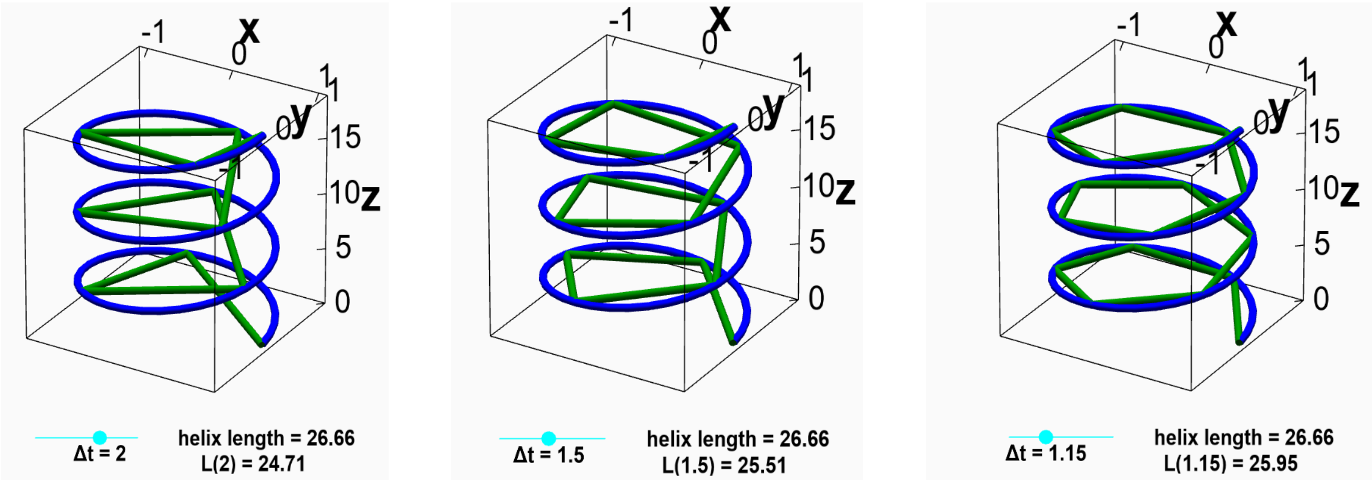
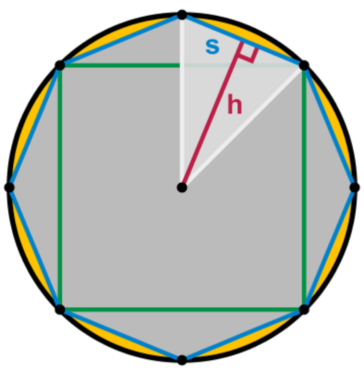
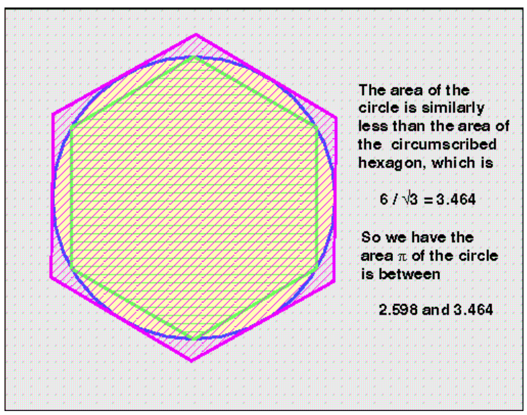
Chapter 13: Integrals and Riemann sums



https://mathinsight.org/parametrized_curve_arc_length

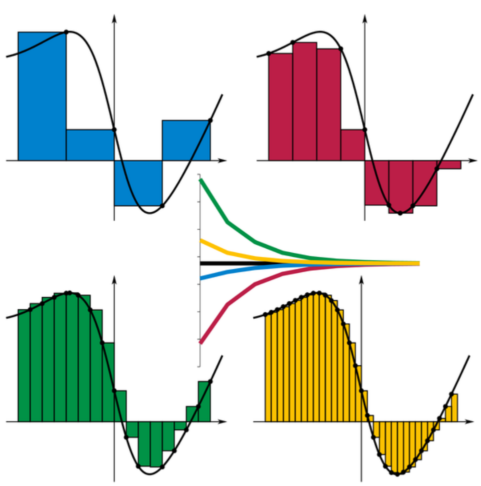




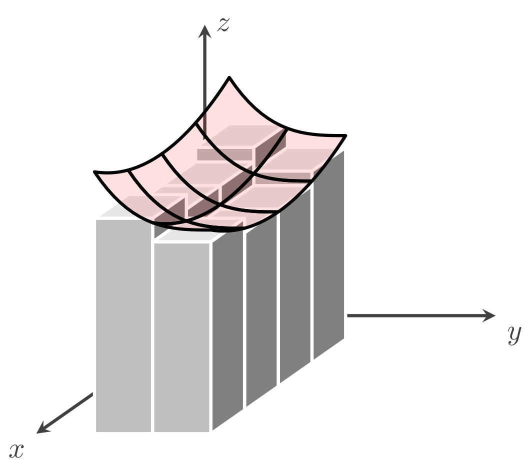
No. of sides	Area of inscribed polygon	Area of circum-scribed poly.
6	2.598076	3.464102
12	3.000000	3.215390
24	3.105829	3.159660
48	3.132629	3.146086
96	3.139350	3.142715
180	3.140955	3.141912
360	3.141433	3.141672
720	3.141553	3.141613
1440	3.141583	3.141598
2880	3.141590	3.141594
5760	3.141592	3.141593

← Archimedes stopped here. (at 96 sides)

www.maa.org/book/export/html/864399 www-users.math.umn.edu/ arnold//calculus/archimedes



en.wikipedia.org/wiki/Riemann_sum



Ex 1: $\int_0^2 \int_4^8 (x^2 - y^2) dxdy = \int_0^2 (\frac{x^3}{3} - y^2 x)|_4^8 dy = \int_0^2 [(\frac{8^3}{3} - 8y^2) - (\frac{4^3}{3} - 4y^2)] dy$

$= \int_0^2 (\frac{8^3}{3} - \frac{4^3}{3} - 4y^2) dy = \int_0^2 (\frac{4^3}{3}(2^3 - 1) - 4y^2) dy = \int_0^2 (\frac{4^3}{3}(7) - 4y^2) dy$

$= \frac{4^3}{3}(7)y - \frac{4y^3}{3}|_0^2 = \frac{4^3}{3}(7)(2) - \frac{4(2)^3}{3} = 2^3(\frac{2^3}{3}(7)(2) - \frac{4}{3}) = (2^3)(4)(\frac{4}{3}(7) - \frac{1}{3})$

$= (2^3)(4)(\frac{27}{3}) = (8)(4)(9) = 320 - 32 = 288$

Ex 1: $\int_4^8 \int_0^2 (x^2 - y^2) dydx = \int_4^8 (x^2 y - \frac{y^3}{3})|_0^2 dx = \int_4^8 (2x^2 - \frac{2^3}{3}) dx$

$= (\frac{2x^3}{3} - \frac{2^3 x}{3})|_4^8 = (\frac{2(8)^3}{3} - \frac{2^3(8)}{3}) - (\frac{2(4)^3}{3} - \frac{2^3(4)}{3}) = \frac{2(2)^3}{3}(4^3 - 4 - 2^3 + 2)$

$= \frac{(2)^5}{3}(2(4^2) - 2 - 2^2 + 1) = \frac{32}{3}(32 - 2 - 4 + 1) = \frac{32}{3}(27) = (32)(9) = 288$

Note since functions are continuous:

$$\int_0^2 \int_4^8 (x^2 - y^2) dxdy = \int_4^8 \int_0^2 (x^2 - y^2) dydx$$