12.4 Partial derivatives

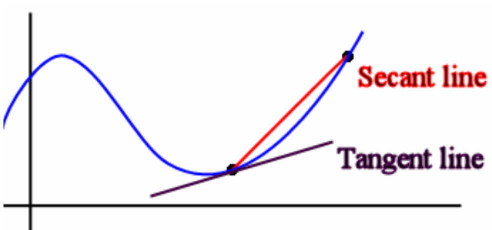
Recall: $\frac{dy}{dt}$ = slope of tangent line = instantaneous rate of change.

If $y = f(t)$,

$$\frac{dy}{dt} = \lim_{\Delta t \to 0} \frac{\Delta y}{\Delta t} = \lim_{h \to 0} \frac{f(t+h) - f(t)}{(t+h) - t} = \lim_{h \to 0} \frac{f(t+h) - f(t)}{h}$$

Slope of tangent line = limit of slope of secant lines.

Instantaneous rate of change = limit of average rate of change.



https://clas.sa.ucsb.edu/staff/lee/secant,%20tangent,%20and%20derivatives.htm

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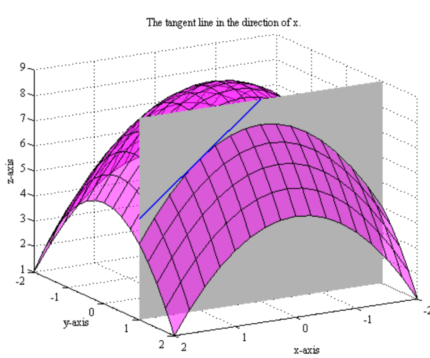
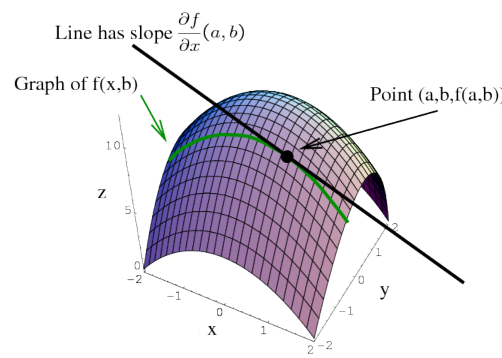
Partial derivatives: If $z = f(x, y)$, then

$$\frac{\partial f}{\partial x} = \frac{\partial f}{\partial x} = f_x(x,y) = D_x[f(x,y)] = D_1[f(x,y)] = \lim_{h \to 0} \frac{f(x+h,y) - f(x,y)}{h}$$

$$\frac{\partial f}{\partial y} = \frac{\partial f}{\partial y} = f_y(x,y) = D_y[f(x,y)] = D_2[f(x,y)] = \lim_{h \to 0} \frac{f(x,y+k) - f(x,y)}{k}$$





https://www.wikihow.com/images/4/4d/OyXsh.png and https://mathinsight.org/partial_derivative_limit_definition

Thus if $g(x) = f(x, b)$ for some fixed b, then $g'(a) = \frac{\partial f}{\partial x}(a, b)$

and if $h(y) = f(a, y)$ for some fixed a, then $h'(b) = \frac{\partial f}{\partial y}(a, b)$