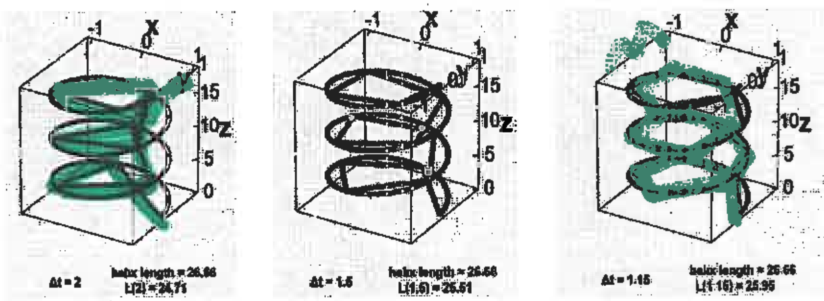
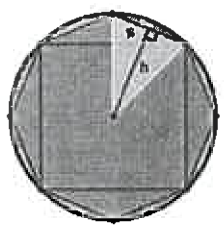
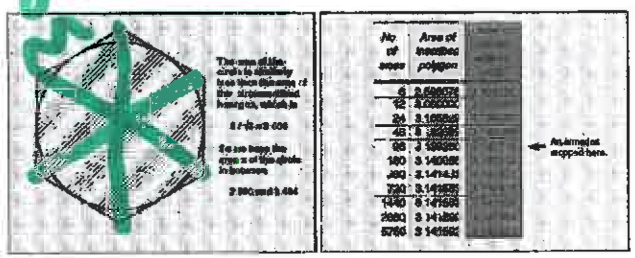
Chapter 13: Integrals and Riemann sums

dx



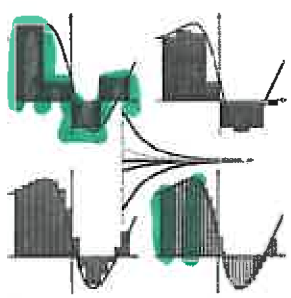
https://mathinsight.org/parametrized_curve_arc_length

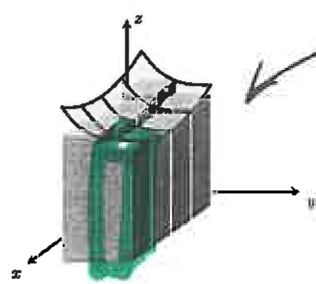




www.maa.org/book/export/html/864399

www-users.math.umn.edu/ arnold//calculus/archimedes





.wikipedia.org/wiki/Riemann_sum

$\int^b \underbrace{h(x)}_{(\text{height})} \cdot \underbrace{dx}_{(\text{width})}$

1

Ex 1: $\int_0^2 \boxed{\int_4^8 (x^2 - y^2)dx}dy = \int_0^2 (\frac{x^3}{3} - y^2x)|_4^8 dy = \int_0^2 [(\frac{8^3}{3} - 8y^2) - (\frac{4^3}{3} - 4y^2)]dy$

$= \int_0^2 (\frac{8^3}{3} - \frac{4^3}{3} - 4y^2)dy = \int_0^2 (\frac{4^3}{3}(2^3 - 1) - 4y^2)dy = \int_0^2 (\frac{4^3}{3}(7) - 4y^2)dy$

$= \frac{4^3}{3}(7)y - \frac{4y^3}{3}|_0^2 = \frac{4^3}{3}(7)(2) - \frac{4(2)^3}{3} = 2^3(\frac{2^3}{3}(7)(2) - \frac{4}{3}) = (2^3)(4)(\frac{4}{3}(7) - \frac{1}{3})$

$= (2^3)(4)(\frac{27}{3}) = (8)(4)(9) = 320 - 32 = 288$

Ex 1: $\int_4^8 \boxed{\int_0^2 (x^2 - y^2)dy}dx = \int_4^8 (x^2y - \frac{y^3}{3})|_0^2 dx = \boxed{\int_4^8 (2x^2 - \frac{2^3}{3})dx}$

$= (\frac{2x^3}{3} - \frac{2^3x}{3})|_4^8 = (\frac{2(8)^3}{3} - \frac{2^3(8)}{3}) - (\frac{2(4)^3}{3} - \frac{2^3(4)}{3}) = \frac{2(2)^3}{3}(4^3 - 4 - 2^3 + 2)$

$= \frac{(2)^5}{3}(2(4^2) - 2 - 2^2 + 1) = \frac{32}{3}(32 - 2 - 4 + 1) = \frac{32}{3}(27) = (32)(9) = 288$

Note since functions are continuous:

$$\int_0^2 \int_4^8 (x^2 - y^2)dxdy = \int_4^8 \int_0^2 (x^2 - y^2)dydx$$

$$\int_c^d \int_a^b \underbrace{h(x,y)}_{\text{height}} \underbrace{dx\,dy}_{\text{area of base}}$$

height

dx

dy

volume = ~~base~~

$\boxed{\text{volume} = \text{height} \cdot \text{base}}$

2