$$g(t) = \begin{cases} 0 & y < 4 \\ 2 & 4 \leq t < 10 \\ t & t \geq 10 \end{cases}$$

Hence $g(t) = 2u_4(t) + (t-2)u_{10}(t)$

$$\text{Solve } 3y'' + y' + y = 2u_4(t) + (t-2)u_{10}(t),$$
$$y(0) = 0,\ y'(0) = 0.$$

$$3\mathcal{L}(y'') + \mathcal{L}(y') + \mathcal{L}(y) = \mathcal{L}(2u_4(t)) - \mathcal{L}((t-2)u_{10}(t))$$

$$3[s^2\mathcal{L}(y) - sy(0) - y'(0)] + s\mathcal{L}(y) - y(0) + \mathcal{L}(y)$$
$$= \mathcal{L}(2u_4(t)) - \mathcal{L}((t-10+8)u_{10}(t))$$

$$\text{Thm: } \mathcal{L}(u_c(t)f(t-c)) = e^{-cs}\mathcal{L}(f(t)).$$

If $f(t) =$ ____________, then $f(t -$ ___$) =$ ____________

If $f(t) =$ ____________, then $f(t -$ ___$) =$ ____________

$$3[s^2\mathcal{L}(y)] + s\mathcal{L}(y) + \mathcal{L}(y)$$
$$= e^{-4s}\mathcal{L}(2) - e^{-10s}\mathcal{L}((t+8))$$

$$\mathcal{L}(y)[3s^2+s+1] = 2e^{-4s}\mathcal{L}(1) - e^{-10s}\mathcal{L}(t) - 8e^{-10s}\mathcal{L}(1)$$

$$\mathcal{L}(y)[3s^2+s+1] = e^{-4s}\frac{2}{s} - e^{-10s}\frac{1}{s^2} - e^{-10s}\frac{8}{s}$$

$$\mathcal{L}(y) = e^{-4s}\frac{2}{s[3s^2+s+1]} - e^{-10s}[\frac{1}{s^2[3s^2+s+1]} - e^{-10s}\frac{8}{s[3s^2+s+1]}$$

$$\mathcal{L}(y) = e^{-4s}\frac{2}{s[3s^2+s+1]} - e^{-10s}\frac{1}{s^2[3s^2+s+1]} - e^{-10s}\frac{8}{s[3s^3+s+1]}$$

$$y = 2\mathcal{L}^{-1}(e^{-4s}\frac{1}{s[3s^2+s+1]}) - \mathcal{L}^{-1}(e^{-10s}\frac{1}{s^2[3s^2+s+1]}) \\ -\mathcal{L}^{-1}(e^{-10s}\frac{8}{s[3s^2+s+1]})$$

$y = u_4(t)f(t-4) - u_{10}f(t-10) - u_{10}h(t-10)$ where

$f(t) = \mathcal{L}^{-1}(\frac{1}{s[3s^2+s+1]})$ and $h(t) = \mathcal{L}^{-1}(\frac{8}{s^2[3s^2+s+1]})$

$$\frac{1}{s[3s^2+s+1]} = \frac{A}{s} + \frac{Bs+C}{3s^2+s+2}$$

$$1 = A(3s^2 + s + 1) + (Bs + C)s$$

$$0s^2 + 0s + 1 = (3A + B)s^2 + (A + C)s + A$$

$$0 = 3A + B,\ 0 = A + C,\ 1 = A$$

Hence $A = 1$, $B = -3A = -3$, $C = -A = -1$

$\frac{8}{s^2[3s^2+s+1]} = \frac{As+D}{s^2} + \frac{Bs+C}{3s^2+s+2}$

$8 = (As + D)(3s^2 + s + 1) + (Bs + C)s^2$

$0s^3 = 0s^2 + 0s + 8 = (3A + B)s^3 + (A + 3D + C)s^2 + (A + D)s + D$

$0 = 3A + B,\ 0 = A + 3D + C,\ 0 = A + D,\ 8 = D.$

Hence $D = 8,\ A = -D = -8,\ C = -A - 3D = 8 - 24 = -16,\ B = -3A = 24.$

$f(t) = \mathcal{L}^{-1}(\frac{1}{s[3s^2+s+1]})$

$= \mathcal{L}^{-1}(\frac{1}{s} + \frac{-3s-1}{3s^2+s+1})$

$= \mathcal{L}^{-1}(\frac{1}{s} + \frac{-3s-1}{3s^2+s+1})$

$= 1 + \mathcal{L}^{-1}(\frac{-3s-1}{3[s^2+\frac{1}{3}s+\frac{1}{3}]}))$

$= 1 + \mathcal{L}^{-1}(\frac{-3s-1}{3[(s^2+\frac{1}{3}s+\underline{\qquad})-\underline{\qquad}+\frac{1}{3}]})$

$= 1 + \mathcal{L}^{-1}(\frac{-3s-1}{3[(s+\frac{1}{6})^2-\frac{1}{36}+\frac{1}{3}]})$

$= 1 + \mathcal{L}^{-1}(\frac{-3(s+\frac{1}{3})}{3[(s+\frac{1}{6})^2+\frac{11}{36}]})$

$$= 1 + \mathcal{L}^{-1}(\frac{-(s+\frac{1}{6}-\frac{1}{6}+\frac{1}{3})}{[(s+\frac{1}{6})^2+\frac{11}{36}]})$$

$$= 1 + \mathcal{L}^{-1}(\frac{-(s+\frac{1}{6}+\frac{1}{6})}{[(s+\frac{1}{6})^2+\frac{11}{36}]})$$

$$= 1 + \mathcal{L}^{-1}(\frac{-(s+\frac{1}{6})}{[(s+\frac{1}{6})^2+\frac{11}{36}]} + \frac{-\frac{1}{6}}{[(s+\frac{1}{6})^2+\frac{11}{36}]})$$

$$= 1 + \mathcal{L}^{-1}(\frac{-(s+\frac{1}{6})}{[(s+\frac{1}{6})^2+\frac{11}{36}]} + \frac{-\frac{1}{6}\frac{6}{\sqrt{11}}\frac{\sqrt{11}}{6}}{[(s+\frac{1}{6})^2+\frac{11}{36}]})$$

$$= 1 + \mathcal{L}^{-1}(\frac{-(s+\frac{1}{6})}{[(s+\frac{1}{6})^2+\frac{11}{36}]} + \frac{-\frac{1}{\sqrt{11}}\frac{\sqrt{11}}{6}}{[(s+\frac{1}{6})^2+\frac{11}{36}]})$$

$$= 1 - e^{-\frac{1}{6}} cos\frac{\sqrt{11}}{6}t - \frac{1}{\sqrt{11}}e^{-\frac{1}{6}} sin\frac{\sqrt{11}}{6}t$$

$$h(t) = \mathcal{L}^{-1}(\frac{8}{s^2[3s^2+s+1]})$$

$$\frac{8}{s^2[3s^2+s+1]} = \frac{-8s+8}{s^2} + \frac{24s-16}{3s^2+s+1}$$

$$= \frac{-8s}{s^2} + \frac{8}{s^2} + \frac{24(s-\frac{2}{3})}{3[(s+\frac{1}{6})^2+\frac{11}{36}]}$$

$$= \frac{-8}{s} + \frac{8}{s^2} + \frac{24(s+\frac{1}{6}-\frac{1}{6}-\frac{2}{3})}{3[(s+\frac{1}{6})^2+\frac{11}{36}]}$$

$$= \frac{-8}{s} + \frac{8}{s^2} + \frac{24(s+\frac{1}{6}-\frac{5}{6})}{3[(s+\frac{1}{6})^2+\frac{11}{36}]}$$

$$= \frac{-8}{s} + \frac{8}{s^2} + \frac{24(s+\frac{1}{6}-\frac{5}{6}\frac{6}{\sqrt{11}}\frac{\sqrt{11}}{6})}{3[(s+\frac{1}{6})^2+\frac{11}{36}]}$$