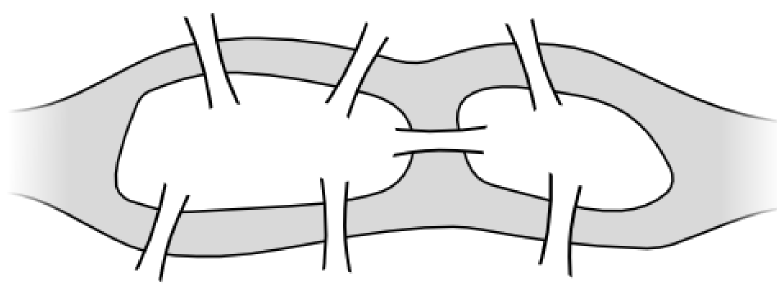
Math 173 March 31, 2011

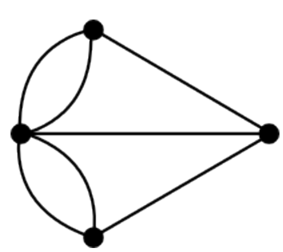
Introduction to Graph Theory Worksheet

Graph Theory is a relatively new area of mathematics, first studied by the super famous mathematician Leonhard Euler in 1735. Since then it has blossomed in to a powerful tool used in nearly every branch of science and is currently an active area of mathematics research. We will begin our study with the problem that started it all: The Seven Bridges of Königsberg.

1. **Königsberg Bridge Problem:** In the time of Euler, in the town of Königsberg in Prussia, there was a river containing two islands. The islands were connected to the banks of the river by seven bridges (as seen below). The bridges were very beautiful, and on their days off, townspeople would spend time walking over the bridges. As time passed, a question arose: was it possible to plan a walk so that you cross each bridge once and only once? Euler was able to answer this question. Are you?

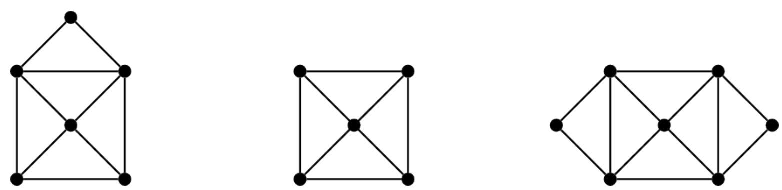


2. Here is another problem: below is a drawing of four dots connected by some lines. Is it possible to trace over each line once and only once (without lifting up your pencil)? You must start and end on one of the dots.



3. Is there a connection between problems 1 and 2? If so, what is it?

4. Pictures like the one for question 2 are called *graphs*. Graphs are made up of a collection of dots (called *vertices*) and lines connecting those dots (called *edges*). If we start at a vertex and trace along edges to get to other vertices, we create a *path* on the graph. If the path travels along every edge exactly once, then the path is called an *Eulerian path*. If in addition, the starting and ending vertices are the same (so you trace along every edge exactly once and end up where you started) then the path is called an *Eulerian circuit*. Which of the graphs below contain an Eulerian path? Which contain an Eulerian circuit?



5. What if I gave you a really big graph? Is there a quick way to check whether or not it contains an Eulerian path or circuit? Make a conjecture, then test it out by drawing some graphs yourself.

6. Give some real life applications where finding an Eulerian path or circuit would be useful.