Planes: We have only covered equations of planes in R^3. The only equation we have used is

$$\mathbf{n}\cdot(\mathbf{x}-\mathbf{p}) = 0 \text{ which simplifies to } ax+by+cz=d.$$

Note $\mathbf{n} = (a,b,c)$ is any vector that is normal to the plane, $\mathbf{p}$ is a fixed point on the plane, and $\mathbf{x}$ is an arbitrary point on the plane.

Lines: My favorite equation of a line is $\mathbf{x} = t\mathbf{v} + \mathbf{p}$ where $\mathbf{v}$ is a vector that specifies the direction of the line, $\mathbf{p}$ is a fixed point on the line, and $\mathbf{x}$ is an arbitrary point on the line. One can also write the line in parametrized form: $x_i = tv_i + p_i$ for $i = 1, ..., n$ where the line is living in $\mathbb{R}^n$. Note this is an equation of a line regardless of which dimension (n) you are working in.

Tangent lines:

If $y = f(x)$, then tangent line at (p_1, p_2) is

$$(x,y) \;=\; t < 1, \tfrac{\partial y}{\partial x} > \;+\; < p_1, p_2 >$$

If $r(t) = (x(t), y(t))$,then tangent line at (p_1, p_2) is

$$(x,y) \;=\; t < \tfrac{dx}{dt}, \tfrac{dy}{dt} > \;+\; < p_1, p_2 >$$

If $r(t) = (x(t), y(t), z(t))$,then tangent line at (p_1, p_2, p_3) is

$$(x,y) \;=\; t < \tfrac{dx}{dt}, \tfrac{dy}{dt}, \tfrac{dz}{dt} > \;+\; < p_1, p_2, p_3 >$$

If $y = f(x,y)$, then two (of infinitely many) tangent lines at (p_1, p_2, p_2) is

$$(x,y,z) \;=\; t < 1, 0, \tfrac{\partial z}{\partial x} > \;+\; < p_1, p_2, p_3 >$$

$$(x,y,z) \;=\; t < 0, 1, \tfrac{\partial z}{\partial y} > \;+\; < p_1, p_2, p_3 >$$

We can take the cross products of the tangent vectors in $\mathbb{R}^3$ to create a normal to the tangent plane:

$$< \tfrac{\partial z}{\partial x}, \tfrac{\partial z}{\partial y}, -1 >$$

Compare to the gradient of f: $\nabla f \;=\; < \tfrac{\partial z}{\partial x}, \tfrac{\partial z}{\partial y} >$

Optimization:

Section 12.5:

- Local vs global extrema.
- Extreme Value theorem: Global extrema exist if domain is closed and bounded. The extrema can be found by checking
  - critical points in the interior of domain
  - boundary points
- Can turn constrained optimization problem into unconstrained optimization problem **IF** one can solve the constraint for one of the variables.
- Section 12.10: Can determine if a critical point is local extrema using the second derivative test (when it applies).

Section 12.9: Solve constrained optimization implicitly by using Lagrange multiplier: Solve $\nabla f(\mathbf{p}) = \lambda \nabla g(\mathbf{p})$. Also check p such that $\nabla g(\mathbf{p}) = 0$.

Section 12.8: Given $z = f(\mathbf{x})$, since $D_{\mathbf{u}}(f) = \nabla f \cdot \mathbf{u} = |\nabla f| cos\theta$, we have

$\nabla f(\mathbf{p})$ = the direction of steepest ascent at point $\mathbf{p}$ = direction in which f increases the most rapidly from point $\mathbf{p}$ = direction of greatest increase starting from point $\mathbf{p}$.

The negative of $\nabla f(\mathbf{p}) = -\nabla f(\mathbf{p})$ = the direction of steepest descent at point $\mathbf{p}$ = direction in which f decreases the most rapidly from point $\mathbf{p}$ = direction of greatest decrease starting from point $\mathbf{p}$.

$|\nabla f(\mathbf{p})|$ = magnitude of rate of change.

NOTE: derivatives only give local information and local approximations. I.e., if you move slightly away from point $\mathbf{p}$, then the direction of steepest ascent can be very different then the direction at point $\mathbf{p}$. And the rate of increase = instantaneous rate of change = approximation of actual rate of change from $\mathbf{p}$ to $\mathbf{q} = \frac{f(\mathbf{q}) - f(\mathbf{p})}{\mathbf{q} - \mathbf{p}}$.