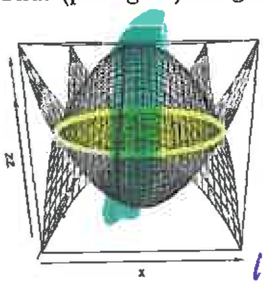
Find the volume between the surfaces: $z = x^2 + y^2 - 9$, $z = 16 - x^2 - y^2$

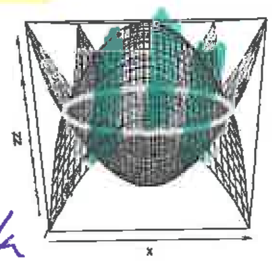
Need to find region over which to integrate:

(1) Find intersection between surfaces: $x^2 + y^2 - 9 = 16 - x^2 - y^2$

Hence intersection is $2x^2 + 2y^2 = 25$

Thus (per figure) integrating over region $x^2 + y^2 \leq \frac{25}{2}$





(2) Height of columns: $\underbrace{16 - x^2 - y^2}_{\text{larger}} - \underbrace{(x^2 + y^2 - 9)}_{\text{smaller}} = 25 - 2x^2 - 2y^2$

(3) Thus need to integrate $\int\int_R (25 - 2x^2 - 2y^2)dA$

Note: this integral is easier to compute using polar coordinates:

Height of columns: $25 - 2x^2 - 2y^2 = 25 - 2r^2$

Region: $x^2 + y^2 \leq \frac{25}{2}$ is equivalent to $0 \leq r \leq \frac{5}{\sqrt{2}}$ and $0 \leq \theta \leq 2\pi$

Hence $\int\int_R (25 - 2x^2 - 2y^2)(dA) = \int_0^{2\pi}\int_0^{\frac{5}{\sqrt{2}}} (25 - 2r^2)(r\,dr\,d\theta)$

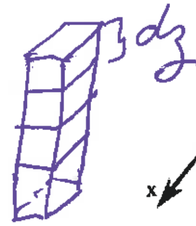
$= \int_0^{\frac{5}{\sqrt{2}}}\int_0^{2\pi} (25r - 2r^3)d\theta dr = \int_0^{\frac{5}{\sqrt{2}}} \int (25r - 2r^3)\theta|_0^{2\pi} dr = \int_0^{\frac{5}{\sqrt{2}}} (25r - 2r^3)(2\pi)dr$

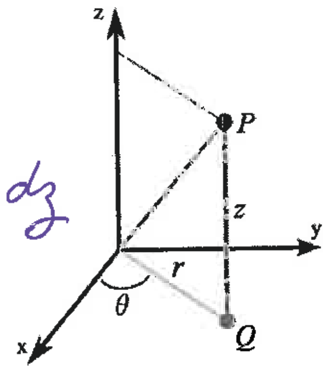
$= (2\pi)(\frac{25}{2}r^2 - \frac{2}{4}r^4)|_0^{\frac{5}{\sqrt{2}}} = (\pi)[25(\frac{5}{\sqrt{2}})^2 - (\frac{5}{\sqrt{2}})^4)] = (\pi)[25(\frac{5^2}{2}) - (\frac{5^4}{4})]$

$= (\pi)[\frac{2(5^4) - 5^4}{4}] = (\pi)[\frac{5^4}{4}] = \frac{625\pi}{4}$

11.8: Cylindrical coordinates and Spherical coordinates

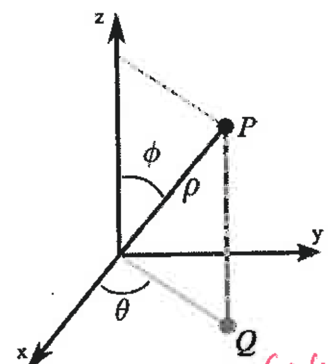
Cylindrical coordinates:





Spherical coordinates:

$$x = \rho \sin\phi \cos\theta$$
$$y = \rho \sin\phi \sin\theta \quad (1)$$
$$z = \rho \cos\phi.$$



$\iint_R [f(x,y) - g(x,y)]dA$

$=$

$\iint_R (25 - 2x^2 - 2y^2)dA = \iint_R \left[\int_{g(x,y)}^{f(x,y)} dz\right] dA$

11.7

Conics in $\mathbb{R}^2$: $Ax^2 + Bxy + Cy^2 + Dx + Ey + F = 0$

for suitable constants $A, \ldots, F$.

In $\mathbf{R}^3$, the analytic analogue of the conic section is called a **quadric surface**. Quadric surfaces are those defined by equations that are polynomials of degree two in three variables:

$$Ax^2 + Bxy + Cxz + Dy^2 + Eyz + Fz^2 + Gx + Hy + Iz + J = 0.$$ ← degree 2, 3 variab

$z\ \|\mathbf{x} - \mathbf{x}_0\| = r$

$(x-x_0)^2 + (y-y_0)^2 + (z-z_0)^2 = r^2$

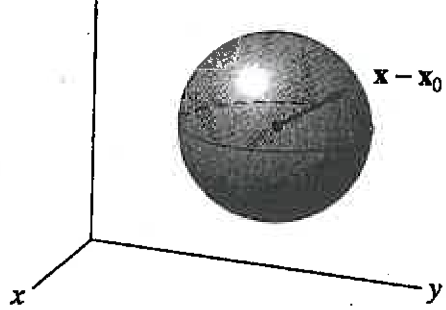


Figure 2.20 The sphere of radius a, centered at (x_0, y_0, z_0).

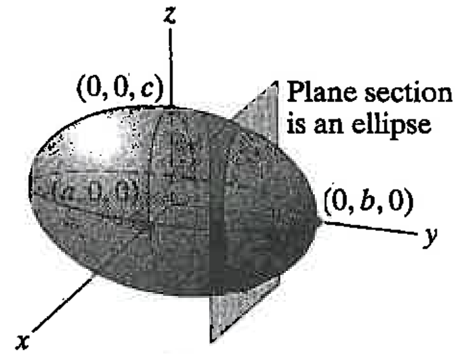
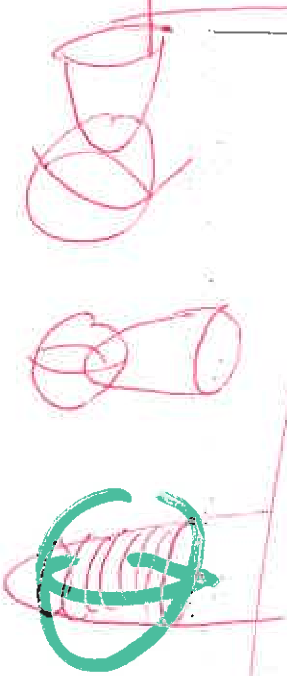


Figure 2.21 The ellipsoid $\dfrac{x^2}{a^2} + \dfrac{y^2}{b^2} + \dfrac{z^2}{c^2} = 1.$



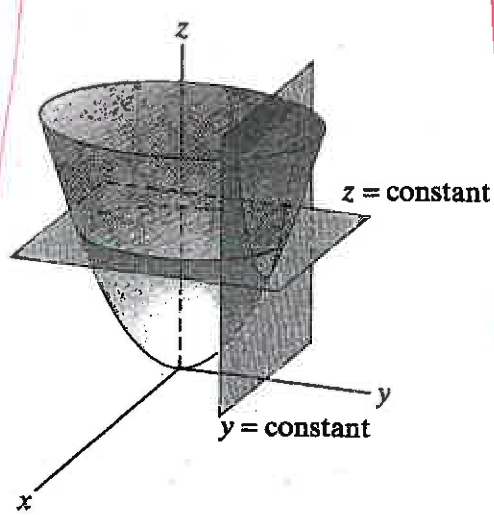


Figure 2.22 The elliptic paraboloid $\dfrac{z}{c} = \dfrac{x^2}{a^2} + \dfrac{y^2}{b^2}$.

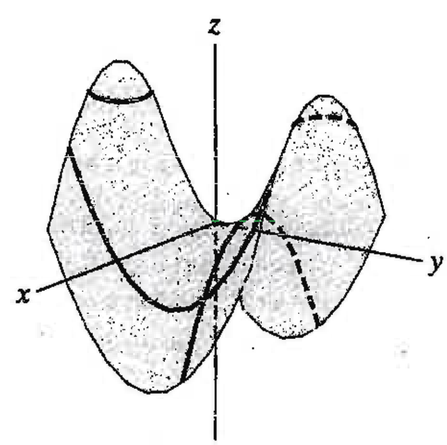


Figure 2.23 The hyperbolic paraboloid $\dfrac{z}{c} = \dfrac{y^2}{b^2} - \dfrac{x^2}{a^2}$.

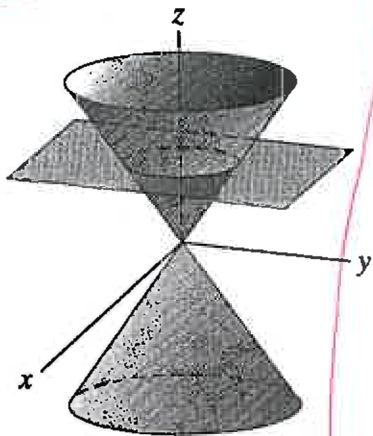


Figure 2.24 The elliptic cone $\frac{z^2}{c^2} = \frac{x^2}{a^2} + \frac{y^2}{b^2}$.

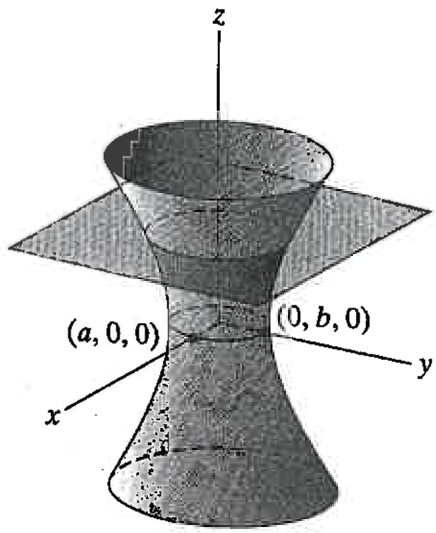


Figure 2.25 The graph of the equation $\frac{x^2}{a^2} + \frac{y^2}{b^2} - \frac{z^2}{c^2} = 1$ is a hyperboloid of one sheet.

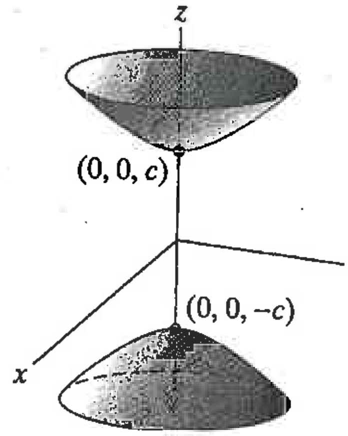


Figure 2.26 The graph of the equation $\frac{z^2}{c^2} - \frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$ is a hyperboloid of two sheets.