Math 3550 Quiz 2 Form A   Feb 21, 2020

1.) Suppose $x = ts + u$, $y = t + su^3$ and $w = x^2y$. If $T(s,t,u) = (x(s,t,u), y(s,t,u))$ and $w = f(x,y)$, then

[1] 1a.) The Jacobian matrix f' has _____ rows and _____ columns.

[1] 1b.) The Jacobian matrix T' has _____ rows and _____ columns.

[1] 1c.) The Jacobian matrix $(f \circ T)'$ has _____ rows and _____ columns.

[7] 1d.) Using matrices, calculate $(f \circ T)'(s,t,u)$

Answer in matrix form: $(f \circ T)'(s,t,u) =$ ________________________________

[10] 2.) State the largest possible domain for the function $f(x,y) = ln(xy)$.

Answer: Domain is ____

Math 3550 Quiz 2 Form B   Feb 21, 2020

1.) Suppose $x = sin(ts) + u$, $y = ln(t + s + u^2)$ and $w = x^2y$. If $T(s, t, u) = (x(s, t, u), y(s, t, u))$ and $w = f(x, y)$, then

[1] 1a.) The Jacobian matrix f' has _____ rows and _____ columns.

[1] 1b.) The Jacobian matrix T' has _____ rows and _____ columns.

[1] 1c.) The Jacobian matrix $(f \circ T)'$ has _____ rows and _____ columns.

[7] 1d.) Using matrices, calculate $(f \circ T)'(s, t, u)$

Answer in matrix form: $(f \circ T)'(s, t, u) =$ ________________________________

[10] 2.) Find the equation of the tangent plane to the surface $z = x^2y$ at the point (1, 2, 2). Note z is a function of x and y. It is NOT a function of s, t, u.

Answer: Equation of tangent plane is ____

Math 3550 Quiz 2 Form B Feb 21, 2020

1.) Suppose $x = sin(ts) + u$, $y = ln(t + s + u^2)$ and $w = x^2y$. If $T(s, t, u) = (x(s, t, u), y(s, t, u))$ and $w = f(x, y)$, then

[1] 1a.) The Jacobian matrix f' has __1__ rows and __2__ columns.

[1] 1b.) The Jacobian matrix T' has __2__ rows and __3__ columns.

[1] 1c.) The Jacobian matrix $(f \circ T)'$ has __1__ rows and __3__ columns.

[7] 1d.) Using matrices, calculate $(f \circ T)'(s, t, u)$

$T(s, t, u) = (sin(ts) + u, ln(t + s + u^2)$ and $w = f(x, y) = x^2y$

$$(f \circ T)'(s, t, u) = f'(T(s, t, u))T'(s, t, u) = [2xy \quad x^2] \begin{bmatrix} t(sin(ts)) & s(sin(ts)) & 1 \\ \frac{1}{t+s+u^2} & \frac{1}{t+s+u^2} & \frac{2u}{t+s+u^2} \end{bmatrix}$$

$$= \begin{bmatrix} 2xyt(sin(ts)) + \frac{x^2}{t+s+u^2} & 2xys(sin(ts)) + \frac{x^2}{t+s+u^2} & 2xy + \frac{2ux^2}{t+s+u^2} \end{bmatrix}$$

Note answer must be in terms of s, t, u. The variables x and y should not appear in your answer.

Answer in matrix form: $(f \circ T)'(s, t, u) =$

$$[2(sin(ts) + u)ln(t + s + u^2)t(sin(ts)) + \frac{(sin(ts)+u)^2}{t+s+u^2} \quad 2(sin(ts) + u)ln(t + s + u^2)s(sin(ts)) + \frac{(sin(ts)+u)^2}{t+s+u^2}$$
$$2(sin(ts) + u)ln(t + s + u^2) + \frac{2u(sin(ts)+u)^2}{t+s+u^2}]$$

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[10] 2.) Find the equation of the tangent plane to the surface $z = x^2y$ at the point (1, 2, 2). Note z is a function of x and y. It is NOT a function of s, t, u.

Tangent vectors: $(\frac{\partial x}{\partial x}, \frac{\partial y}{\partial x}, \frac{\partial z}{\partial x}) = (1, 0, \frac{\partial z}{\partial x}) = (1, 0, 2xy)$ and $(\frac{\partial x}{\partial y}, \frac{\partial y}{\partial y}, \frac{\partial z}{\partial y}) = (0, 1, \frac{\partial z}{\partial y}) = (0, 1, x^2)$

Thus by taking the cross product of these two vectors, we get a vector normal to the tangent plane: $(\frac{\partial z}{\partial x}, \frac{\partial z}{\partial y}, -1) = (2xy, x^2, -1)$

Thus normal to tangent plan at (1, 2, 2) is $(2(1)(2), 1^2, -1) = (4, 1, -1)$

Equation of plane: $\mathbf{n} \cdot (\mathbf{x} - \mathbf{p}) = 0$

$(4, 1, -1) \cdot ((x, y, z) - (1, 2, 2)) = 4(x - 1) + y - 2 - (z - 2) = 0$

Answer: Equation of tangent plane is __$4x + y - z = 4$__

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[10] 2.) State the largest possible domain for the function $f(x, y) = ln(xy)$.

Answer: Domain is $(x > 0$ and $y > 0)$ OR $(x < 0$ and $y < 0)$