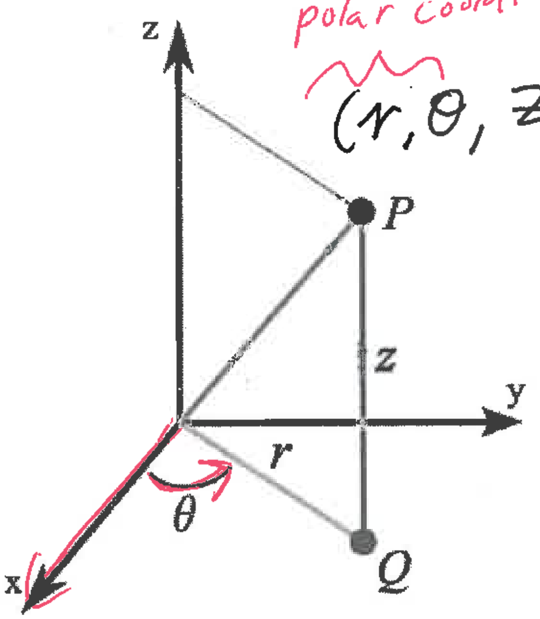
11.8: Cylindrical coordinates and Spherical coordinates

Cylindrical coordinates:

polar coordinates

(r, θ, z)

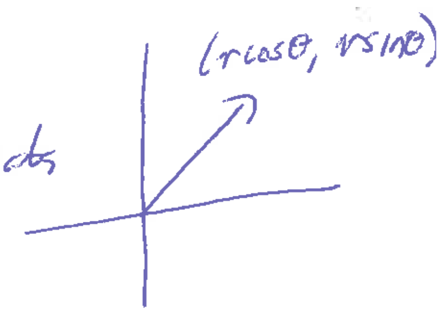


$x = r\cos\theta$

$y = r\sin\theta$

$z = z$

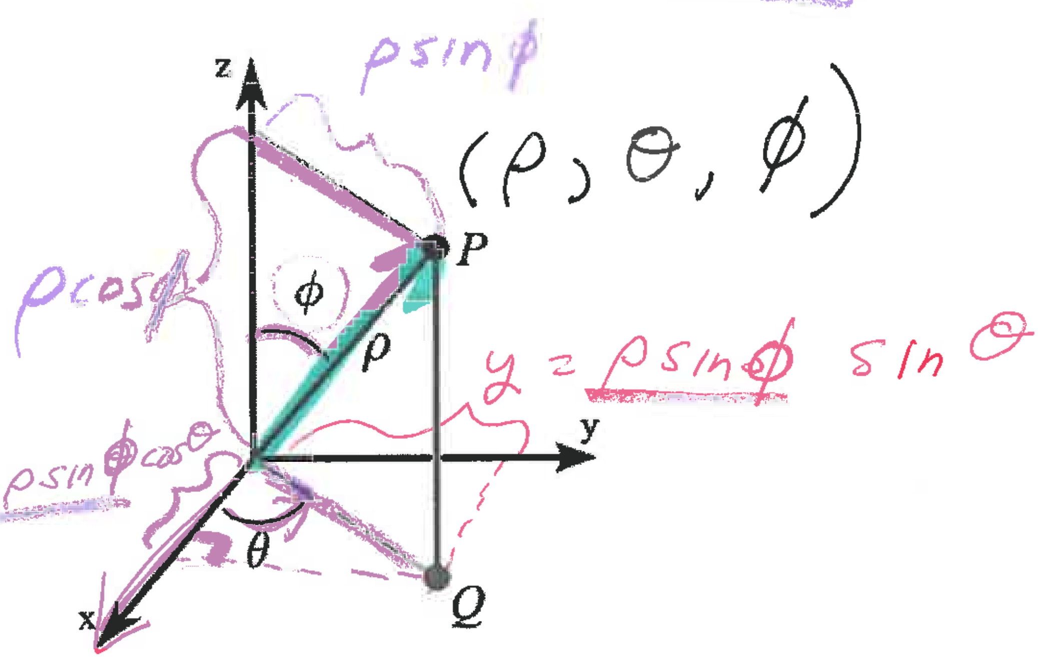
compare to polar coordinates



Spherical coordinates:

$$
\begin{aligned}
x &= \rho \sin\phi \cos\theta \\
y &= \rho \sin\phi \sin\theta \\
\boxed{z} &\boxed{= \rho \cos\phi.}
\end{aligned} \tag{1}
$$

(ρ, θ, ϕ)



Potential Exam 1 problem

Find the volume between the surfaces: $z = x^2 + y^2 - 9$, $z = 16 - x^2 - y^2$

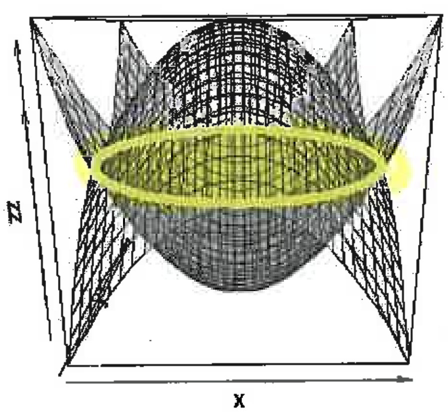
Need to find region over which to integrate:

Find intersection between surfaces: $x^2 + y^2 - 9 = 16 - x^2 - y^2$

larger value $\boxed{16 - x^2 - y^2}$

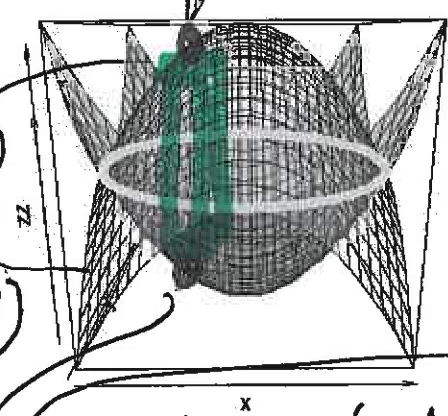
Hence intersection is $2x^2 + 2y^2 = 25$

Thus (per figure) integrating over region $x^2 + y^2 \leq \frac{25}{2}$



height of column $= \text{larger} - \text{smaller}$

$(16 - x^2 - y^2) - (x^2 + y^2 - 9)$



smaller value: $x^2 + y^2 - 9$

Height of columns: $16 - x^2 - y^2 - (x^2 + y^2 - 9) = 25 - 2x^2 - 2y^2$

Thus need to integrate $\int\int_R (25 - 2x^2 - 2y^2) dA$

Note: this integral is easier to compute using polar coordinates:

Height of columns: $25 - 2x^2 - 2y^2 = 25 - 2r^2$

Region: $x^2 + y^2 \leq \frac{25}{2}$ is equivalent to $0 \leq r \leq \frac{5}{\sqrt{2}}$ and $0 \leq \theta \leq 2\pi$

Hence $\int\int_R (25 - 2x^2 - 2y^2)(dA) = \int_0^{2\pi} \int_0^{\frac{5}{\sqrt{2}}} (25 - 2r^2)(r dr d\theta)$

$= \int_0^{\frac{5}{\sqrt{2}}} \int_0^{2\pi} (25r - 2r^3) d\theta dr = \int_0^{\frac{5}{\sqrt{2}}} \int (25r - 2r^3)\theta|_0^{2\pi} dr = \int_0^{\frac{5}{\sqrt{2}}} (25r - 2r^3)(2\pi) dr$

$= (2\pi)(\frac{25}{2}r^2 - \frac{2}{4}r^4)|_0^{\frac{5}{\sqrt{2}}} = (\pi)[25(\frac{5}{\sqrt{2}})^2 - (\frac{5}{\sqrt{2}})^4)] = (\pi)[25(\frac{5^2}{2}) - (\frac{5^4}{4})]$

$= (\pi)[\frac{2(5^4) - 5^4}{4}] = (\pi)[\frac{5^4}{4}] = \frac{625\pi}{4}$