2.3: Modeling with differential equations.

Ex.: $F = ma = mv'$

a = acceleration $= v' = x''$
v = velocity $= x'$
x = position
m = mass $\qquad\qquad$ mg = weight

Model 1: Falling ball near earth, neglect air resistance.

F_g = Gravitational force $= -mg$
IF the positive direction points up.
Note in some examples in the book, the positive direction points down ($F_g = +mg$) while in other examples in the book, the positive direction points up ($F_g = -mg$)

$mv' = -mg$ implies $v' = -g$. Thus $v = -gt + C$.

IVP: $v(0) = v_0$ implies $v_0 = -g(0) + C$ implies $C = v_0$. Thus $v = -gt + v_0$

$x' = v = -gt + v_0$ implies $x = -\frac{1}{2}gt^2 + v_0 t + C$.

IVP: $x(0) = x_0$ implies $x_0 = -\frac{1}{2}g(0)^2 + v_0(0) + C$ implies $C = x_0$.

Thus $x = -\frac{1}{2}gt^2 + v_0 t + x_0$.

Note when ball reaches maximum height $v = 0$

Model 2: Falling ball near earth, include air resistance.

Let $A(v)$ = the force due to air resistance.

$mv' = F_g + R(v) = -mg + A(v)$

Model 3: Far from earth.

$F_g = -mg\frac{R^2}{(R+x)^2}$ where R = radius of the earth.

If x is small, $\frac{R^2}{(R+x)^2} \sim 1$ and thus $F_g = -mg$ when close to earth.

For large x, $mv' = -mg\frac{R^2}{(R+x)^2}$ where R constant.

$\frac{dv}{dt} = -mg\frac{R^2}{(R+x)^2}$ with 3 variables: v, t, x

To eliminate one variable: $\frac{dv}{dt} = \frac{dv}{dx}\frac{dx}{dt} = v\frac{dv}{dx}$

Note this trick can also be used to simplify some problems.