Math 3550 Midterm 1 Name:

March 3, 2020 Show all work. Circle section number: 91 (9:30 am) 131 (1:30pm)

1.) For the plane curve, $x(t) = t$, $y(t) = ln(t)$,

[6] 1a.) State the integral that gives the arclength from $t = 1$ to $t = e$ (you do not need to calculate the integral).

Answer: ____

[8] 1b.) Find the unit tangent and unit normal to this curve at the point (e, 1).

Answer: $\mathbf{T} =$ ____________________ and $\mathbf{N} =$ ____________________

2.) Circle T for true and F for false.

[2] 2a.) If $\mathbf{a}$ and $\mathbf{b}$ represent adjacent sides of a parallelogram $PQRS$, so that $\mathbf{a} = \overrightarrow{RQ}$ and $\mathbf{b} = \overrightarrow{RS}$, then the area of $PQRS$ is $|\mathbf{a} \cdot \mathbf{b}|$.

T F

[2] 2b.) If $\mathbf{a}$ and $\mathbf{b}$ represent adjacent sides of a parallelogram $PQRS$, so that $\mathbf{a} = \overrightarrow{RQ}$ and $\mathbf{b} = \overrightarrow{RS}$, then the area of $PQRS$ is $|\mathbf{a} \times \mathbf{b}|$.

T F

[2] 2c.) If f is integrable, then the Riemann sum $\sum_{i=1}^{k} f(x_i^*, y_i^*)\Delta A_i$ can be made arbitrarily close to the value of the double integral $\int\int_R f(x, y)dA$ by choosing an inner partition of R with sufficiently small norm.

T F

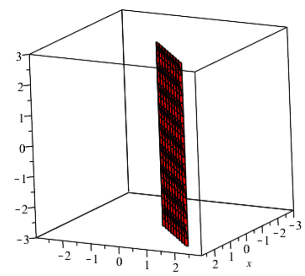
3.) Match the function to its graph by circling the appropriate letter. Recall (r, θ, z) refers to cylindrical coordinates, while (ρ, θ, ϕ) refers to spherical coordinates.

[3] 3i.) $r = 4$ A B C D E F G H I

[3] 3ii.) $\rho = 4$. A B C D E F G H I

[3] 3iii.) $\theta = \frac{\pi}{4}$ A B C D E F G H I

[3] 3iv.) $\phi = \frac{\pi}{4}$ A B C D E F G H I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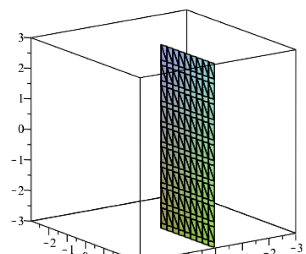
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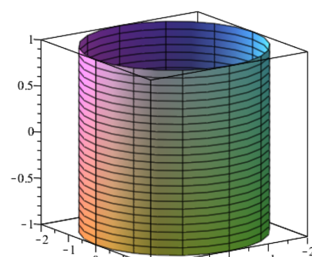
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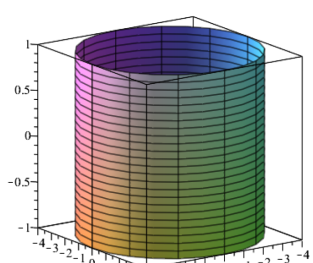
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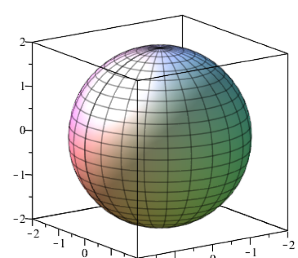
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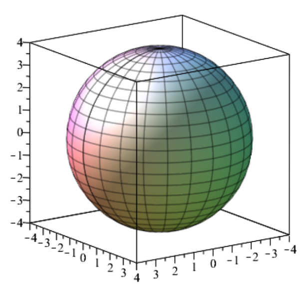
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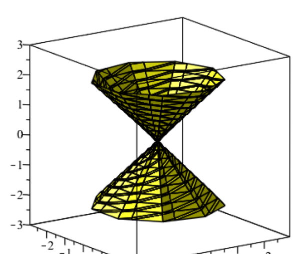
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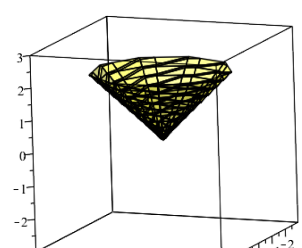
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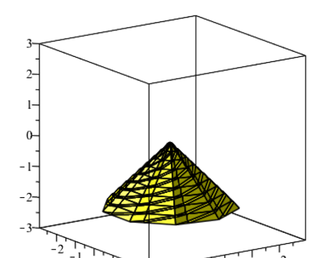
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4.) For the surface $f(x, y) = x^3 - 3x^2 + 3xy^2 - 3y^2$,

[4] 4a.) $\nabla f =$ ______________________________

[2] 4b.) If f represents elevation, the direction of steepest ascent starting at $(x, y) = (1, 2)$ is

[2] 4c.) If f represents elevation, the direction of steepest descent starting at the point $(x, y) = (1, 2)$ is

[5] 4d.) The equation of the tangent plane at $(1, 2, -2)$ is ______________________________

[4] 4e.) Use differentials to estimate $f(1.1, 1.8)$.

4 continued.) For the surface $f(x,y) = x^3 - 3x^2 + 3xy^2 - 3y^2$,

[8] 4f.) Find and classify the critical points using the 2nd derivative test.

The critical point ____________ is a ____

The critical point ____________ is a ____

The critical point ____________ is a ____

The critical point ____________ is a ____

[15] 5.) Use the method of Lagrange multipliers to find the point(s) on the surface $z = 3xy + 6$ closest to the origin.

Answer: ______________________________

[12] 6.) Let $R = [0,3] \times [0,1]$. Use a partition consisting of 3 unit squares to estimate the volume of the solid under the surface $z = xy + 2y$ and above the rectangle $R = [0,3] \times [0,1]$ in the xy-plane. Use the upper right corner of each of the 3 unit squares to estimate the height of the 3 rectangular columns.

Answer: ______________________________

7.) Write the four double integrals specified below for determining the volume of the solid under the surface $z = \frac{2y}{x^2+y^2}$ and above the region in the xy-plane bounded by the curves $y = 0$ and $y = \sqrt{9 - x^2}$. You do NOT need to evaluate the integral. CIRCLE your answer.

[4] 7a.) Use polar coordinates, integrating first with respect to r, and then with respect to θ.

[4] 7b.) Use polar coordinates, integrating first with respect to θ, and then with respect to r.

[4] 7c.) Use Euclidean coordinates, integrating first with respect to x, and then with respect to y.

[4] 7d.) Use Euclidean coordinates, integrating first with respect to y, and then with respect to x.