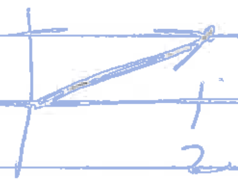
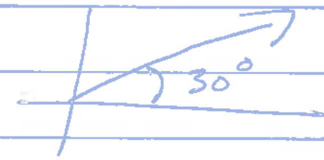
Vectors in $\mathbb{R}^2$ (1,1) { $\mathbb{R}^3$ (1, 2) or $\vec{\mathbb{R}}$

(2, 1) $\begin{bmatrix} 2 \\ 1 \end{bmatrix}$ $2\vec{i} + \vec{j}$

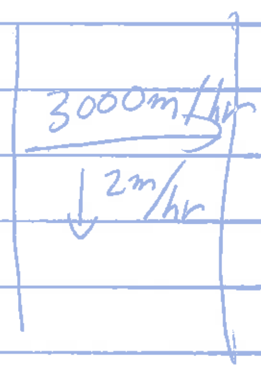


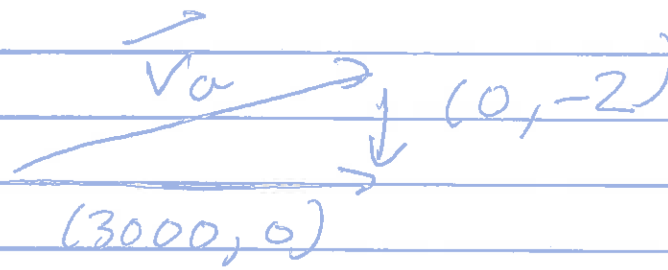
length: $\sqrt{2^2+1^2} = \sqrt{5}$

direction:



How should one swim to directly cross a river at 3 km/hr if current is 2 m/hr





$\vec{V_a} = (3000, 0) - (0, -2)$

$= (3000, 2)$ = velocity vector

speed: $\sqrt{3000^2 + 2^2} = \sqrt{9000004}$

If possible write (-6, -5) as a linear combination of (4, 1), (-2, 3)

$$r\begin{bmatrix}4\\1\end{bmatrix} + s\begin{bmatrix}-2\\3\end{bmatrix} = \begin{bmatrix}-6\\-5\end{bmatrix}$$

$$\begin{bmatrix}4 & -2\\1 & 3\end{bmatrix}\begin{bmatrix}r\\s\end{bmatrix} = \begin{bmatrix}-6\\-5\end{bmatrix}$$

$$\left[\begin{array}{cc|c}4^{-4} & -2^{-12} & -6^{+20}\\1 & 3 & -5\end{array}\right] \rightarrow \left[\begin{array}{cc|c}1 & 3 & -5\\0 & -14 & 14\end{array}\right]$$

$$\rightarrow \left[\begin{array}{cc|c}1 & 3^{-3} & -5^{+3}\\0 & 1 & -1\end{array}\right] \rightarrow \left[\begin{array}{cc|c}1 & 0 & -2\\0 & 1 & -1\end{array}\right]$$

$$\begin{bmatrix}-6\\-5\end{bmatrix} = -2\begin{bmatrix}4\\1\end{bmatrix} - \begin{bmatrix}-2\\3\end{bmatrix}$$

(3)

Defn: $\vec{a} \cdot \vec{b} = \sum a_i b_i$

$= |a|\,|b| \cos\theta$

Ex: $(3,4,5)\cdot(6,8,0) = 18+32+0 = 50$

$|(3,4,5)| = \sqrt{9+16+25} = \sqrt{50} = 5\sqrt{2}$

$|(6,8,0)| = \sqrt{36+64+0} = \sqrt{100} = 10$

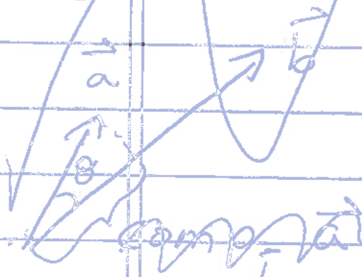
$$\cos\theta = \frac{50}{(5\sqrt{2})(10)} = \frac{1}{\sqrt{2}} = \frac{\sqrt{2}}{2}$$

$$\Rightarrow \theta = \frac{\pi}{4}$$

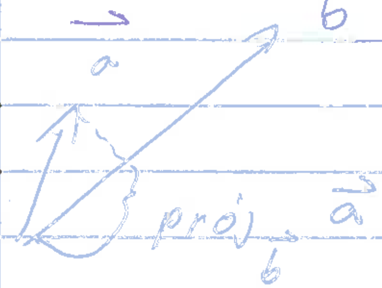
Ex: $(0,1)\cdot(1,0) = 0+0 = 0$

$\Rightarrow$ perpendicular $\Leftrightarrow \vec{a}\cdot\vec{b} = 0$

Defn: $\text{comp}_{\vec{b}}\, \vec{a} = \text{proj}_{b}\, \vec{a} = \dfrac{\vec{a}\cdot\vec{b}}{|b|}$



$$\frac{|\text{comp}_{\vec{b}}\,\vec{a}|}{|a|} = \cos\theta$$



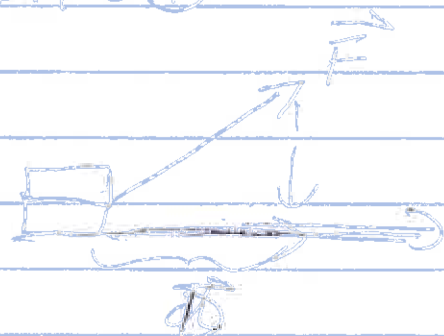
$$\frac{comp_{\vec{b}}\,\vec{a}}{|a|} = \cos\theta$$

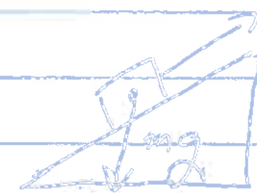
$$\Rightarrow comp_{\vec{b}}\,\vec{a} = \frac{|a|\,|b|\cos\theta}{|b|}$$

$$comp_{\vec{b}}\,\vec{a} = |proj_{\vec{b}}\,\vec{a}| = \frac{\vec{a}\cdot\vec{b}}{|b|}$$

In 1D : Work = $W = Fd$

In 3D $\qquad W = \vec{F}\cdot\vec{D}$





$$1D:\; comp_{\vec{D}}\vec{F}\cdot|D| = \frac{\vec{F}\cdot\vec{D}}{|D|}\cdot|D|$$

11.3 Cross Product ($\mathbb{R}^3$)

$\vec{a}, \vec{b}$ in $\mathbb{R}^3$

$$\vec{a} \times \vec{b} = \begin{vmatrix} \vec{i} & \vec{j} & \vec{k} \\ a_1 & a_2 & a_3 \\ b_1 & b_2 & b_3 \end{vmatrix}$$

EX:

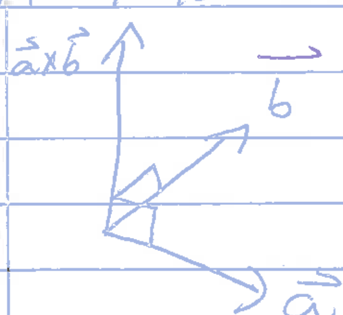
$$(1,2,3) \times (4,5,6) = \begin{vmatrix} \vec{i} & \vec{j} & \vec{k} \\ 1 & 2 & 3 \\ 4 & 5 & 6 \end{vmatrix}$$

$$= \vec{i} \begin{vmatrix} 2 & 3 \\ 5 & 6 \end{vmatrix} - \vec{j} \begin{vmatrix} 1 & 3 \\ 4 & 6 \end{vmatrix} + \vec{k} \begin{vmatrix} 1 & 2 \\ 4 & 5 \end{vmatrix}$$

$$= -3\vec{i} + 6\vec{j} - 3\vec{k}$$

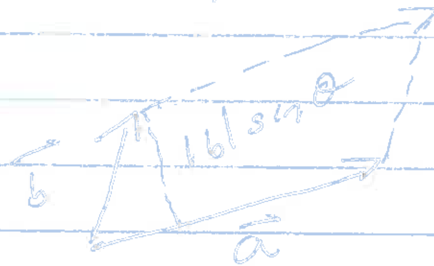
$$= (-3, 6, -3)$$

Right-hand rule



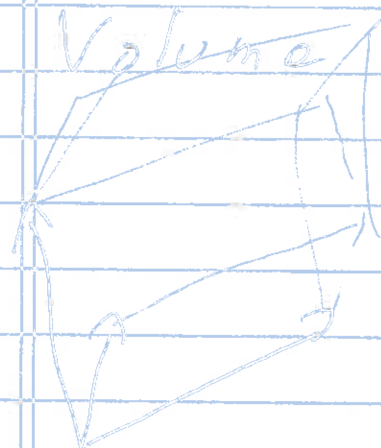
$|a \times b| = |a||b| \sin\theta$

Area of parallelogram



$$\text{Area} = |a||b|\sin\theta$$
$$= |a \times b|$$

Volume of parallelpiped



$$\text{Volume} = |\vec{a} \cdot (\vec{b} \times \vec{c})|$$

scalar triple product

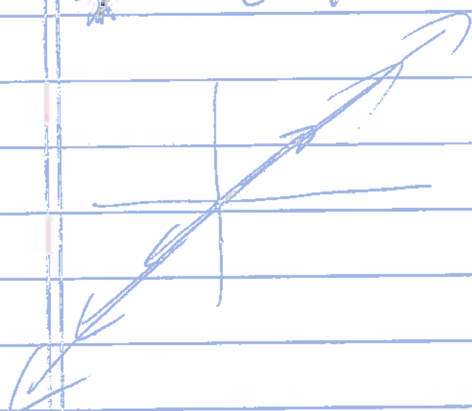
11.4 : Lines & Planes

Lines

(vector format)

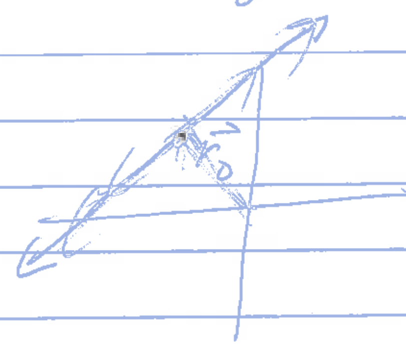
thru origin

$\vec{r} = t\vec{v}$



thru $\vec{r}_0$

$\vec{r} = \vec{r}_0 + t\vec{v}$



Parametric equation of line

if $\vec{r}_0 = (x_0, y_0, z_0)$, $\vec{v} = (a, b, c)$

$$x = x_0 + ta$$
$$y = y_0 + tb$$
$$z = z_0 + tc$$

Symmetric equation (eliminate t)

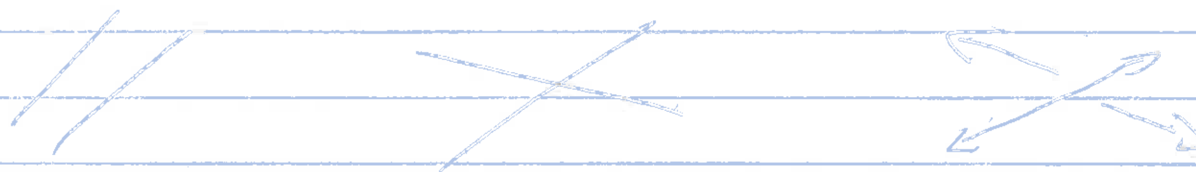
$$t = \boxed{\frac{x - x_0}{a} = \frac{y - y_0}{b} = \frac{z - z_0}{c}}$$ if $abc \neq 0$

Ex: If $a = 0$, $bc \neq 0$

$$x = x_0, \quad \frac{y - y_0}{b} = \frac{z - z_0}{c}$$

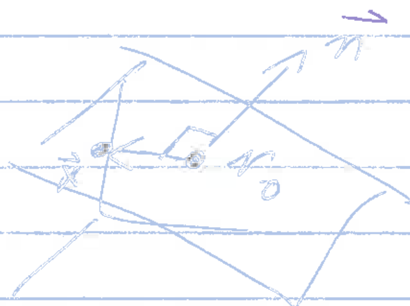
In $\mathbb{R}^3$

parallel intersecting skew



Planes

Plane thru $\vec{r}_0$ and $\perp$ to $\vec{n}$



$n \cdot (\vec{x} - \vec{r}_0) = 0$

If $\vec{n} = (a, b, c)$

$ax + by + cz = d$

11.5: Curves & Motions

Parametric curve:

$x = f(t) \quad y = g(t) \quad z = h(t)$

only 1 independent variable t

$\Rightarrow$ 1D curve

Ex: $x = \cos t$

$y = \sin t$

$z = t$

Vector-valued fn

$\vec{r} = (x, y, z) = \vec{i}\cos t + \vec{j}\sin t + t\vec{k}$

Tangent line to curve at $(1,0,0)$

$\vec{r}'(t) = -\vec{i}\sin t + \vec{j}\cos t + \vec{k}$

$r'(0) = (0\vec{i} + \vec{j} + \vec{k})$

tangent line: $\vec{x} = (1,0,0) + t(0,1,1)$

velocity

position vector $\vec{r}(t)$
velocity " $\vec{r}'(t)$
accelerat " $\vec{r}''(t)$

magnitude of vel = speed $= |r'(t)|$

acc = scalar accelerati $= |r''(t)|$