$\times$ = wrong

$\checkmark$ = right

Math 34 Differential Equations Exam #1
October 9, 2003

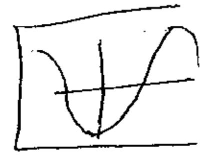
SHOW ALL WORK

Solve the following differential equations

[15] 1a.) $t^2y' + 2ty = t\ sin(t)$.

$$\int (t^2 y)' = \int t \sin(t)$$

Let $u = t$ $dv = \sin t$
$du = dt$ $v = -\cos t$



$$t^2 y = -t\cos(t) + \int \cos(t)\,dt$$

$$t^2 y = -t\cos(t) + \sin(t) + C$$

$$y = -t^{-1}\cos(t) + t^{-2}\sin(t) + ct^{-2}$$

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or $t^2y' + 2ty = t\sin(t)$

$$y' + \frac{2}{t}y = \frac{\sin(t)}{t}$$

$$u = e^{\int \frac{2}{t}dt} = e^{2\ln|t|} = e^{\ln|t|^2} = t^2$$

$$t^2\left[y' + \frac{2}{t}y\right] = \left[\frac{\sin(t)}{t}\right]t^2$$

$$t^2y' + 2ty = t\sin(t)$$

Answer 1a.) 1 $y = -t^{-1}\cos(t) + t^{-2}\sin(t) + c$ 2 t^{-2}

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[15] 1b.) $y'' - 4y' + 4y = 0$, $y(0) = 2$, $y'(0) = 3$.

$$r^2 - 4r + 4 = 0$$

$$(r-2)^2 = 0 \Rightarrow r = 2$$

$$y = c_1 e^{2t} + c_2 te^{2t}$$

$$y' = 2c_1 e^{2t} + c_2(e^{2t} + 2te^{2t})$$

$$y(0) = 2: \quad 2 = c_1 + 0 \Rightarrow c_1 = 2$$

$$y'(0) = 3: \quad 3 = 2c_1 + c_2(1+0)$$

$$3 = 2(2) + c_2 \Rightarrow c_2 = -1$$

Answer 1b.) $y = 2e^{2t} - te^{2t}$

[15] 1c.) $y'' - 4y' + 4y = 5t + 1$.

Guess $\psi = At + B$

$$\psi' = A$$

$$\psi'' = 0$$

$$0 - 4A + 4(At + B) = 5t + 1$$

$$\underbrace{4At} + 4B - 4A = \underbrace{5t} + 1$$

$$4A = 5 \Rightarrow A = 5/4$$

$$4B - 4A = 1 \Rightarrow 4B = 6 \Rightarrow B = 6/4 = 3/2$$

$$4B - 5 = 1$$

Answer 1c.) $c_1 e^{2t} + c_2 te^{2t} + \frac{5}{4}t + \frac{3}{2}$

[15] 1d.) $2y'' - 3y^2 = 0$, $y(0) = 1$, $y'(0) = 1$..

Let $v = y' = dy/dt$

$\frac{dv}{dt} = v' = y''$

$2\frac{dv}{dt} - 3y^2 = 0$

$2v\frac{dv}{dy} - 3y^2 = 0$

$2v\frac{dv}{dy} = 3y^2$

$\int 2v\,dv = \int 3y^2\,dy$

$v^2 = y^3 + C$

$(y')^2 = y^3 + C$

$\frac{dv}{dt} = \frac{dv}{dy}\cdot\frac{dy}{dt} = \frac{dv}{dy}v$

$y(0) = 1 \quad y'(0) = 1$

$(1)^2 = (1)^3 + C$

$\Rightarrow C = 0$

$(y')^2 = y^3$

$\frac{dy}{dt} = y^{3/2}$

$\int y^{-3/2}dy = \int dt \Rightarrow -2y^{-1/2} = t + C$

$t = 0, y = 1: \quad -2(1)^{-1/2} = 0 + C \Rightarrow C = -2$

$-2y^{-1/2} = t - 2 \Rightarrow y^{-1/2} = \frac{t-2}{-2} \Rightarrow (y^{1/2})^2 = \left(\frac{-2}{t-2}\right)^2$

Answer 1d.) $y = \frac{4}{(t-2)^2}$

[5] 2.) Use Euler's formula to write e^{2+3i} is the form of $a + ib$.

$e^{2+3i} = e^2 e^{3i} = e^2(\cos 3 + i\sin 3)$

$= e^2\cos 3 + ie^2\sin 3$

Answer 2.) $e^2\cos(3) + ie^2\sin(3)$

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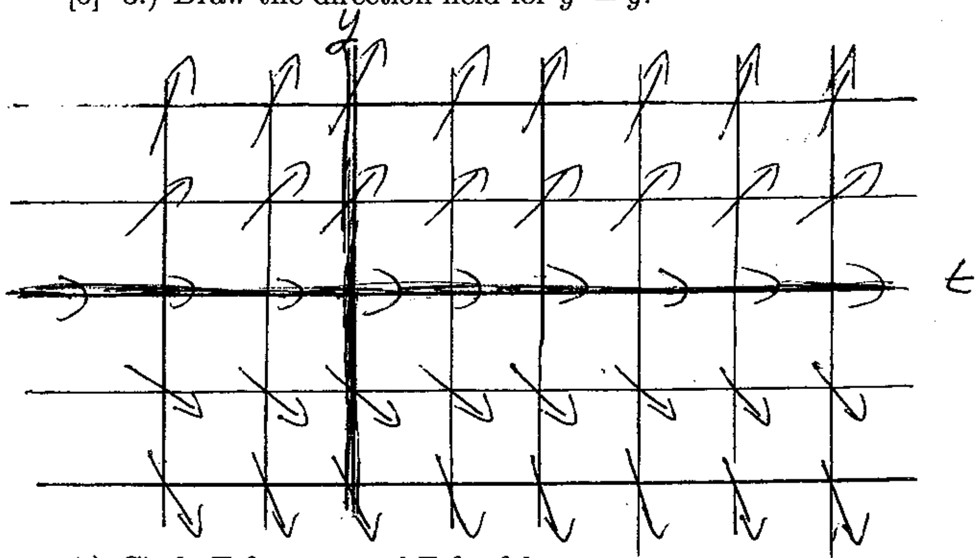
[5] 3.) Draw the direction field for $y' = y$.



4.) Circle T for true and F for false.

[3] 4a.) Suppose ψ_1 and ψ_2 are solutions to the linear equation, $ay'' + by' + cy = g(t)$, then $\psi_1 + \psi_2$ must also be a solution to $ay'' + by' + cy = g(t)$.

T F

[3] 4b.) Suppose ψ_1 and ψ_2 are solutions to the equation, $ay'' + by' + cy^2 = 0$, then $\psi_1 + \psi_2$ must also be a solution to $ay'' + by' + cy^2 = 0$.

T F

[3] 4c.) Suppose ψ_1 is a solution to the linear equation, $ay'' + by' + cy = g(t)$, and ψ_2 is a solution to the linear equation, $ay'' + by' + cy = f(t)$, then $5\psi_1 + 3\psi_2$ must also be a solution to $ay'' + by' + cy = 5g(t) + 3f(t)$.

T F

[3] 4d.) If p, q, and g are continuous, then there exists a unique solution to

$$y'' + p(t)y' + q(t)y = g(t),\ y(t_0) = y_0,\ y(t_1) = y_1.$$

T F

[3] 4e.) If p, q, and g are continuous, then there exists a unique solution to

$$y'' + p(t)y' + q(t)y = g(t),\ y(t_0) = y_0,\ y'(t_0) = y_0'.$$

T F

[3] 4f.) Given an initial value, there always exists a unique solution to any first order differential equation.

T F

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5.) Choose one of the two following problems. Clearly indicate which problem you have chosen.

5A.) Suppose the equation $\frac{dp}{dt} = \gamma p$ describes the population of field mice. If the population of field mice doubles in 10 years, how long will it take the population to quadruple.

5B.) Find the escape velocity for a body projected upward with an initial velocity v_0 from a point 3R above the surface of the earth, where R is the radius of the earth. Neglect air resistance. Recall that the equation of motion is $m\frac{dv}{dt} = -\frac{mgR^2}{(R+x)^2}$ where x is the distance from the earth's surface.

5A.) $\frac{dp}{dt} = \gamma p \Rightarrow \frac{dp}{p} = \gamma dt \Rightarrow e^{\ln|p|} = e^{(\gamma t + C)}$

$|p| = e^{c} e^{\gamma t} \Rightarrow |p| = c_1 e^{\gamma t} \Rightarrow p = c_2 e^{\gamma t}$ 6

$t = 0,\ p = p_0 : \quad p_0 = c_2 e^{0} \Rightarrow c_2 = p_0$

$t = 10,\ p = 2p_0 : \quad 2p_0 = p_0 e^{10\gamma} \Rightarrow 2 = e^{10\gamma} \Rightarrow \ln 2 = 10\gamma \Rightarrow \gamma = \ln 2/10$

$t = ?,\ p = 4p_0 : \quad 4p_0 = p_0 e^{\gamma t}$

$4\cancel{p_0} = \cancel{p_0} e^{\left[\frac{\ln 2}{10}\right]t}$

$4 = e^{\frac{t}{10}\ln 2} = e^{\ln 2^{t/10}} = 2^{t/10}$

$\ln 4 = \frac{t}{10}\ln 2$

$\ln 2^2 = \frac{t}{10}\ln 2$

$2\ln 2 = \frac{t}{10}\ln 2 \Rightarrow 2 = \frac{t}{10} \Rightarrow t = 20$

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Answer 5A or 5B.) 20 yrs

for 5B see Example 4 in sect 2.3 (p 58, 59)

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