

Divergence of a Vector Field.

The divergence of $\mathbf{F} = \text{div } \mathbf{F} = \nabla \cdot \mathbf{F}$.

Thus in 2-dimensions where $\mathbf{F} = \langle P, Q \rangle$ and $\nabla = \langle \frac{\partial}{\partial x}, \frac{\partial}{\partial y} \rangle$,

$$\text{div } \mathbf{F} = \nabla \cdot \mathbf{F} = \langle \frac{\partial}{\partial x}, \frac{\partial}{\partial y} \rangle \cdot \langle P, Q \rangle = \frac{\partial P}{\partial x} + \frac{\partial Q}{\partial y}.$$

Thus in 3-dimensions where $\mathbf{F} = \langle P, Q, R \rangle$ and $\nabla = \langle \frac{\partial}{\partial x}, \frac{\partial}{\partial y}, \frac{\partial}{\partial z} \rangle$,

$$\text{div } \mathbf{F} = \nabla \cdot \mathbf{F} = \langle \frac{\partial}{\partial x}, \frac{\partial}{\partial y}, \frac{\partial}{\partial z} \rangle \cdot \langle P, Q, R \rangle = \frac{\partial P}{\partial x} + \frac{\partial Q}{\partial y} + \frac{\partial R}{\partial z}.$$

Suppose a $\mathbf{F} = \langle P(x, y), Q(x, y) \rangle$ represents the velocity vector of water flowing on a flat surface.

I.e., the water at the location (x, y) is flowing in the directions $\langle P(x, y), Q(x, y) \rangle$ with speed $\| \langle P(x, y), Q(x, y) \rangle \|$ *meters per second*