Let G be a connected graph.

If $G = \{\{w\}, \emptyset\} = \bullet$, then $\lambda(G) = 0$

In all other cases:

$\lambda(G)$ = the size of a minimal edge cut

= the minimum number of edges one can remove such that the resulting subgraph is disconnected.

$\boxed{\text{Thm 2.4: } \lambda(G) \leq min\{\delta(v) \mid v \in V(G)\}}$

Choose $w \in V(G)$ such that
$\delta(w) \leq \delta(v)$ for all $v \in V(G)$.

I.e., choose $w \in V(G)$ such that
$\delta(w) = min\{\delta(v) \mid v \in V(G)\}$

Case 1: $G = \bullet = \{\underbrace{\{w\}}_{V(G)}, \underbrace{\emptyset}_{E(G)}\}$

$\lambda(G) = 0 \leq 0 = min\{\delta(v) \mid v \in V(G)\} = \delta(w)$ since $V(G) = \{w\}$

Case 2: Let $E^* = \{e \mid e \text{ is incident to } w\}$

$= \{<w,v> \mid <w,v> \in E(G)\}$

$= \{<w,v> \mid v \in N(w)\}$

= the set of all edges having w as one of its vertices.

Note $|E^*| = \delta(w)$. Claim: E^* is an edge cut.

Note $(\{w\}, \emptyset)$ is a component of $G - E^*$.

Thus $G - E^*$ is disconnected since $\delta(w) > 0$

$\Rightarrow \exists v \in V(G)$ st v is not connected to w in $G - E^*$

Thus $\lambda(G) \leq min\{\delta(v) \mid v \in V(G)\}$

$G = (V, E)$ is *k-edge-connected* if removing any set of $k - 1$ edges in G does not disconnect it and $|V| \geq 2$.

k edge connected implies $k - 1$ edge connected if $k > 1$

disconnected $\Rightarrow$ only 0-edge connected

connected = 1-edge connected

$\lambda(G)$ = connectivity of $G = max\{k \mid G\ k\text{--}connected\}$ ■

$G = \bullet$ is 1-edge connected