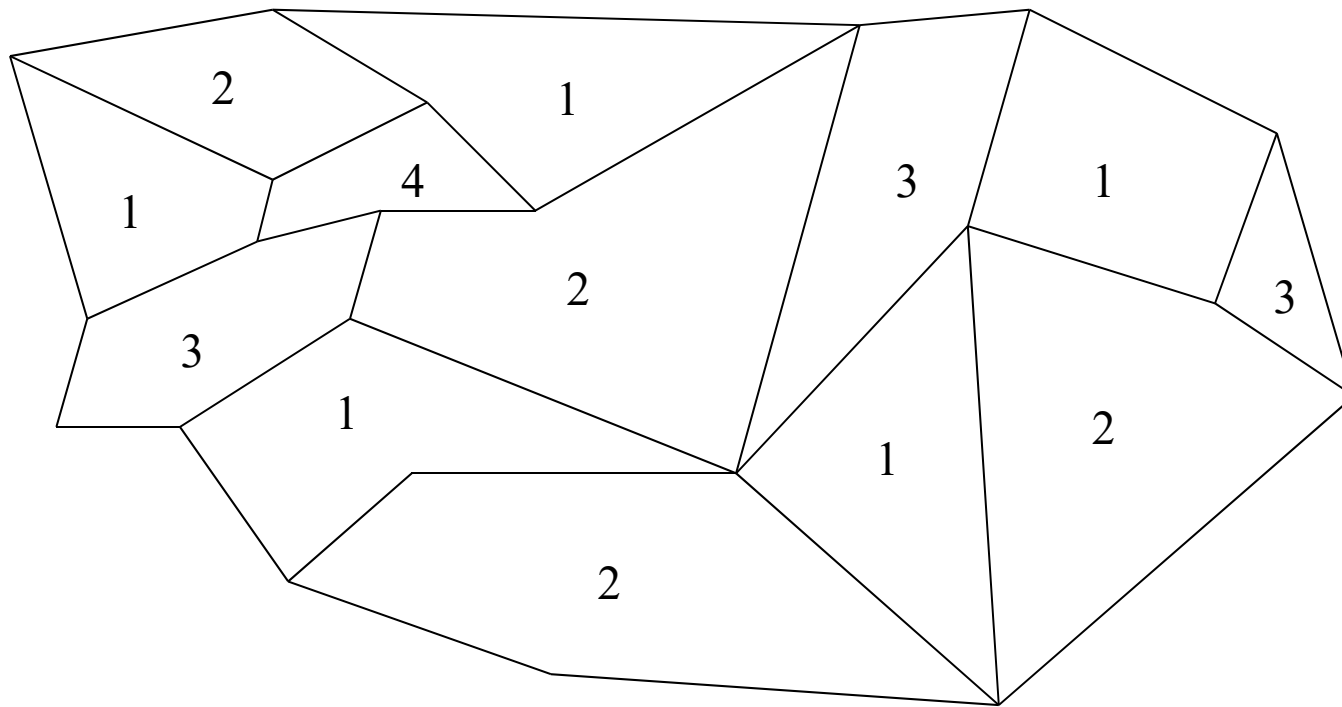
Countries with a common boundary must have different colors.



Four Color Problem

1852 letter by Augustus de Morgan to
Sir William Hamilton:

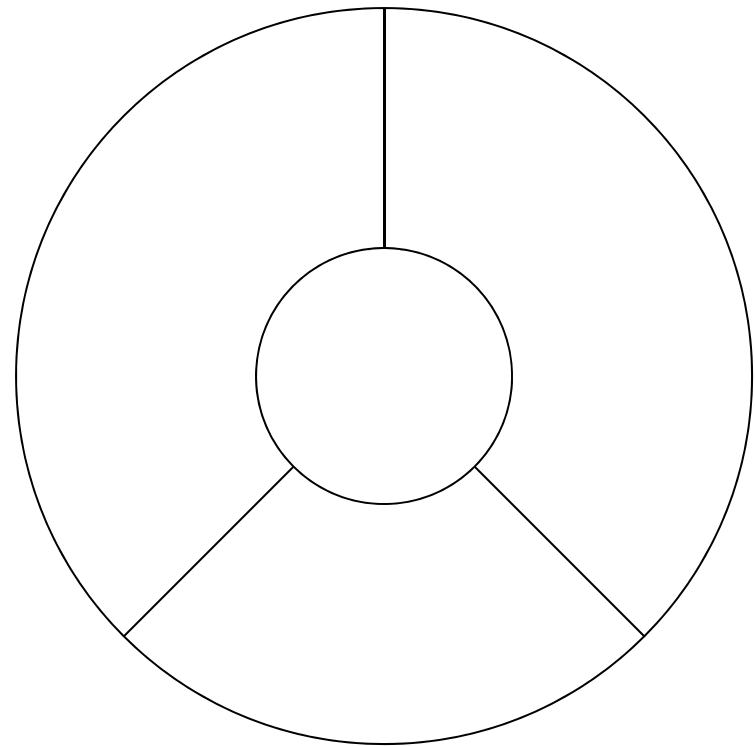
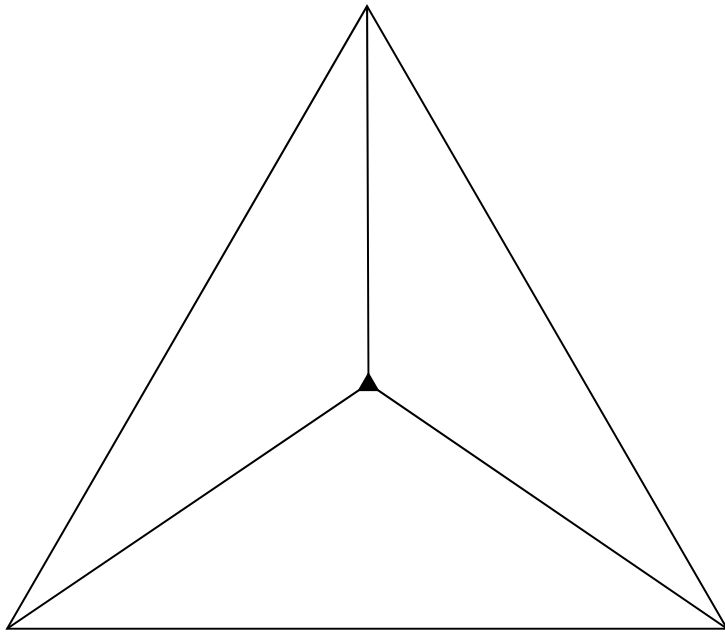
Four colors are required. Do 4 colors suffice?



1976: Appel and Haken proved it using an intricate case analysis on a computer.

Exercise:

Draw a map that requires four colors.

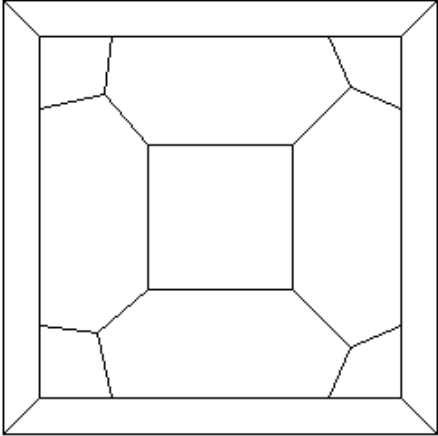


3-Coloring Maps

Computer Science project by Malvika Rao (student), McGill U.

<http://www.cs.mcgill.ca/~rao/cs507/MapColoring.html>

3-Coloring Cubic Maps - by Malvika Rao



X: 375 Y: 207

Cubic Map 4

Draw vertices Draw edges

Validate Map

Red Green Blue

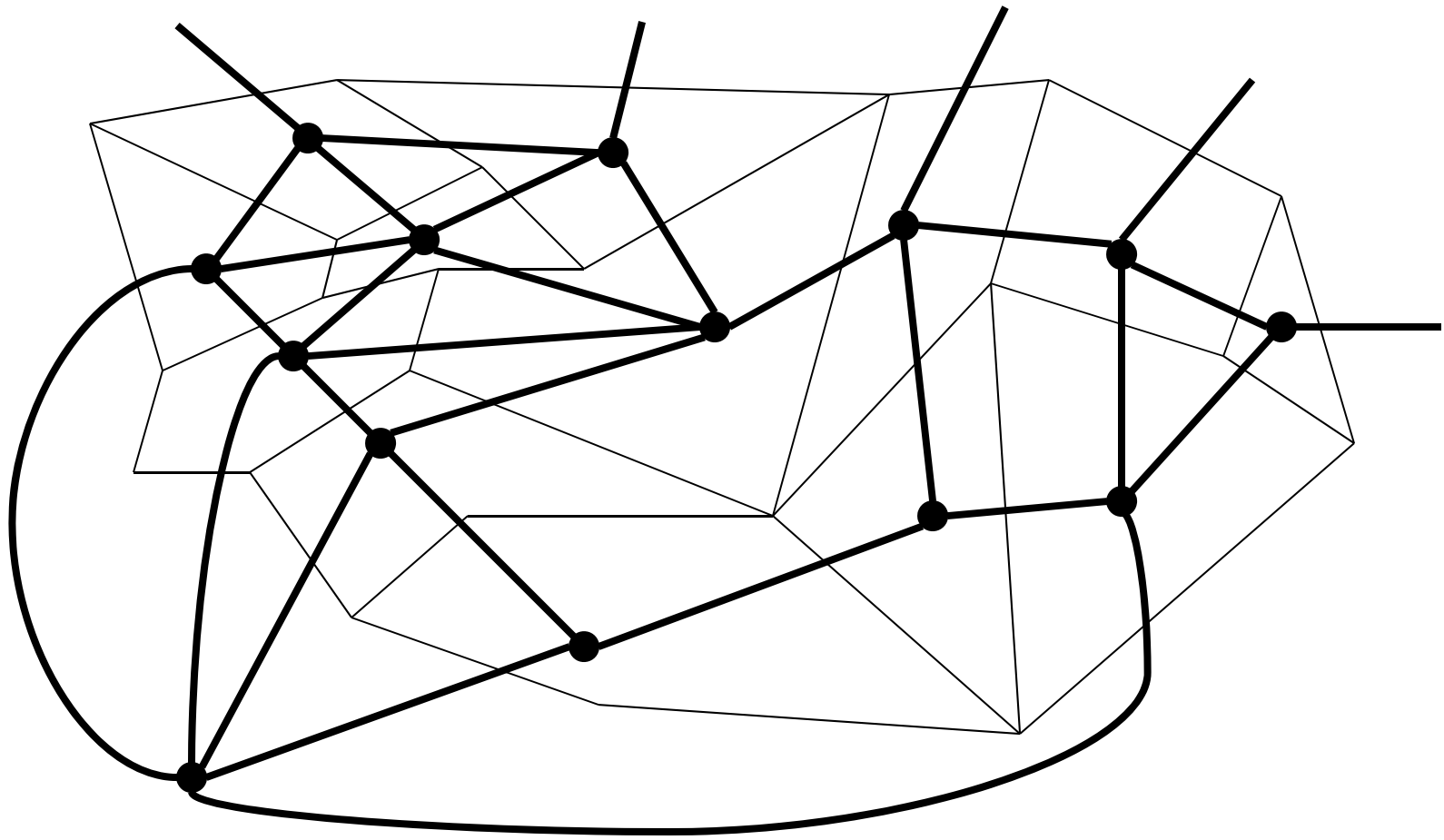
Validate Color

Run Coloring Algorithm

Reset

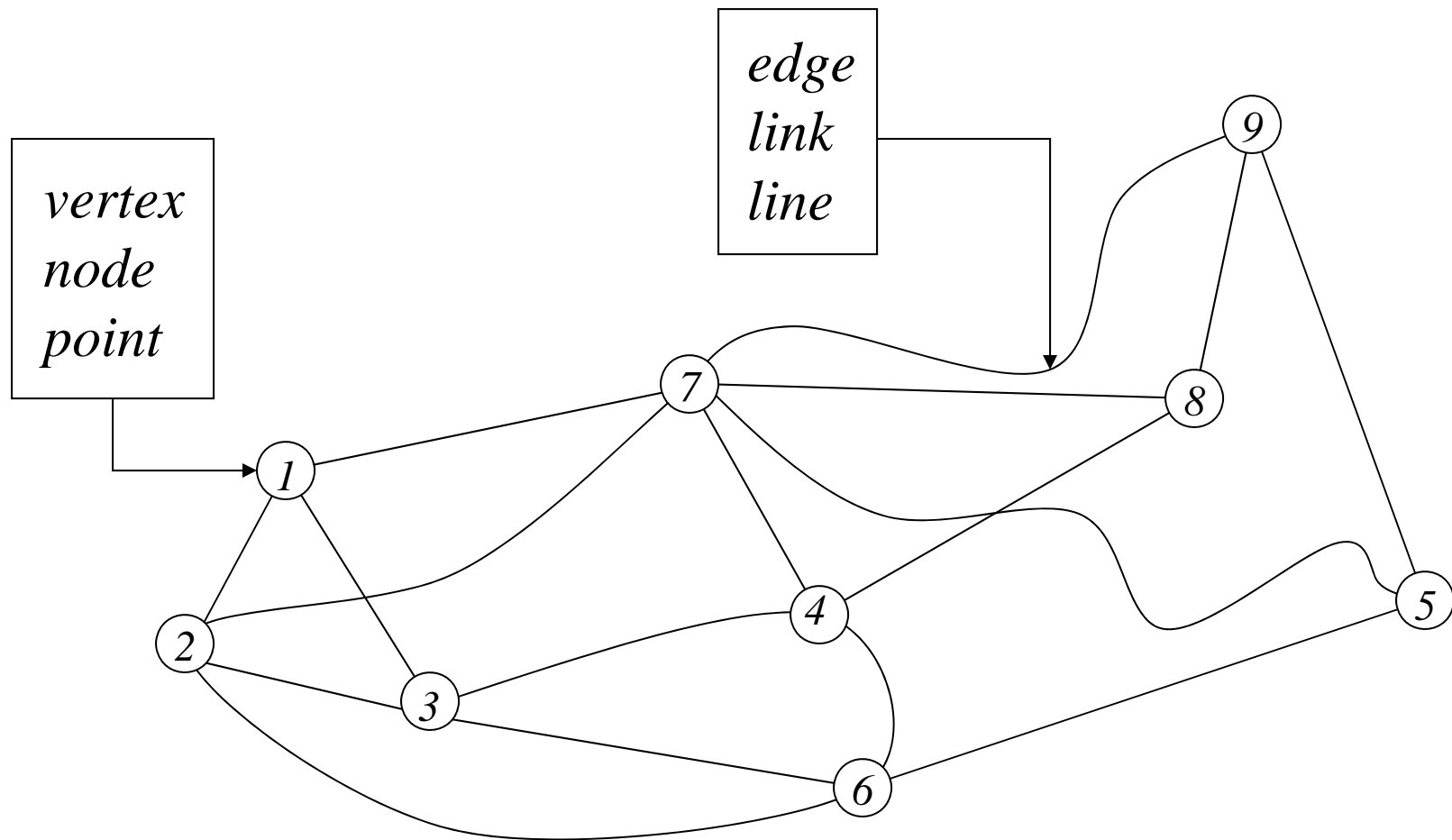
Welcome! Select a map or draw one.

The Dual is a Planar Graph.



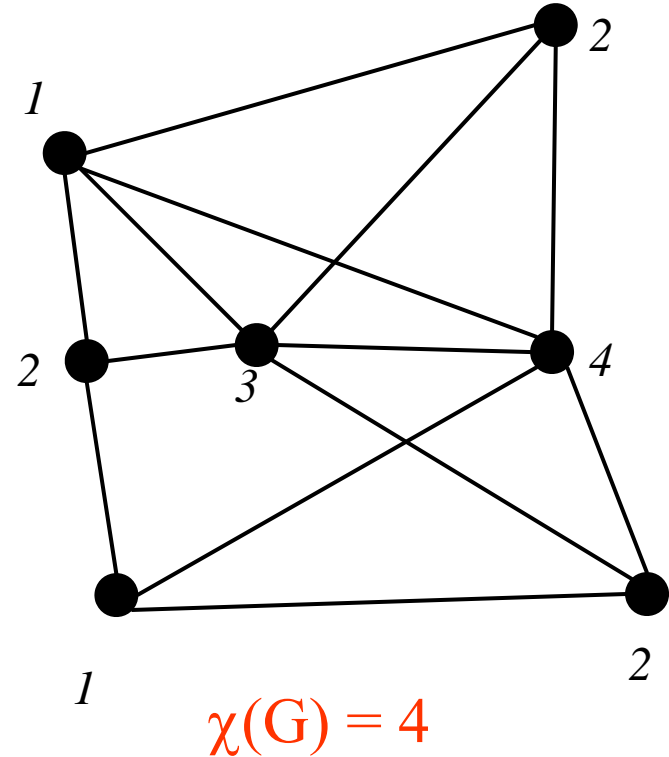
Graph $G=(V,E)$

consists of a set **V** of objects called vertices
and a set **E** of unordered pairs of vertices.



Vertex Coloring

- A *k-coloring* is a labeling $f:V(G) \rightarrow \{1,2,\dots,k\}$.
- A k-coloring is *proper* if $xy \in E(G)$ implies $f(x) \neq f(y)$.
- G is *k-colorable* if it has a proper k-coloring.
- The *chromatic number* $\chi(G)$ is the smallest k such that G is k-colorable.



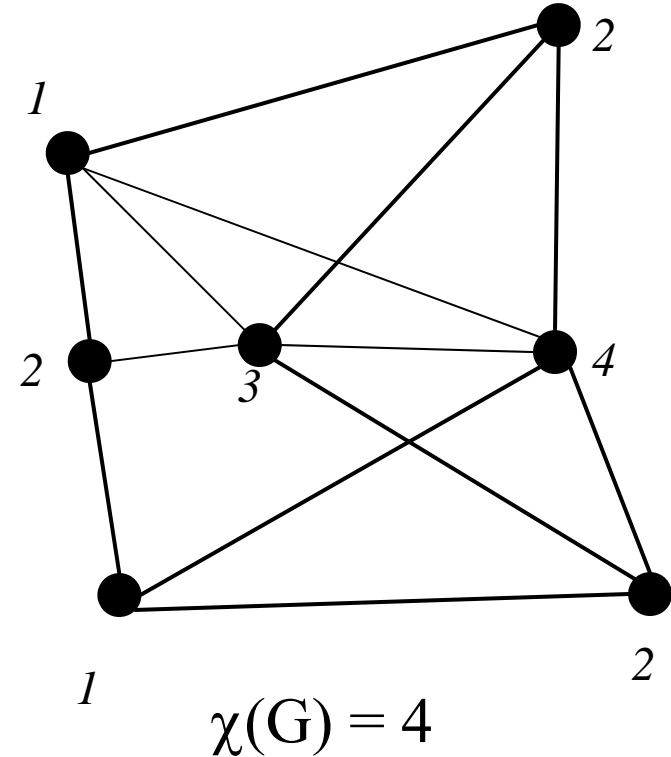
Party Problem

- People P_1, P_2, \dots, P_n meet for a party, but certain pairs are incompatible.
- *Goal:* Assign people to rooms so that no two people in the same room are incompatible.
- How many rooms are needed?

Solution to the Party Problem

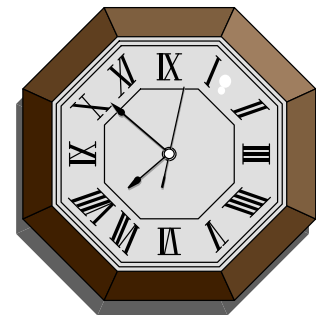
Construct a conflict graph G .

- $V(G) = \{P_1, P_2, \dots, P_n\}$.
- $P_i, P_j \in E(G)$ iff P_i and P_j are incompatible.
- The **chromatic number** $\chi(G)$ is the least number of rooms.



Scheduling Problem

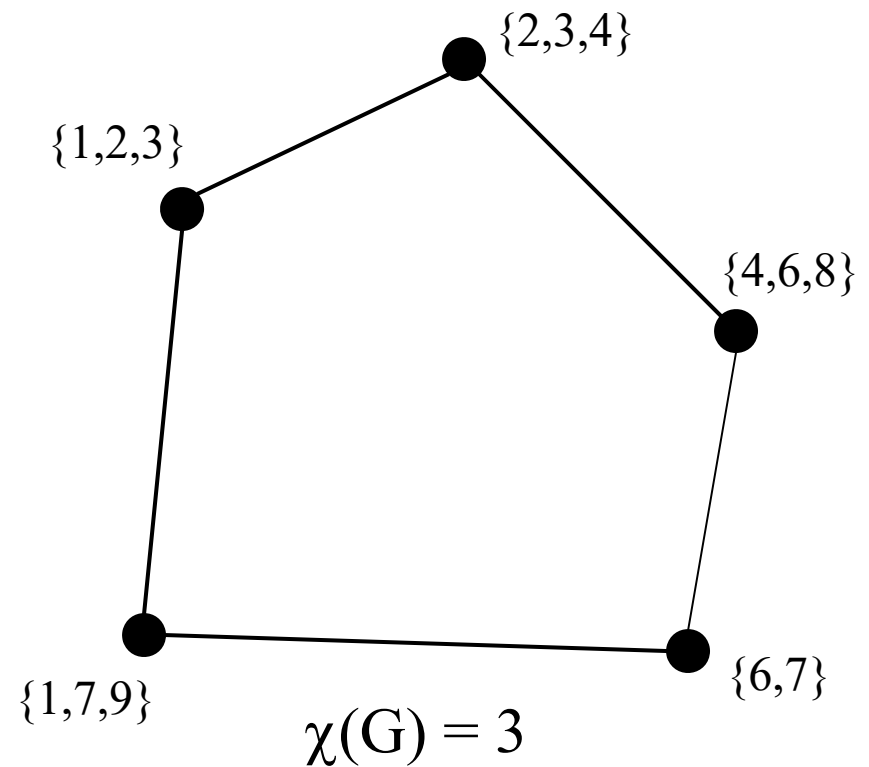
- Five different groups of students $\{1,2,3\}$, $\{6,7\}$, $\{1,7,9\}$, $\{4,6,8\}$, $\{2,3,4\}$ must take exams in the following engineering courses S_1, S_2, S_3, S_4, S_5 , respectively.
- *Goal:* Schedule the exams using a minimum number of time periods.



Solution to the Scheduling Problem

Construct a conflict graph G .

- $V(G) = \{S_1, S_2, S_3, S_4, S_5\}$.
- $S_i, S_j \in E(G)$ iff $S_i \cap S_j \neq \emptyset$.
- The **chromatic number** $\chi(G)$ is the minimum number of time periods.





Future Work

- Allow student to enter any arbitrary map or (nonplanar) graph.
- Add a map library, and allow transferring between map and corresponding graph.
- Develop algorithms with student.
- Explore applications.

Thanks!



Frank



David

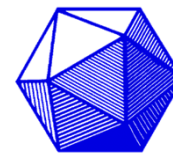


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