Note the following review problems DO NOT cover all problem types which may appear on the final.

6.3 preliminaries:

1a.) Suppose $f(t) = t^2$, then $f(t-2) = \underline{(t-2)^2}$

1b.) Suppose $f(t) = t^2 + 3t + 4$, then $f(t-2) = \underline{(t-2)^2 + 3(t-2) + 4}$

1c.) Suppose $f(t) = sin(t) + e^{8t}$, then $f(t-2) = \underline{sin(t-2) + e^{8(t-2)}}$

2a.) Suppose $f(t-2) = (t-2)^2$, then $f(t) = \underline{t^2}$

2b.) Suppose $f(t-2) = (t-2)^2 + 3(t-2) + 4$, then $f(t) = \underline{t^2 + 3t + 4}$

2c.) Suppose $f(t-2) = sin(t-2) + e^{8(t-2)}$, then $f(t) = \underline{sin(t) + e^{8t}}$

3a.) Suppose $f(t-2) = t^2 + 2t + 5$, then $f(t) = \underline{t^2 + 6t + 13}$

$t^2 + 2t + 5 = (t-2)^2 + 4t - 4 + 2t + 5 = (t-2)^2 + 6t + 1 = (t-2)^2 + 6(t-2) + 12 + 1$
$= (t-2)^2 + 6(t-2) + 13$

Check: $f(t-2) = (t-2)^2 + 6(t-2) + 13 = t^2 - 4t + 4 + 6t - 12 + 13 = t^2 + 2t + 5$

3b.) Suppose $f(t-2) = 3t^2 + 8t + 1$, then $f(t) = \underline{3t^2 + 20t + 29}$

$3t^2 + 8t + 1 = 3(t-2)^2 - 3(-4t + 4) + 8t + 1 = 3(t-2)^2 + 12t - 12 + 8t + 1$
$= 3(t-2)^2 + 20t - 11 =$
$3(t-2)^2 + 20(t-2) + 40 - 11 = 3(t-2)^2 + 20(t-2) + 29$

Check: $f(t-2) = 3(t-2)^2 + 20(t-2) + 29 = 3(t^2 - 4t + 4) + 20t - 40 + 29 = 3t^2 - 12t + 12 + 20t - 11 =$
$3t^2 + 8t + 1$

3c.) Suppose $f(t-2) = cos(t) + e^{8t}$, then $f(t) = \underline{cos(t+2) + e^{8t+16}}$

$cos(t) + 4^{8t} = cos(t - 2 + 2) + e^{8(t-2)+16}$

Check: $f(t-2) = cos(t-2+2) + e^{8(t-2)+16} = cos(t) + e^{8t-16+16} = cos(t) + e^{8t}$

Chapter 6:

4.) Find the LaPlace transform of the following: (used $\mathcal{L}(u_c(t)f(t-c)) = e^{-cs}\mathcal{L}(f(t))$)

4a.) $\mathcal{L}(u_3(t^2 - 2t + 1)) = \underline{e^{-3s}(\frac{2}{s^3} + 4\frac{1}{s^2} + \frac{4}{s})}$

$\mathcal{L}(u_3(t^2 - 2t + 1)) = \mathcal{L}(u_3((t-3)^2 + 6t - 9 - 2t + 1)) = \mathcal{L}(u_3((t-3)^2 + 4t - 8)) = \mathcal{L}(u_3((t-3)^2 + 4(t-3) + 12 - 8)) = \mathcal{L}(u_3((t-3)^2 + 4(t-3) + 4)) = e^{-3s}\mathcal{L}(t^2 + 4t + 4)) = e^{-3s}(\frac{2}{s^3} + 4\frac{1}{s^2} + \frac{4}{s})$

4b.) $\mathcal{L}(u_4(e^{-8t})) = \underline{e^{-4s-32}\frac{1}{s+8}}$

$\mathcal{L}(u_4 e^{-8t}) = \mathcal{L}(u_4 e^{-8(t-4)-32}) = e^{-4s}\mathcal{L}(e^{-8t-32}) = e^{-4s}e^{-32}\mathcal{L}(e^{-8t}) = e^{-4s-32}\frac{1}{s+8}$

4c.) $\mathcal{L}(u_2(t^2e^{3t})) = \underline{e^{-2s+6}(\frac{2}{(s-3)^3} + \frac{4}{(s-3)^2} + \frac{4}{(s-3)})}$

$\mathcal{L}(u_2(t^2e^{3t})) = \mathcal{L}(u_2([(t-2)^2+4t-4]e^{3(t-2)+6})) = \mathcal{L}(u_2([(t-2)^2+4(t-2)+8-4]e^{3(t-2)+6})) = \mathcal{L}(u_2([(t-2)^2+4(t-2)+4]e^{3(t-2)+6})) = e^{-2s}\mathcal{L}([t^2+4t+4]e^{3t+6})) = e^{-2s}e^6\mathcal{L}([t^2+4t+4]e^{3t})) = e^{-2s}e^6\mathcal{L}(t^2e^{3t} + 4te^{3t} + 4e^{3t})) = e^{-2s+6}(\frac{2}{(s-3)^3} + 4\frac{1}{(s-3)^2} + \frac{4}{(s-3)})$

5.) Find the inverse LaPlace transform of the following: (usually used $u_c(t)f(t-c) = \mathcal{L}^{-1}(e^{-cs}\mathcal{L}(f(t)))$

5a.) $\mathcal{L}^{-1}(e^{-8s}\frac{1}{s-3}) = \underline{u_8(t)e^{3(t-8)}}$

$\mathcal{L}^{-1}(e^{-8s}\frac{1}{s-3}) = u_8 f(t-8)$ where

$\mathcal{L}(f(t)) = \frac{1}{s-3})$. Hence $f(t) = \mathcal{L}^{-1}(\frac{1}{s-3}) = e^{3t}$

5b.) $\mathcal{L}^{-1}(e^{4s}\frac{1}{s^2-3}) = \underline{u_{-4}(t)sinh(\sqrt{3}(t+4))}$

$\mathcal{L}^{-1}(e^{4s}\frac{1}{s^2-3}) = u_{-4}(t)f(t+4)$ where

$\mathcal{L}(f(t)) = \frac{1}{s^2-3})$. Hence $f(t) = \frac{1}{\sqrt{3}}\mathcal{L}^{-1}(\frac{\sqrt(3)}{s^2-3}) = sinh(\sqrt{3}t)$

5c.) $\mathcal{L}^{-1}(e^{s}\frac{1}{(s-3)^2+4}) = \underline{\frac{1}{2}u_{-1}(t)e^{3(t+1)}sin(2(t+1))}$

$\mathcal{L}^{-1}(e^{s}\frac{1}{(s-3)^2+4}) = u_{-1}(t)f(t+1)$ where

$\mathcal{L}(f(t)) = \frac{1}{(s-3)^2+4})$. Hence $f(t) = \frac{1}{2}\mathcal{L}^{-1}(\frac{2}{(s-3)^2+4}) = \frac{1}{2}e^{3t}sin(2t)$

5d.) $\mathcal{L}^{-1}(e^{-s}\frac{5}{(s-3)^4}) = \underline{u_1(t)\frac{1}{6}(t-1)^3e^{3(t-1)}}$

$\mathcal{L}^{-1}(e^{-s}\frac{5}{(s-3)^4}) = u_1(t)f(t-1)$ where

$\mathcal{L}(f(t)) = \frac{1}{(s-3)^4})$. Hence $f(t) = \frac{1}{6}\mathcal{L}^{-1}(\frac{6}{(s-3)^4}) = \frac{1}{6}t^3e^{3t}$

5e.) $\underline{\frac{\mathcal{L}^{-1}(e^s}{4s)=\frac{1}{4}u_{-1}(t)}}$

$\frac{\mathcal{L}^{-1}(e^s}{4s)} \frac{=\frac{1}{4}\mathcal{L}^{-1}(e^s}{s)} = \frac{1}{4}u_{-1}(t)f(t+1)$ where

$\mathcal{L}(f(t)) = \frac{1}{s}$. Hence $f(t) = 1$. Thus $f(t+1) = 1$

5f.) $\mathcal{L}^{-1}(e^s) = \underline{\delta(t+1)}$

6.) Use the definition and not the table to find the LaPlace transform of the following:

6a.) $\mathcal{L}(t^2) = \underline{\frac{2}{s^3}}$

$\int_0^\infty e^{-st}t^2dt = t^2\frac{e^{-st}}{-s}|_0^\infty - \int_0^\infty 2t\frac{e^{-st}}{-s} = lim_{t\to\infty}t^2\frac{e^{-st}}{-s} - 0^2\frac{e^0}{-s} - [2t\frac{e^{-st}}{s^2}|_0^\infty - \int_0^\infty 2\frac{e^{-st}}{s^2}]$
$= 0 - 0 - [lim_{t\to\infty}2t\frac{e^{-st}}{s^2} - 2(0)\frac{e^0}{s^2} - 2\frac{e^{-st}}{-s^3}|_0^\infty] = -[0 - 0 - (lim_{t\to\infty}2\frac{e^{-st}}{-s^3} - 2\frac{e^0}{-s^3})] = (0 - \frac{2}{-s^3})]$
$= \frac{2}{s^3}$

Let $u = t^2,\ dv = e^{-st}$

$du = 2t,\ v = \frac{e^{-st}}{-s}$

$d^2u = 2,\ \int v = \frac{e^{-st}}{s^2}$

8.) Find $f * g$

8a.) $4t * 5t^4 = \underline{\frac{2}{3}t^6}$

$\int_0^t 4(t-s)5s^4ds = \int_0^t 20(ts^4 - s^5)ds = 4ts^5 - \frac{20}{6}s^6|_0^t = 4t^6 - \frac{10}{3}t^6 = \frac{2}{3}t^6$

8b.) $5t^4 * 4t = \underline{\frac{2}{3}t^6}$

$5t^4 * 4t = 4t * 5t^4 = \frac{2}{3}t^6$

Chapter 3:

9.) Solve the following initial problems:

9a.) $y'' + 6y' + 8y = 0,\ y(0) = 0,\ y'(0) = 0$

Suppose $y = e^{rt}$. Then $y' = re^{rt},\ y'' = r^2e^{rt}$

$r^2e^{rt} + 6re^{rt} + 8e^{rt} = 0$ Hence $r^2 + 6r + 8 = 0$. Thus, (r + 2)(r + 4) $= 0. Hence$ r = -2, -4

Hence general solution is $y = c_1e^{-2t} + c_2e^{-4t}$

$y(0) = 0 : 0 = c_1 + c_2$. Thus $c_2 = -c_1$

$y'(0) = 0 : y' = -2c_1e^{-2t} - 4c_2e^{-4t}$

$0 = -2c_1 - 4c_2 = 2c_2 - 4c_2 = -2c_2$ Thus $c_2 = 0, c_1 = 0$

Thus, $y = 0$

Make sure you also study exam 1 and 2 as well as everything else. Remember the above list is INCOMPLETE.