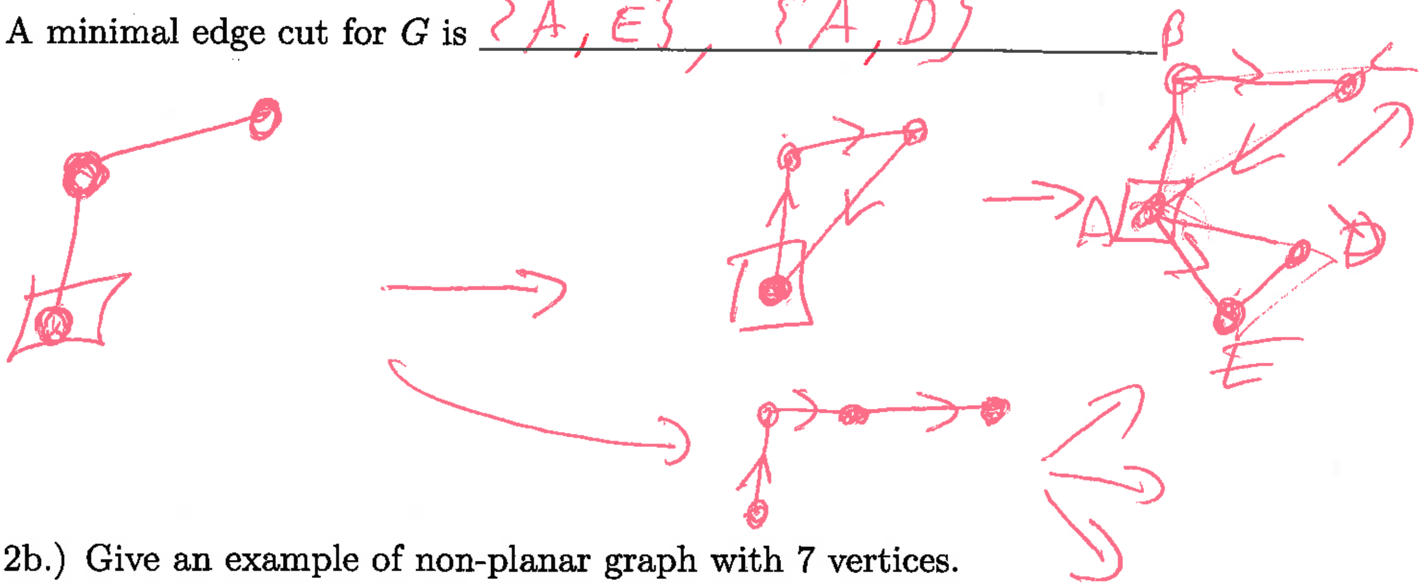
[14] 2.) Choose **2** from the following 3 problems. **Clearly indicate your choices.** You may attempt all problems for additional partial credit as discussed in class.

2a.) Give an example of a planar graph, G, with 5 vertices that contains an Eulerian circuit where $\kappa(G) = 1$ and $\lambda(G) = 2$.

What is the Eulerian circuit? A B C D E A

A minimal vertex cut for G is {A}

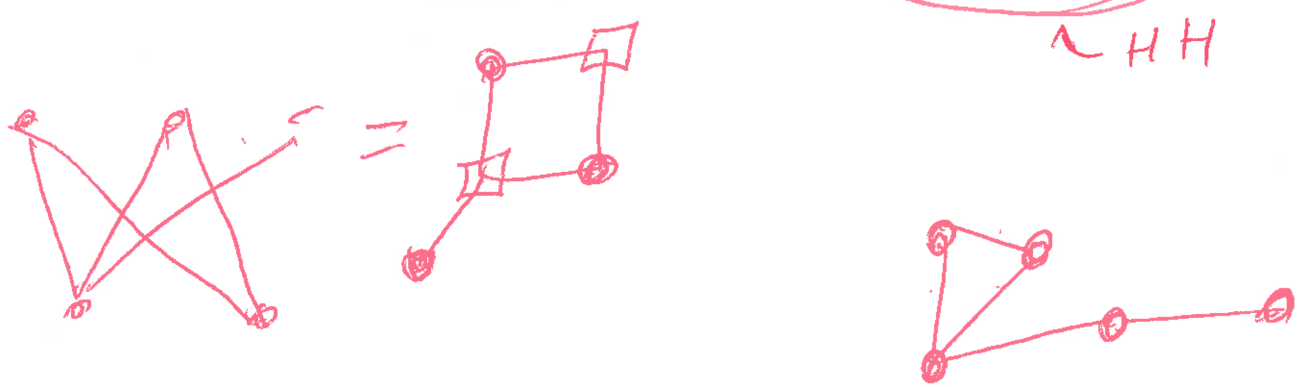
A minimal edge cut for G is {A, E}, {A, D}



2b.) Give an example of non-planar graph with 7 vertices.

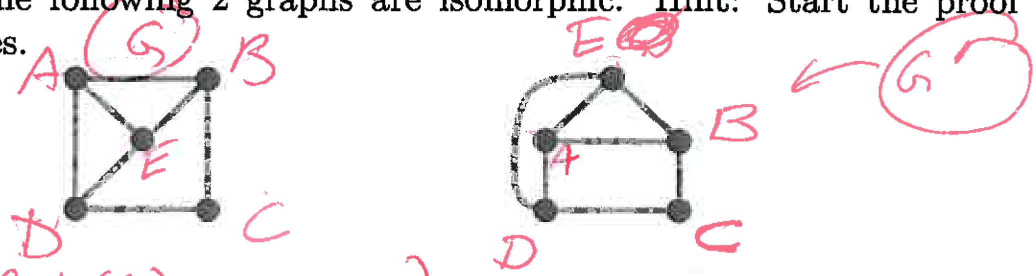
2c.) Give an example of two non-isomorphic graphs with degree sequence [3, 2, 2, 2, 1] where one of the graphs is bipartite while the other is not bipartite.

HH



[10] 3.) Choose **1** from the following 2 problems. **Clearly indicate your choice.** You may attempt both problems for additional partial credit as discussed in class.

3a.) Prove that the following 2 graphs are isomorphic. Hint: Start the proof by labeling the vertices.



f bij $f: V(G) \to V(G')$

induces $\bar{f}: E(G) \to E(G')$ bij

Pf By using above labeling, we have a bijection btwn vertices and not edges in G are sent to edges in G' via induced bijectn $f(\langle u,v\rangle) = \langle f(u), f(v)\rangle$

3b.) Prove that a graph $G = (V, E)$ where $V = \{v_1, ..., v_n\}$ is bipartite if and only if the vertices can be ordered so that the adjacency matrix is of the form $\begin{pmatrix} 0_{k\times k} & A \\ B & 0_{\ell\times\ell} \end{pmatrix}$ where $0_{m\times m}$ is an $m \times m$ matrix whose entries are all 0.

Note $V(G) = \{A, B, C, D, E\} = V(G')$

$E(G) = \{\langle A,B\rangle, \langle A,D\rangle, \langle A,E\rangle, \langle B,E\rangle, \langle E,D\rangle, \langle B,C\rangle, \langle D,E\rangle\} = E(G')$

Thus $G = G'$

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