3.7 Variation of Parameters

$$\text{Solve } y'' - 2y' + y = e^t ln(t)$$

1) Find homogeneous solutions: Solve $y'' - 2y' + y = 0$

Guess: $y = e^{rt}$, then $y' = re^{rt}$, $y'' = r^2e^{rt}$, and

$r^2e^{rt} - 2re^{rt} + e^{rt} = 0$

$$r^2 - 2r + 1 = 0$$

$(r-1)^2 = 0$, and hence $r = 1$

General homogeneous solution: $y = c_1e^t + c_2te^t$

since have two linearly independent solutions: $\{e^t, te^t\}$

2.) Find a non-homogeneous solution:

Sect. 3.5 method: Educated guess

Sect.3.6: **Guess** $y = u_1(t)e^t + u_2te^t$ **and solve for** u_1 **and** u_2

$y' = u_1'(t)e^t + u_1(t)e^t + u_2'te^t + u_2(e^t + te^t)$.

Two unknown functions, u_1 and u_2, but only one equation ($y'' - 2y' + y = e^t ln(t)$). Thus might be OK to choose 2nd eq'n.

Avoid 2nd derivative in y''**:** $\quad u_1'(t)e^t + u_2'te^t = 0$

Hence $y' = u_1(t)e^t + u_2(e^t + te^t)$.

and $y'' = u_1'(t)e^t + u_1(t)e^t + u_2'(e^t + te^t) + u_2(e^t + e^t + te^t)$.