6.5: Impulse functions

Unit impulse function = Dirac delta function is a generalized function with the properties

$$\delta(t) = 0, \qquad t \neq 0$$

$$\int_{-\infty}^{\infty} \delta(t)dt = 1$$

Let $d_c(t) = \begin{cases} \frac{1}{2c} & -c < t < c \\ 0 & t \leq -c \text{ or } t \geq c \end{cases}$

Note $lim_{c\to 0} d_c(t) = 0$ if $t \neq 0$

and $lim_{c\to 0} \int_{-\infty}^{\infty} d_c(t) = lim_{c\to 0} 1 = 1 = \int_{-\infty}^{\infty} \delta(t)dt$

$sin(t) = \frac{e^{it}-e^{-it}}{2i}$ $\qquad$ $cos(t) = \frac{e^{it}+e^{-it}}{2}$

$sinh(t) = \frac{e^{t}-e^{-t}}{2}$ $\qquad$ $cosh(t) = \frac{e^{t}+e^{-t}}{2}$

$[sinh(t)]' =$ $\qquad$ $[cosh(t)]' =$

$sinh(0) = \frac{e^{0}-e^{0}}{2} = 0$ $\qquad$ $cosh(0) = \frac{e^{0}+e^{0}}{2} = 1$

Define the * product on R^2 by

$(x_1, y_1) * (x_2, y_2) = (x_1x_2 - y_1y_2, x_1y_2 - y_1x_2)$

Note $*$ is

1.) commutative:
$(x_1, y_1) * (x_2, y_2) = (x_1x_2 - y_1y_2, x_1y_2 - y_1x_2) =$
$(x_2x_1 - y_2y_1, x_2y_1 - y_2x_1) = (x_2, y_2) * (x_1, y_1)$

2.) associative: $(f * g) * h = f * (g * h)$

3.) distributive w.r.t +: $f * (g_1 + g_2) = f * g_1 + f * g_2$

4.) $(x_1, y_1) * (0, 0) = (0, 0)$

Note $(0, 1) * (0, 1) = (-1, 0)$

6.6: The Convolution Integral

Defn: The convolution of f and g is the function $f * g$ defined by

$(f * g)(t) = \int_0^t f(t - s)g(s)ds = \int_0^t f(s)g(t - s)ds$

Note $*$ is

1.) commutative: $f * g = g * f$

2.) associative: $(f * g) * h = f * (g * h)$

3.) distributive w.r.t +: $f * (g_1 + g_2) = f * g_1 + f * g_2$

4.) $f * 0 = 0 * f = 0$