

2.4 Solve Bernoulli's equation,

$$y' + p(t)y = g(t)y^n,$$

when $n \neq 0, 1$ by changing it

$$y^{-n}y' + p(t)y^{1-n} = g(t)$$

when $n \neq 0, 1$ by changing it to a linear equation by substituting $v = y^{1-n}$

Solve $ty' + 2t^{-2}y = 2t^{-2}y^5$

$$ty^{-5}y' + 2t^{-2}y^{-4} = 2t^{-2}$$

Let $v = y^{-4}$. Thus $v' = -4y^{-5}y'$

$$-4ty^{-5}y' - 8t^{-2}y^{-4} = -8t^{-2}$$

$$tv' - 8t^{-2}v = -8t^{-2}$$

Make coefficient of $v' = 1$

$$v' - 8t^{-3}v = -8t^{-3}$$

An antiderivative of $-8t^{-3}$ is $4t^{-2}$

Multiply equation by $e^{4t^{-2}}$

$$e^{4t^{-2}}v' - 8t^{-3}e^{4t^{-2}}v = -8t^{-3}e^{4t^{-2}}$$

$(e^{4t^{-2}}v)' = -8t^{-3}e^{4t^{-2}}$ by PRODUCT rule.

$$\int(e^{4t^{-2}}v)'dt = -8 \int t^{-3}e^{4t^{-2}}dt$$

$$e^{4t^{-2}}v = -8 \int t^{-3}e^{4t^{-2}}dt.$$

Let $u = 4t^{-2}$. Then $du = -8t^{-3}dt$

$$e^{4t^{-2}}v = \int e^u du = e^u + C$$

$$e^{4t^{-2}}v = e^{4t^{-2}} + C$$

$$v = 1 + Ce^{-4t^{-2}}$$

$$y^{-4} = 1 + Ce^{-4t^{-2}} \text{ implies } y = \pm(1 + Ce^{-4t^{-2}})^{-\frac{1}{4}}$$

$$y' + \frac{2}{t-3}y = 1$$

An anti-derivative of $\frac{2}{t-3} = 2\ln(t-3)$

$$e^{2\ln(t-3)} = e^{\ln[(t-3)^2]} = (t-3)^2$$

$$y' + \frac{2}{t-3}y = 1$$

$$(t-3)^2y' + 2(t-3)y = (t-3)^2$$

$$\int[(t-3)^2y]' = \int(t-3)^2$$

$$(t-3)^2y = \frac{(t-3)^3}{3} + C \text{ implies } y = \frac{(t-3)}{3} + C(t-3)^{-2}$$