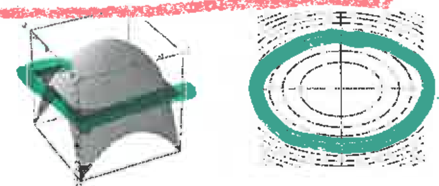
12.2 Functions of several variables: $z = f(x_1, ..., x_n)$

Level set: $f(x_1, ..., x_n) = c$ for some constant c.



$f(x,y) = c$

Nykamp DQ, Level sets. From Math Insight. http://mathinsight.org/level_sets

A topographical map shows level sets:

https://sciencing.com/read-topographic-maps-4577366.html

Can use level sets to understand graphs of functions with 3 variables:

Example: $f(x, y, x) = x^2 + y^2 + z^2$

Example: $g(x, y, x) = x^2 + y^2 + z^2 - 8z$

Cross section = the intersection of the graph of $z = f(x, y)$ with a plane.

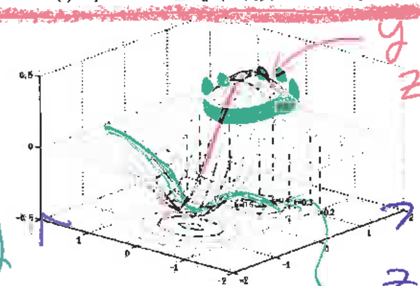
Examples:

$z = f(c, y)$ $x = c$

$z = f(x, c)$ $y = c$

$c = f(x, y)$ $z = c$

level sets



$y = 0$

$z = f(x, 0)$

$x = -\frac{1}{2}$

$z = f(-\frac{1}{2}, y)$

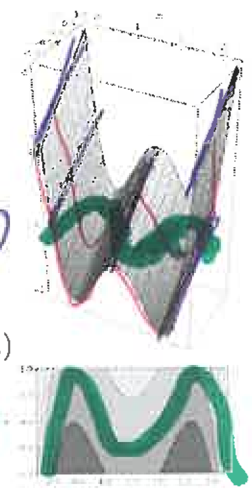
https://www.math.tamu.edu/~mpilant/math696/m696_240/jsamayoa/public_html/levelcurves.html

Match the following graphs to one of the functions below:

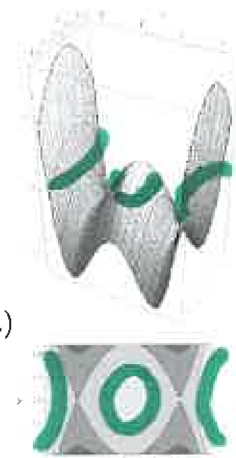
$y = c$

$x = c$

1.)



2.)



https://www.wolframalpha.com/

a.) ~~$f(x, y) = 6x^3 + 11x^2 - 6x + y$~~

b.) $f(x, y) = x^4 - 6x^3 + 11x^2 - 6x + y$ $x = c \Rightarrow z = k + y$

c.) $f(x, y) = x^4 - 6x^3 + 11x^2 - 6x - y^2$

d.) $f(x, y) = x^4 - 6x^3 + 11x^2 - 6x + y^2$

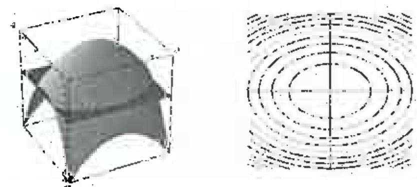
$f(x, y) = h(x) + k(y)$

level set $z = c = f(x,y)$ $c =$

$y = c - x^4 + 6x^3 - 11x^2 + 6x$

12.2 Functions of several variables: $z = f(x_1, \ldots, x_n)$

Level set: $f(x_1, \ldots, x_n) = c$ for some constant c.



Nykamp DQ, Level sets. From Math Insight. http://mathinsight.org/level_sets

A topographical map shows level sets:

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Can use level sets to understand graphs of functions with 3 variables:

Example: $f(x, y, x) = x^2 + y^2 + z^2$

Level set $c = x^2 + y^2 + z^2$

Sphere of radius $\sqrt{c}$ centered at origin

Example: $g(x, y, x) = x^2 + y^2 + z^2 - 8z$

complete the square $\rightarrow$ $c = x^2 + y^2 + (z-4)^2 - 16$

levelsets = spheres centered at $(0, 0, 4)$

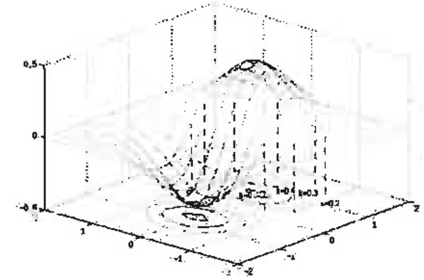
Cross section = the intersection of the graph of $z = f(x, y)$ with a plane.

Examples:

$z = f(c, y)$

$z = f(x, c)$

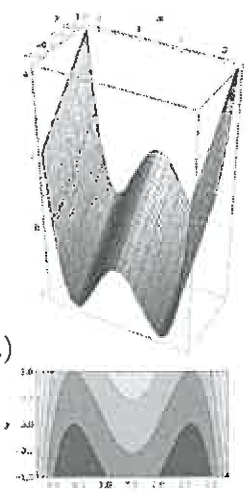
$c = f(x, y)$



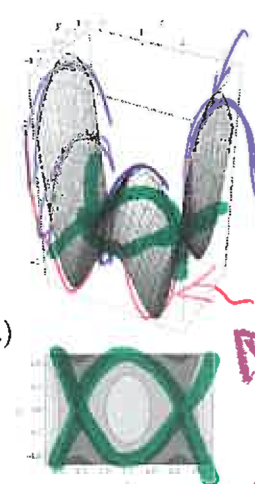
https://www.math.tamu.edu/~mpilant/math696/m696_240/jsamayoa/public_html/levelcurves.html

Match the following graphs to one of the functions below:

1.)



2.)



parabola $\frown$ $-y^2$

$y = c$

https://www.wolframalpha.com/

a.) $f(x, y, z) = 6x^3 + 11x^2 - 6x + y$

b.) $f(x, y, z) = x^4 - 6x^3 + 11x^2 - 6x + y$

c.) $f(x, y, z) = x^4 - 6x^3 + 11x^2 - 6x - y^2$ $z = k - y^2$

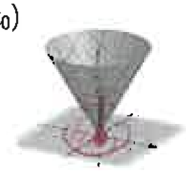
d.) $f(x, y, z) = x^4 - 6x^3 + 11x^2 - 6x + y^2$

$c = x^4 - \cdots - y^2$

12.3: Limits and continuity

$z = f(\mathbf{x})$ is **continuous** at $\mathbf{x}_0$ if $\lim_{\mathbf{x}\to\mathbf{x}_0} f(\mathbf{x}) = f(\mathbf{x}_0)$

Ex: $\lim_{(x,y)\to(0,0)} \sqrt{3x^2+3y^2} = \sqrt{3(0)^2+3(0)^2}$
$= 0$



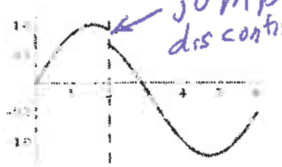
https://math.stackexchange.com/questions/2203478/triple-integrals-in-spherical-coordinates-z-sqrt3x23y2

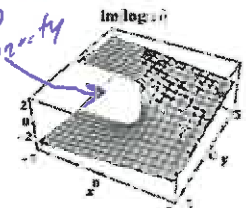
Defn: $\lim_{\mathbf{x}\to\mathbf{x}_0} f(\mathbf{x}) = L$ iff for all $\epsilon > 0$, there exists $\delta > 0$ such that if $|\mathbf{x}-\mathbf{x}_0| < \delta$, then $|f(\mathbf{x}) - L| < \epsilon$

***** In other words if $\mathbf{x}$ is close to $\mathbf{x}_0$, then $\mathbf{f}(\mathbf{x})$ is close to L *****

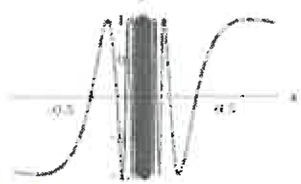
Discontinuous functions:

jump discontinuity





$y = sin(\frac{1}{x})$



Understand 12.3: 51 HW # 2

Ex 9: Let $f(x,y) = \frac{xy}{x^2+y^2}$. $\lim_{(x,y)\to(0,0)} \frac{xy}{x^2+y^2} =$ DNE

Suppose we travel to (0, 0) along a line with slope m. Thus $y = mx$:

$$\lim_{(x,mx)\to(0,0)} \frac{xy}{x^2+y^2} = \lim_{(x,mx)\to(0,0)} \frac{x(mx)}{x^2+(mx)^2} = \lim_{x\to 0} \frac{mx^2}{(m^2+1)x^2}$$

$$= \lim_{x\to 0} \underbrace{\frac{m}{m^2+1}}_{\text{constant}} = \frac{m}{m^2+1}$$

Note $f(0,y) = 0$, $f(x,0) = 0$, $f(x,x) = \frac{1}{2}$, $f(x,-x) = -\frac{1}{2}$

$f(x, mx) = \frac{m}{m^2+1}$, $x \neq 0$

$y = mx$

Ex 9: $z = \frac{xy}{x^2+y^2}$

missing $(0,0,z)$

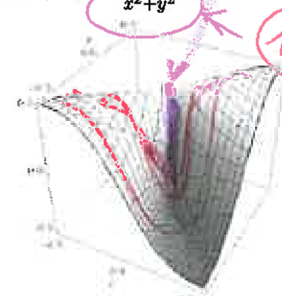
$(x,y) = (r\cos\theta, r\sin\theta)$

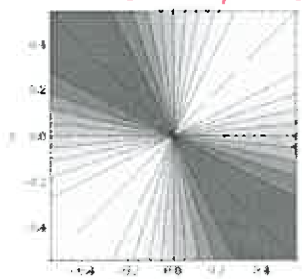
$r = \sqrt{x^2+y^2}$

(r, θ)

r^2

r = radius





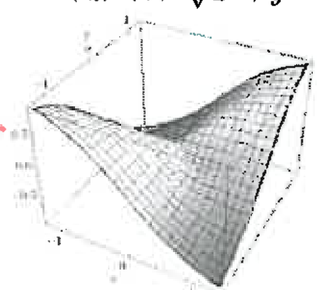
Change to polar coordinates: Let $(x,y) = (rcos(\theta), rsin(\theta))$.

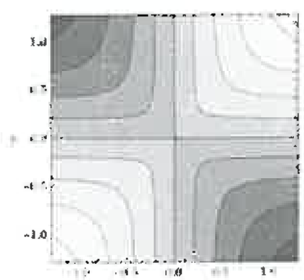
$\frac{xy}{x^2+y^2} = \frac{rcos(\theta)rsin(\theta)}{r^2} = cos(\theta)sin(\theta) = \frac{sin(2\theta)}{2}$

Thus $f(x,y) = \frac{xy}{x^2+y^2}$ in polar coordinates is

$$f(r,\theta) = \frac{sin(2\theta)}{2}, \quad r \geq 0, \theta \in [0, 2\pi]$$

Ex 8: $\lim_{(x,y)\to(0,0)} \frac{xy}{\sqrt{x^2+y^2}} =$





Change to polar coordinates: Let $(x,y) = (rcos(\theta), rsin(\theta))$.

$\frac{xy}{\sqrt{x^2+y^2}} = \frac{rcos(\theta)rsin(\theta)}{r} = rcos(\theta)sin(\theta) = \frac{rsin(2\theta)}{2}$

$$\lim_{(x,y)\to(0,0)} \frac{xy}{\sqrt{x^2+y^2}} = \lim_{r\to 0} \frac{rsin(2\theta)}{2} = 0$$

$r = |(x,y)| \to 0$
$\Leftrightarrow (x,y) \to (0,0)$