

Linear algebra pre-requisites you must know.

$\mathbf{b}_1, \dots, \mathbf{b}_n$  are linearly independent if

$$c_1 \mathbf{b}_1 + c_2 \mathbf{b}_2 + \dots + c_n \mathbf{b}_n = d_1 \mathbf{b}_1 + d_2 \mathbf{b}_2 + \dots + d_n \mathbf{b}_n$$

implies  $c_1 = d_1, c_2 = d_2, \dots, c_n = d_n$ .

or equivalently,

$\mathbf{b}_1, \dots, \mathbf{b}_n$  are linearly independent if

$$c_1 \mathbf{b}_1 + c_2 \mathbf{b}_2 + \dots + c_n \mathbf{b}_n = 0 \text{ implies } c_1 = c_2 = \dots = c_n = 0.$$

Example 1:  $\mathbf{b}_1 = (1, 0, 0), \mathbf{b}_2 = (0, 1, 0), \mathbf{b}_3 = (0, 0, 1)$ . ■

$$(1, 2, 3) \neq (1, 2, 4).$$

If  $(a, b, c) = (1, 2, 3)$  then  $a = 1, b = 2, c = 3$ .

Example 2:  $\mathbf{b}_1 = 1, \mathbf{b}_2 = t, \mathbf{b}_3 = t^2$ .

$$1 + 2t + 3t^2 \neq 1 + 2t + 4t^2.$$

If  $a + bt + ct^2 = 1 + 2t + 3t^2$  then  $a = 1, b = 2, c = 3$ .

## Application: Partial Fractions

$$\begin{aligned}\frac{4}{(x^2+1)(x-3)} &= \frac{Ax+B}{x^2+1} + \frac{C}{x-3} \\ &= \frac{(Ax+B)(x-3)+C(x^2+1)}{(x^2+1)(x-3)}\end{aligned}$$

$$\text{Hence } \frac{4}{(x^2+1)(x-3)} = \frac{(Ax+B)(x-3)+C(x^2+1)}{(x^2+1)(x-3)}$$

$$\text{Thus } 4 = (Ax + B)(x - 3) + C(x^2 + 1)$$

$$4 = Ax^2 + Bx - 3Ax - 3B + Cx^2 + C$$

$$4 = (A + C)x^2 + (B - 3A)x - 3B + C$$

$$\text{I.e., } 0x^2 + 0x + 4 = (A + C)x^2 + (B - 3A)x - 3B + C$$

$$\text{Thus } 0 = A + C, 0 = B - 3A, 4 = -3B + C.$$

$$C = -A, B = 3A,$$

$$4 = -3(3A) + -A \text{ implies } 4 = -10A.$$

$$\text{Hence } A = -\frac{2}{5}, B = 3\left(-\frac{2}{5}\right) = -\frac{6}{5}, C = \frac{2}{5}.$$

$$\begin{aligned}\text{Thus, } \frac{4}{(x^2+1)(x-3)} &= \frac{-\frac{2}{5}x - \frac{6}{5}}{x^2+1} + \frac{\frac{2}{5}}{x-3} \\ &= \frac{-2x-6}{5(x^2+1)} + \frac{2}{5(x-3)}\end{aligned}$$