Math 3550 Quiz 3 Form A   Feb 28, 2020

1.) Circle T for true and F for false.

[4] 1a.) The derivative of the polar coordinate transformation $T : \mathbb{R}^2 \to \mathbb{R}^2$, $T(r, \theta) = (x(r, \theta), y(r, \theta))$ has determinant $|T'(r, \theta)| = r$. T F

[4] 1b.) The derivative of the polar coordinate transformation $T : \mathbb{R}^2 \to \mathbb{R}^2$, $T(r, \theta) = (x(r, \theta), y(r, \theta))$ has determinant $|T'(r, \theta)| = r^2$. T F

[12] 2.) The base radius r and height h of a right circular **cylinder** are measured as 4m and 20m respectively. There is a possible error of as much as 10% in each of the measurements. Use differentials to estimate the maximum possible error in computing the volume of the **cylinder**.

Answer: ____

Math 3550 Quiz 3 Form B   Feb 28, 2020

1.) Circle T for true and F for false.

[4] 1a.) The maximum value of $f(x, y)$ subject to the constraint $g(x, y) = 0$ could occur at a point P where $\nabla g(P) = 0$. T F

[4] 1b.) The maximum value of $f(x, y)$ subject to the constraint $g(x, y) = 0$ could occur at a point P where $\nabla f(P) = 0$. T F

[12] 2.) Find $\frac{\partial z}{\partial x}$ as functions of x, y, and z, where $z = f(x, y)$ satisfies the equation $xz + yx^2 + yz^2 = y^2$

Answer: ______________________________

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[12] 2.) The base radius r and height h of a right circular **cylinder** are measured as 4m and 20m respectively. There is a possible error of as much as 10% in each of the measurements. Use differentials to estimate the maximum possible error in computing the volume of the **cylinder**.

$V = (base)(height) = \pi r^2 h$ $\qquad dr = 0.4, \quad dh = 2$

$\nabla V = <\frac{\partial V}{\partial r}, \frac{\partial V}{\partial h}> = <2\pi rh, \pi r^2> \qquad \nabla V(4.20) = <2(4)(20)\pi, (4)^2\pi> = <160\pi, 16\pi>$

$$dV = [\frac{\partial V}{\partial r} \quad \frac{\partial V}{\partial h}] \begin{bmatrix} dr \\ dh \end{bmatrix} = \frac{\partial V}{\partial r}dr + \frac{\partial V}{\partial h}dh = 160(0.4)\pi + 16(2)\pi = 16(6)\pi = 96\pi$$

Answer: 96π

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[12] 2.) Find $\frac{\partial z}{\partial x}$ as functions of x, y, and z, where $z = f(x,y)$ satisfies the equation $xz + yx^2 + yz^2 = y^2$

Note: x, y are independent variables, while z depends on x and y. Thus z is a function of x, while y is treated as a constant when computing $\frac{\partial}{\partial x}$

$\frac{\partial}{\partial x}(xz + yx^2 + yz^2) = \frac{\partial}{\partial x}(y^2)$

$$\frac{\partial}{\partial x}(xz) + \frac{\partial}{\partial x}(yx^2) + \frac{\partial}{\partial x}(yz^2) = 0$$

$$(z + x\frac{\partial z}{\partial x}) + (2yx) + (2y\frac{\partial z}{\partial x}) = 0$$

Note due to chain rule, it will always be easy to solve for $\frac{\partial z}{\partial x}$ by factoring it out:

$\frac{\partial z}{\partial x}(x + 2y) = -z - 2xy \qquad$ Thus $\frac{\partial z}{\partial x} = \frac{-z-2xy}{x+2y}$

Answer: $\frac{\partial z}{\partial x} = \frac{-z-2xy}{x+2y}$