Note the following problems are from Maarten van Steen's Graph Theory and Complex Networks Exercises: http://www.van-steen.net/my-data/gtcn/mainexer.pdf

Answers to these questions are available at the above website, but I strongly recommend doing them on your own before consulting the answers.

HW 2: Turn in hard copies of a few of these for feedback next week (2nd week of classes, and no later than Friday Jan 25th). Note full credit will be given even if your answer is wrong. The purpose of this assignment is for you to get feedback on your proofs for which you are uncertain.

Q 4: Prove that for any graph, the sum of its vertex degrees is even.

Q 10: Show that two graphs with the same degree sequence need not be isomorphic.

Q 11: Show that there is no simple graph with 12 vertices and 28 edges in which

Q 12: Show that there is no simple graph with four vertices such that three vertices have degree 3 and one vertex has degree 1.

Q 13: Show that the number of vertices in a k-regular graph is even if k is odd.

Q 14: Let $v = [d_1, d_2, \ldots, d_n]$ and $w = [w_n, w_{n-1}, \ldots, w_2, w_1]$, where $w_i = n - 1 - d_i$. Show that v is graphic if and only if w is graphic.

Q 15: Show that there is no simple graph with six vertices of which the degrees of five vertices are 5, 5, 3, 2, and 1.

Q 16: Find k if $[8, k, 7, 6, 6, 5, 4, 3, 3, 1, 1, 1]$ is graphic.

Q 17: Show that an ordered sequence of nonincreasing numbers in which no two numbers are equal cannot be graphic.

Q 18: Show that in a simple graph, there are at least two vertices with equal degrees.

Q 19: Show that there exists a simple graph with 12 vertices and 28 edges such that the degree of each vertex is either 3 or 5. Draw this graph.

Q 20: Show that there exists a simple graph with seven vertices and 12 edges such that the degree of each vertex is 2 or 3 or 4.