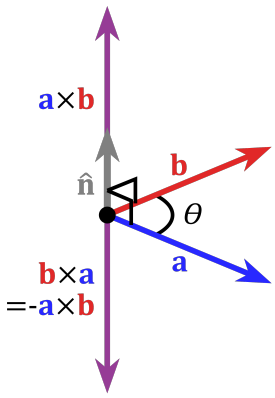
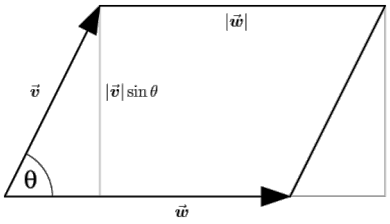
$$\vec{a} \times \vec{b} = \begin{vmatrix} \vec{i} & \vec{j} & \vec{k} \\ a_1 & a_2 & a_3 \\ b_1 & b_2 & b_3 \end{vmatrix}$$

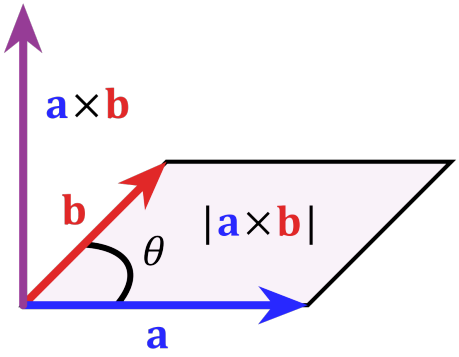
$$\vec{a} \times \vec{b} = \begin{vmatrix} a_2 & a_3 \\ b_2 & b_3 \end{vmatrix} \vec{i} - \begin{vmatrix} a_1 & a_3 \\ b_1 & b_3 \end{vmatrix} \vec{j} + \begin{vmatrix} a_1 & a_2 \\ b_1 & b_2 \end{vmatrix} \vec{k}$$

$$\vec{a} \times \vec{b} = \begin{vmatrix} \vec{i} & \vec{j} & \vec{k} \\ a_1 & a_2 & a_3 \\ b_1 & b_2 & b_3 \end{vmatrix} \begin{matrix} \vec{i} & \vec{j} \\ a_1 & a_2 \\ b_1 & b_2 \end{matrix}$$

$$\vec{a} \times \vec{b} = \begin{matrix} +\mathbf{i}a_2b_3 \\ +a_1b_2\mathbf{k} \\ +b_1\mathbf{j}a_3 \\ -b_1a_2\mathbf{k} \\ -\mathbf{i}b_2a_3 \\ -a_1\mathbf{j}b_3 \end{matrix} \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ a_1 & a_2 & a_3 \\ b_1 & b_2 & b_3 \end{vmatrix} \begin{matrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ a_1 & a_2 & a_3 \end{matrix}$$




Direction: Perpendicular to $\vec{a}$ and $\vec{b}$.

Length: $|\vec{a} \times \vec{b}| = |\vec{a}||\vec{b}|sin\theta$ = area of parallelogram with sides $\vec{a}$ & $\vec{b}$.

Color figures from: https://en.wikipedia.org/wiki/Cross_product,

BW figure from https://www.maa.org/sites/default/files/images/upload_library/4/vol6/Dray2/cross.gif

The volume of the parallelepiped is

$$\text{Volume} = |\mathbf{a} \times \mathbf{b}|\ |\mathbf{c}|\ |\cos\phi| = |(\mathbf{a} \times \mathbf{b}) \cdot \mathbf{c}|.$$

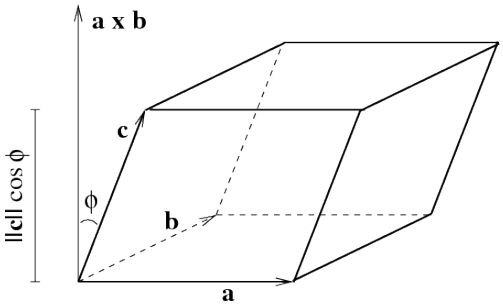



Figure: Nykamp DQ, The scalar triple product. From Math Insight. http://mathinsight.org/scalar_triple_product

Scalar triple product:

$$(\vec{a} \times \vec{b}) \cdot \vec{c} = \left( \begin{vmatrix} a_2 & a_3 \\ b_2 & b_3 \end{vmatrix} \vec{i} - \begin{vmatrix} a_1 & a_3 \\ b_1 & b_3 \end{vmatrix} \vec{j} + \begin{vmatrix} a_1 & a_2 \\ b_1 & b_2 \end{vmatrix} \vec{k} \right) \cdot (c_1\vec{i} + c_2\vec{j} + c_3\vec{k})$$

$$= c_1 \begin{vmatrix} a_2 & a_3 \\ b_2 & b_3 \end{vmatrix} - c_2 \begin{vmatrix} a_1 & a_3 \\ b_1 & b_3 \end{vmatrix} + c_3 \begin{vmatrix} a_1 & a_2 \\ b_1 & b_2 \end{vmatrix}$$