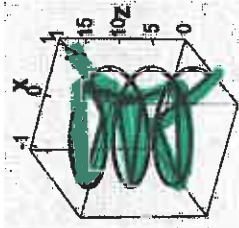
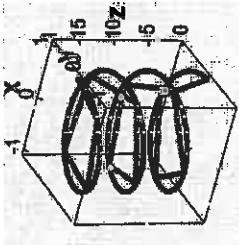


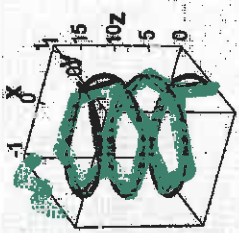
Chapter 13: Integrals and Riemann sums



$\Delta t = 2$
helix length = 26.86
 $L(1/2) = 24.75$

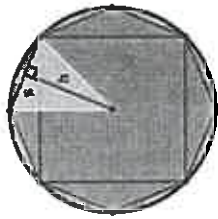


$\Delta t = 1.5$
helix length = 26.60
 $L(1/3) = 25.61$



$\Delta t = 1.16$
helix length = 26.66
 $L(1/4) = 25.96$

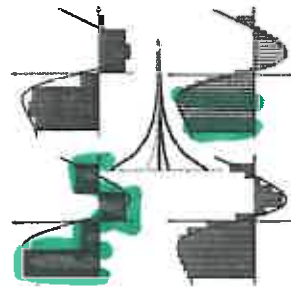
<https://mathinsight.org/parametric-arc-length>



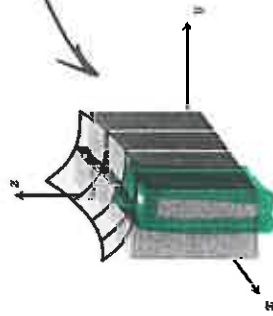
No. of vertices	Area of surface polygon
4	2.000000
6	2.598076
8	3.464102
10	4.702381
12	6.211657
14	7.927061
16	9.859813
18	11.989905
20	14.299435
22	16.774314
24	19.399644
26	22.161525
28	25.045956
30	28.039027

www.maa.org/book/export/html/864399

www-users.math.umn.edu/~arnold//calculus/archimedes



wikipedia.org/wiki/Riemann-sum



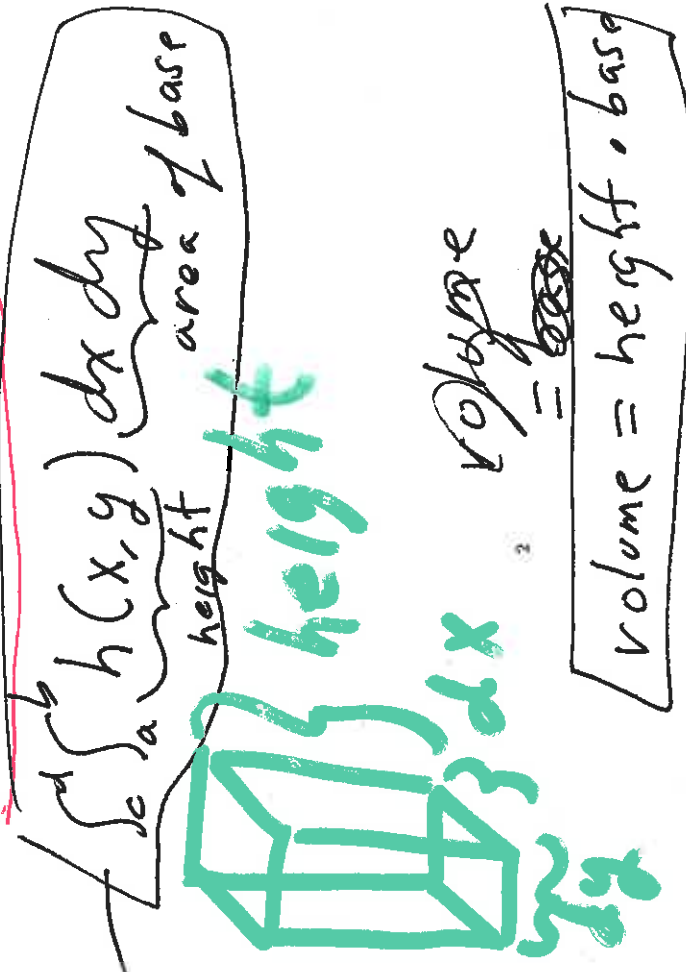
Δx

Ex 1: $\int_0^8 \int_4^{x^2-y^2} dx dy = \int_0^2 (\frac{x^3}{3} - y^2 x) \Big|_4^{x^2-y^2} dy = \int_0^2 [(\frac{8^3}{3} - 8y^2) - (\frac{4^3}{3} - 4y^2)] dy$
 $= \int_0^2 (\frac{6^3}{3} - \frac{4^3}{3} - 4y^2) dy = \int_0^2 (\frac{4^3}{3}(2^3 - 1) - 4y^2) dy = \int_0^2 (\frac{4^3}{3}(7) - 4y^2) dy$
 $= \frac{4^3}{3}(7)y - \frac{4y^3}{3} \Big|_0^2 = \frac{4^3}{3}(7)(2) - \frac{4(2)^3}{3} = 2^3(\frac{2^3}{3}(7)(2) - \frac{4}{3}) = (2^3)(4)(\frac{4}{3}(7) - \frac{1}{3})$
 $= (2^3)(4)(\frac{27}{3}) = (8)(4)(9) = 320 - 32 = 288$

Ex 1: $\int_4^8 \int_0^{x^2-y^2} dy dx = \int_4^8 (x^2 y - \frac{y^3}{3}) \Big|_0^{x^2-y^2} dx = \int_4^8 (2x^2 - \frac{2^3}{3}) dx$
 $= (\frac{2x^3}{3} - \frac{2^3 x}{3}) \Big|_4^8 = (\frac{2(8)^3}{3} - \frac{2^3(8)}{3}) - (\frac{2(4)^3}{3} - \frac{2^3(4)}{3}) = \frac{2(2)^3}{3}(4^3 - 4 - 2^3 + 2)$
 $= \frac{(2)^5}{3}(2(4^2) - 2 - 2^2 + 1) = \frac{32}{3}(32 - 2 - 4 + 1) = \frac{32}{3}(27) = (32)(9) = 288$

Note since functions are continuous:

$\int_0^2 \int_4^8 (x^2 - y^2) dx dy = \int_4^8 \int_0^2 (x^2 - y^2) dy dx$



$\int h(x) \cdot dx$
 (height * width)